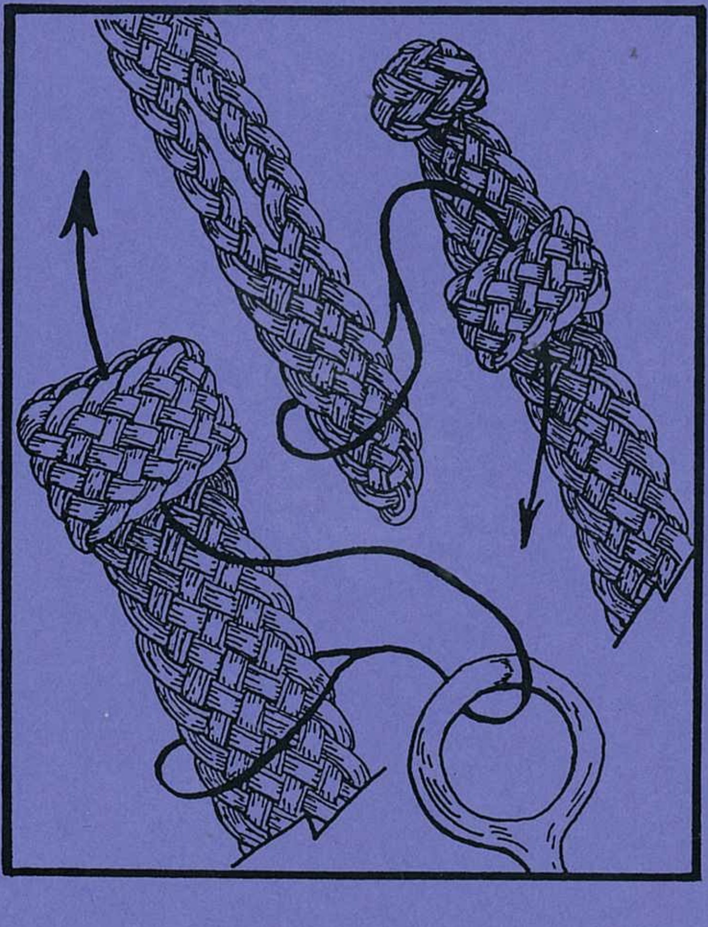
SPECIAL BRAID FORMS Pt.1

END-KEEPERS AND MID-KEEPERS



A.G. Schaake

T. Hall

J.C. Turner

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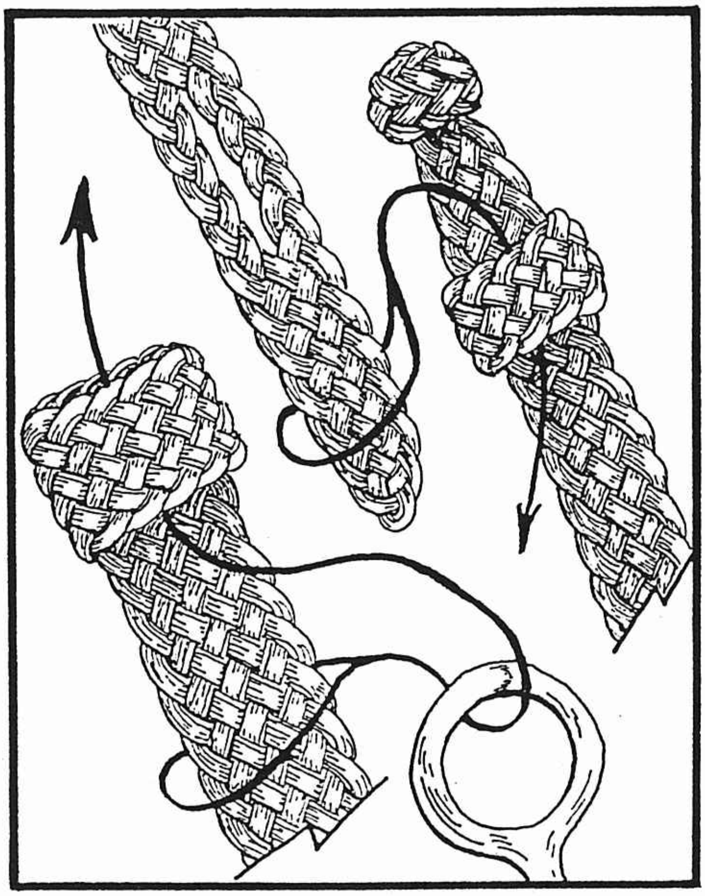

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Special Braid Forms Pt.1

A.G. Schaake
T. Hall
J.C. Turner

Abstract:

The properties and construction procedures associated with integrated Regular End-keepers and Regular Mid-keepers are discussed in some detail. Pictorial drawings of the various construction steps for most of the Examples dealt with are shown, in addition to their grid-diagram representations. General string-run diagrams are provided in which are indicated the valid values of the governing parameters for the most commonly encountered End- and Mid-keepers.

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March, 1995.

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INTRODUCTION

Braiding projects consist, in most cases, of one or more "*braid-forms*". For example, a braid-form can be a simple Regular Flat Braid, it can be a braided knot, etc. Often, some of the various braid-forms in a project have to be 'adjusted' size-wise in order to achieve the proper fits. Even the braiding material used demands, in most cases, some 'adjustments' to the braid-forms in order to obtain an overall satisfactory result. It is therefore important that a braiding artisan is thoroughly familiar with the properties and construction methods of various braid-forms.

In this part of Pamphlet No. 10 we shall discuss in some detail the properties and construction methods of integrated Regular End-keepers and Mid-keepers. The pictorial drawings on the front cover clearly show what these braid-forms look like: the left-hand drawing shows an integrated Regular End-keeper, and the right-hand drawing shows an integrated Regular Mid-keeper. These end-keepers and mid-keepers should not be confused with their "bush" counter parts. The "bush" end-keeper is an end-keeper constructed at the *end* of a braiding stage, whereas a proper end-keeper is constructed at the *beginning* of a braiding stage. Hence with a "bush" end-keeper, one has to work away the ends of the braiding-strings near such an end-keeper, while this is not the case with a proper integrated end-keeper. A "bush" mid-keeper is braided from one or more strings which do *not* take part in the adjoining braid-work. A proper integrated mid-keeper on the other hand is an integral part of the adjoining braid-work. In good braid-work of a high standard, such "bush" end-keepers and mid-keepers should have no place.

In this Pamphlet use is made of various concepts which have been discussed in previous Pamphlets, and hence when the reader is not conversant with them, he or she must refer to these earlier publications. In all calculation procedures associated with braiding, modular arithmetic plays a major role, and consequently it is assumed that the reader is proficient in it. If this is not the case, then the relevant chapter in Pamphlet No. 7 entitled *"Some Braiding Mathematics"* should first be studied thoroughly. No braider will be able to advance to the craftsman's stage without a proper understanding of this important braiding-tool.

As is the case with all our publications, the contents of this one require serious study, and hence this Pamphlet is aimed at those braiders whose desire it is to become braiding artisans. In order to improve the presentation of the material discussed, we would appreciate comments and suggestions; but we ask the reader to bear in mind that concepts fully treated in earlier publications cannot be dealt with again.

REGULAR END-KEEPERS

A **regular end-keeper** is a Regular Cylindrical Braid with p-parts and b-bights, which at one of the bight-boundaries goes over into a Flat Braid. If the Flat Braid has $2x$ parts (hence is a $2x$ lead Flat Braid), then the transition between the Regular Cylindrical Braid and the Flat braid is at x **adjacent** "bight-points" on one of the bight-boundaries of the Regular Cylindrical Braid. Hence such a regular end-keeper is an x-string Regular Cylindrical Braid in which the starting-points and end-points of the x-strings fall in x adjacent "bight-points". It will therefore be obvious that the g.c.d. (greatest common divisor) of the parts and bights of a regular end-keeper for a $2x$-part ($2x$-lead) Flat Braid must be equal to or less than x. This can readily be seen in the general string-run diagrams of regular end-keeper braids for 4-part, 6-part, 8-part, 10-part and 12-part Flat Braids on pgs. 81 — 91.

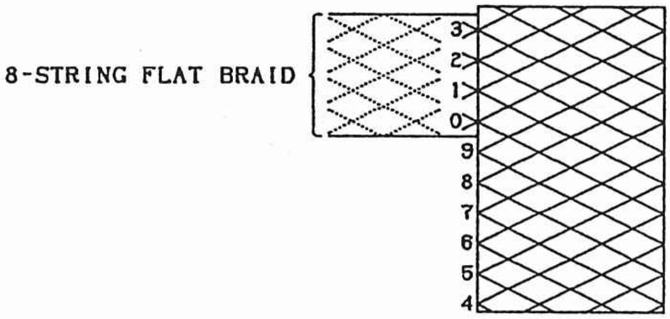
The transition points between the Regular Cylindrical Braid (p-parts/b-bights) and the Flat Braid ($2x$-parts) are given, in the upwards direction, the sequential transition index-numbers $0, 1, 2, \cdots, (x-1)$. The further bight-points on the transition bight-boundary are given the sequential bight index-numbers $x, (x+1), \cdots, (b-1)$ in a continuing upwards direction.

Regular end-keepers can be braided in several different ways. Not only will different braiders employ different methods due to personal preferences, but often a braider will select that method which in his opinion is the most convenient for a particular case. The various braiding methods can be divided into two groups: one which is based on *raising procedures* and one which is based on *half-cycle braiding-algorithms*. The braiding methods based on raising procedures are generally only of practical value when the weaving-pattern is of the Casa type (Turk's Head type). In fact for the Casa type coding, the braiding methods based on raising procedures are of great value since all such coded regular end-keepers can be braided by at least one of these methods. We shall first discuss the braiding methods based on the various raising procedures.

Since a regular end-keeper for a $2x$-part Flat Braid is braided from x strings, its string-run consists of x components. Each component has a number of **parts** and a number of **bights**, which we shall indicate by p_c and b_c respectively. Generally not all components of a regular end-keeper have the same numerical value for their respective parts, nor do they have the same numerical value for their respective bights. Only when the g.c.d. of the total parts and bights of the regular end-keeper is equal to x do we have the case where all its components are identical with respect to their parts/bights specifications. The total number of parts of a regular end-keeper is equal to the sum of the parts of its components, and similarly the total number of bights of a regular end-keeper is equal to the sum of the bights of its components.

Before we are able to decide how to braid a regular end-keeper, we must know the parts/bights specifications of its components. To obtain these specifications we must first calculate the string-run details of each component. The string-run details and the parts/bights specifications of the components are assembled in the **component-table** of the regular end-keeper. A row in the component-table specifies the sequential index-numbers on the string-run of the associated component, whereby each component string-run starts at a transition index-number in an upwards direction from left to right.

Say a component string-run starts at transition index-number i, then the next index-number on its string-run is $|i + p|_b$. If this index-number is not a transition index-number, then the next index-number on its string-run is $|i + 2p|_b$. If this index-number is not a transition index-number, then the next index-number on its string-run is $|i + 3p|_b$. And so on, till we reach a transition index-number (here the string-run of the component exits from the regular end-keeper and goes over into the Flat Braid). See Example below. We shall in the following text shorten regular end-keeper to end-keeper.



6-PART/10-BIGHT END-KEEPER

THE TRANSITION INDEX-NUMBERS ARE 0,1,2,3.
THESE ARE THE INDEX-NUMBERS WHICH ARE ASSOCIATED WITH THE TRANSITION CROSSINGS BETWEEN THE END-KEEPER AND THE FLAT BRAID.

THE NUMBER OF TRANSITION INDEX-NUMBERS IS EQUAL TO HALF THE NUMBER BY WHICH THE FLAT BRAID IS INDICATED.

6/10 END-KEEPER

0 - 6 - 2	1/2
1 - 7 - 3	1/2
2 - 8 - 4 - 0	2/3
3 - 9 - 5 - 1	2/3

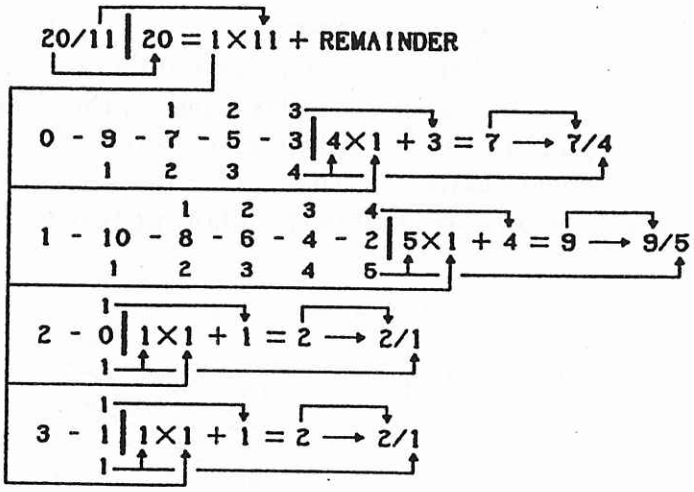
THE COMPONENT-TABLE OF AN END-KEEPER CONSISTS OF ROWS. WHERE A ROW INDICATES THE STRING-RUN OF THE COMPONENT ASSOCIATED WITH THE TRANSITION INDEX-NUMBER AT THE START OF THE ROW. THE STRING-RUN OF THE COMPONENT IS HEREBY THOUGHT TO BEGIN IN AN UPWARDS DIRECTION FROM LEFT TO RIGHT. THE FINAL INDEX-NUMBER OF A ROW IS THE TRANSITION INDEX-NUMBER WHERE THE STRING-RUN OF THE COMPONENT EMERGES FROM THE END-KEEPER.
THE PARTS/BIGHTS SPECIFICATION OF A COMPONENT IS GIVEN AT THE END OF A ROW, AFTER THE VERTICAL LINE IN THE TABLE.

The parts/bights specifications of the components are calculated according to the scheme as set out in the examples on pg. 4. There, the numbers above each row of string-run specification form the counting procedure used in calculating the parts of the associated component. Only the *in sequence decreasing* index-numbers take part in this counting procedure; however, the highest index numbers in these decreasing sequences are not counted. The numbers below each row of string-run specification form the counting procedure used in calculating the parts and bights of the associated component. The first index number is neglected in this counting procedure.

It is here an opportune moment to stress and illustrate the importance of **parts-equivalency** and **bights-equivalency** in braids. We have mentioned in earlier publications that the "New Knot Theory" is much concerned with various equivalencies between braids, and that the term equivalent must always be preceeded by a qualifying term (see for example Pamphlet No. 8). The most commonly encountered equivalencies are the parts-equivalency and the bights-equivalency in braids. Two braids with different number of parts, but with similar string-runs are said to be parts-equivalent; two braids with different number of bights, but with similar string-runs are said to be bights-equivalent.

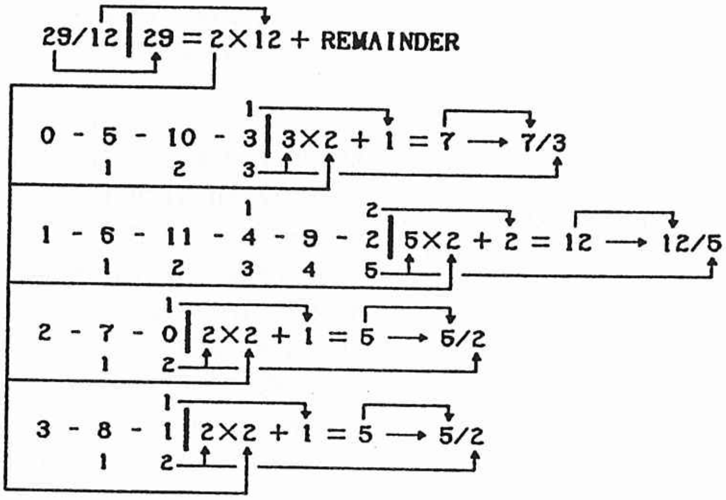
20/11 END-KEEPER

0 - 9 - 7 - 5 - 3	7/4
1 - 10 - 8 - 6 - 4 - 2	9/5
2 - 0	2/1
3 - 1	2/1



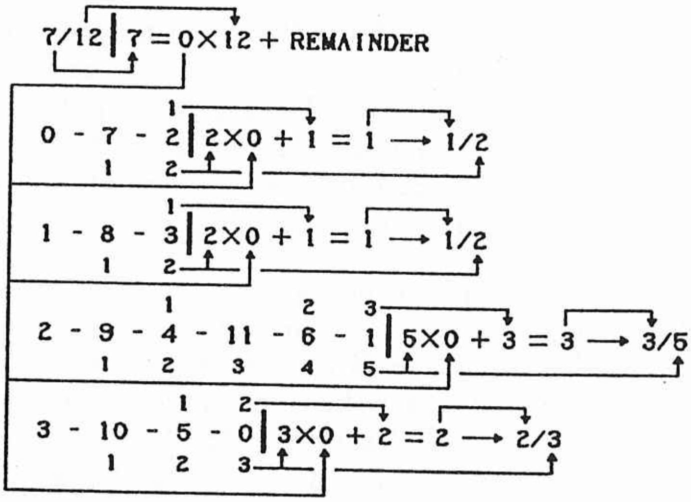
29/12 END-KEEPER

0 - 5 - 10 - 3	7/3
1 - 6 - 11 - 4 - 9 - 2	12/5
2 - 7 - 0	5/2
3 - 8 - 1	5/2



7/12 END-KEEPER

0 - 7 - 2	1/2
1 - 8 - 3	1/2
2 - 9 - 4 - 11 - 6 - 1	3/5
3 - 10 - 5 - 0	2/3



Calculation procedures for the parts/bights specifications of the components.

For Regular Cylindrical Braids, two of such braids are parts-equivalent when they have both the same number of bights, but their parts differ by a multiple of their bights; and two of such braids are bights-equivalent when they have both the same number of parts, but their bights differ by a multiple of their parts. The following examples give an illustration of these equivalencies concerning end-keepers. Compare the string-run specifications between the parts-equivalent end-keepers, and compare the string-run specifications between the bights-equivalent end-keepers.

7/9 END-KEEPER	
0 - 7 - 5 - 3	2/3
1 - 8 - 6 - 4 - 2	3/4
2 - 0	1/1
3 - 1	1/1

16/9 END-KEEPER	
0 - 7 - 5 - 3	5/3
1 - 8 - 6 - 4 - 2	7/4
2 - 0	2/1
3 - 1	2/1

25/9 END-KEEPER	
0 - 7 - 5 - 3	8/3
1 - 8 - 6 - 4 - 2	11/4
2 - 0	3/1
3 - 1	3/1

34/9 END-KEEPER	
0 - 7 - 5 - 3	11/3
1 - 8 - 6 - 4 - 2	15/4
2 - 0	4/1
3 - 1	4/1

PARTS-EQUIVALENCY

7/16 END-KEEPER	
0 - 7 - 14 - 5 - 12 - 3	2/5
1 - 8 - 15 - 6 - 13 - 4 - 11 - 2	3/7
2 - 9 - 0	1/2
3 - 10 - 1	1/2

7/23 END-KEEPER	
0 - 7 - 14 - 21 - 5 - 12 - 19 - 3	2/7
1 - 8 - 15 - 22 - 6 - 13 - 20 - 4 - 11 - 18 - 2	3/10
2 - 9 - 16 - 0	1/3
3 - 10 - 17 - 1	1/3

BIGHTS-EQUIVALENCY

There are **three types** of raising procedures for end-keepers: one in which the raising procedures do not increase the number of transition points, and two in which the raising procedures do increase the number of transition points.

First type — Raising procedures in which the number of transition points is not increased. These are akin to the two Enlargement Methods I and II (which have been discussed in Pamphlet No. 4), and hence will be called **enlargements**.

Second type — A size-increasing stage, although based on the Enlargement Method I or II, is carried out with a new string, resulting in the introduction of an additional transition point. We designate this type by either TIE I (TIE Method I) or TIE II (TIE Method II).

Third type — A size-increasing procedure, which consists of following an already existing component string-run with a new string. We designate this type by TIF p_c/b_c, where p_c/b_c is the parts/bights specification of the component string-run being followed.

These three types of raising procedures are illustrated on pgs. 7; 10; 13 for a $p/b = 3/5$ end-keeper. The component-tables belonging to the associated end-keepers are presented on pg. 6. Since raising procedures are mainly used for Casa coded end-keepers, which require sets of two raising processes of identical magnitude, we have shown the application of the three types of raising procedures to Casa coded end-keepers on pgs. 8, 9; 11, 12; 14, 15, 16.

3/5 END-KEEPER	
0 - 3 - 1	1/2
1 - 4 - 2 - 0	2/3

METHOD I →

4/7 END-KEEPER	
0 - 4 - 1	1/2
1 - 5 - 2 - 6 -3 - 0	3/5

METHOD I →

5/9 END-KEEPER	
0 - 5 - 1	1/2
1 - 6 - 2 - 7 -3 - 8 - 4 - 0	4/7

3/5 END-KEEPER	
0 - 3 - 1	1/2
1 - 4 - 2 - 0	2/3

METHOD II →

5/8 END-KEEPER	
0 - 5 - 2 - 7 - 4 - 1	3/5
1 - 6 - 3 - 0	2/3

METHOD II →

7/11 END-KEEPER	
0 - 7 - 3 - 10 - 6 - 2 - 9 - 5 - 1	5/8
1 - 8 - 4 - 0	2/3

3/5 END-KEEPER	
0 - 3 - 1	1/2
1 - 4 - 2 - 0	2/3

METHOD II →

4/6 END-KEEPER	
0 - 4 - 2 - 0	2/3
1 - 5 - 3 - 1	2/3

METHOD II →

5/7 END-KEEPER	
0 - 5 - 3 - 1	2/3
1 - 6 - 4 - 2 - 0	3/4

ENLARGEMENTS —— SEE PAGE 7 FOR ASSOCIATED STRING-RUN DIAGRAMS

3/5 END-KEEPER	
0 - 3 - 1	1/2
1 - 4 - 2 - 0	2/3

TIE I →

4/7 END-KEEPER	
0 - 4 - 1	1/2
1 - 5 - 2	1/2
2 - 6 - 3 - 0	2/3

TIE I →

5/9 END-KEEPER	
0 - 5 - 1	1/2
1 - 6 - 2	1/2
2 - 7 - 3	1/2
3 - 8 - 4 - 0	2/3

3/5 END-KEEPER	
0 - 3 - 1	1/2
1 - 4 - 2 - 0	2/3

TIE II →

5/8 END-KEEPER	
0 - 5 - 2	1/2
1 - 6 - 3 - 0	2/3
2 - 7 - 4 - 1	2/3

TIE II →

7/11 END-KEEPER	
0 - 7 - 3	1/2
1 - 8 - 4 - 0	2/3
2 - 9 - 5 - 1	2/3
3 - 10 - 6 - 2	2/3

3/5 END-KEEPER	
0 - 3 - 1	1/2
1 - 4 - 2 - 0	2/3

TIE II →

4/6 END-KEEPER	
0 - 4 - 2	1/2
1 - 5 - 3 - 1	2/3
2 - 0	1/1

TIE II →

5/7 END-KEEPER	
0 - 5 - 3	1/2
1 - 6 - 4 - 2	2/3
2 - 0	1/1
3 - 1	1/1

TIE PROCEDURES —— SEE PAGE 10 FOR ASSOCIATED STRING-RUN DIAGRAMS

3/5 END-KEEPER	
0 - 3 - 1	1/2
1 - 4 - 2 - 0	2/3

TIF 1/2 →

4/7 END-KEEPER	
0 - 4 - 1	1/2
1 - 5 - 2	1/2
2 - 6 - 3 - 0	2/3

TIF 1/2 →

5/9 END-KEEPER	
0 - 5 - 1	1/2
1 - 6 - 2	1/2
2 - 7 - 3	1/2
3 - 8 - 4 - 0	2/3

3/5 END-KEEPER	
0 - 3 - 1	1/2
1 - 4 - 2 - 0	2/3

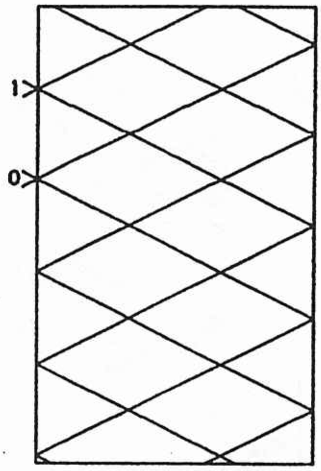
TIF 2/3 →

5/8 END-KEEPER	
0 - 5 - 2	1/2
1 - 6 - 3 - 0	2/3
2 - 7 - 4 - 1	2/3

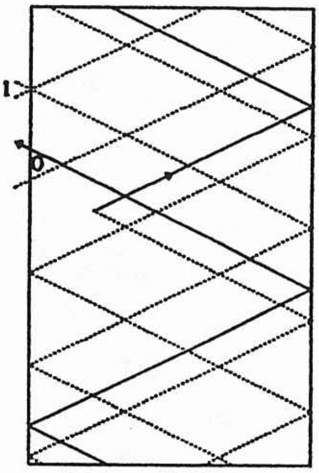
TIF 2/3 →

7/11 END-KEEPER	
0 - 7 - 3	1/2
1 - 8 - 4 - 0	2/3
2 - 9 - 5 - 1	2/3
3 - 10 - 6 - 2	2/3

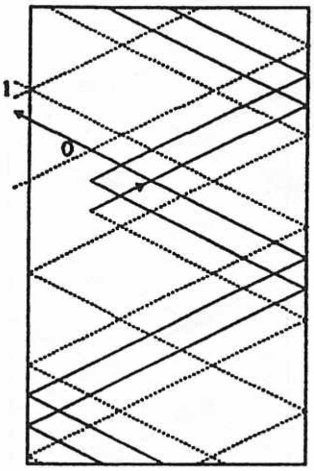
TIF PROCEDURES —— SEE PAGE 13 FOR ASSOCIATED STRING-RUN DIAGRAMS



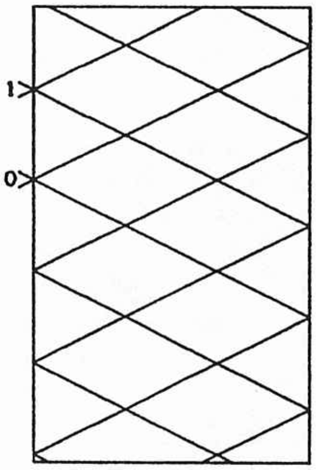
3/5



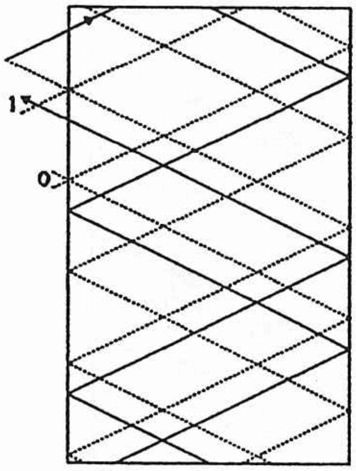
4/7 — METHOD I



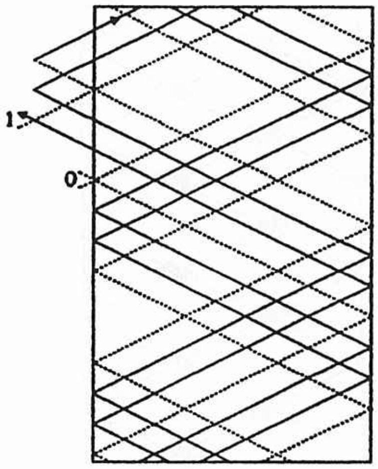
5/9 — 2X METHOD I



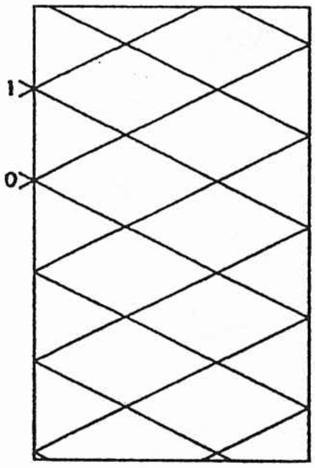
3/5



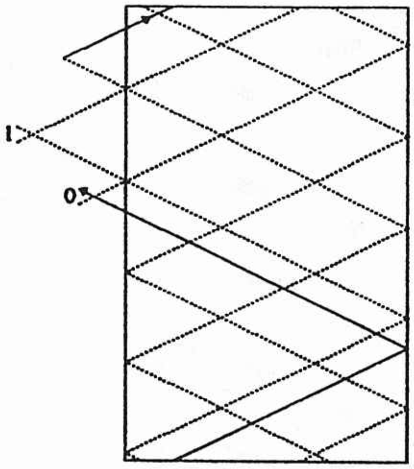
5/8 — METHOD II



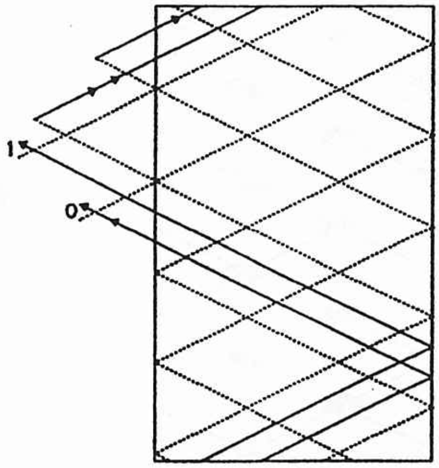
7/11 — 2X METHOD II



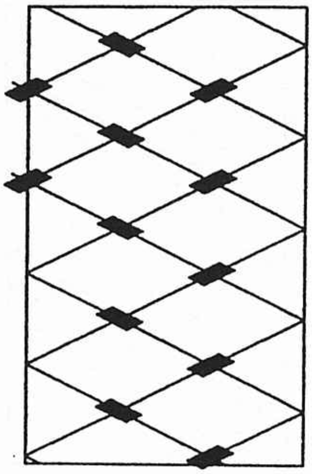
3/5



4/6 — METHOD II



5/7 — 2X METHOD II



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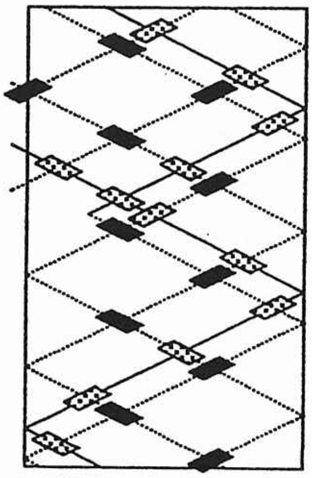
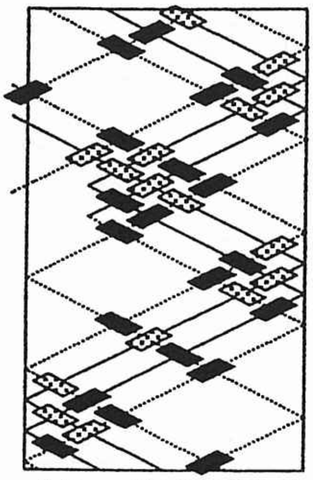
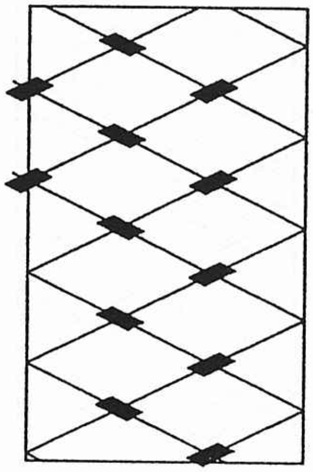


4/7 — METHOD I



5/9 — 2X METHOD I



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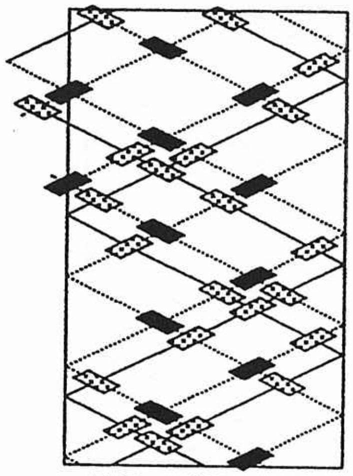
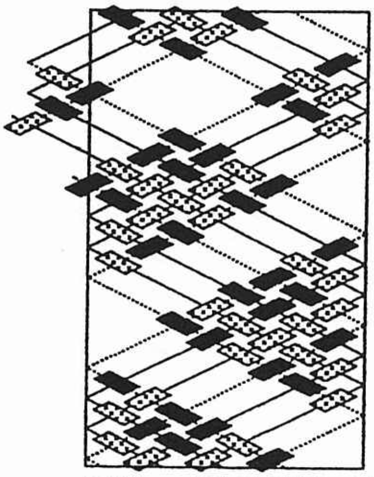
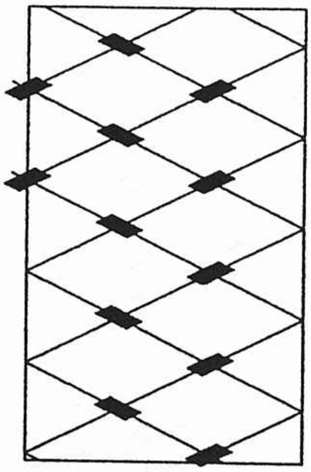


5/8 — METHOD II



7/11 — 2X METHOD II



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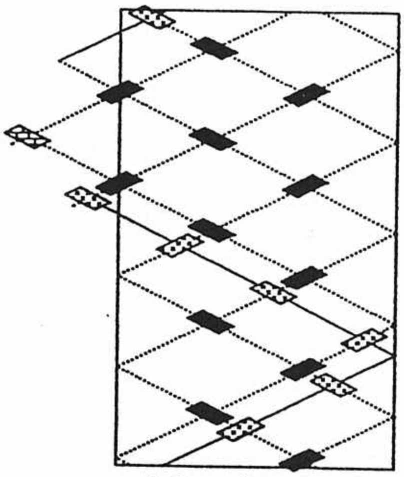
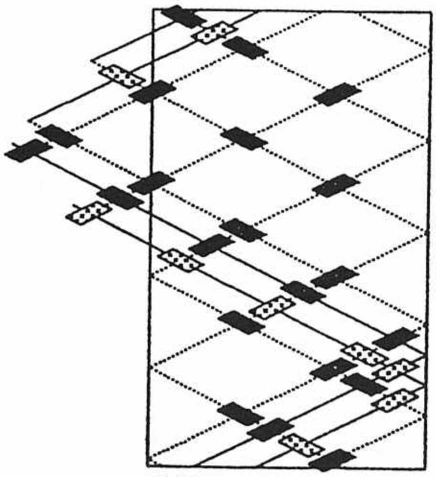
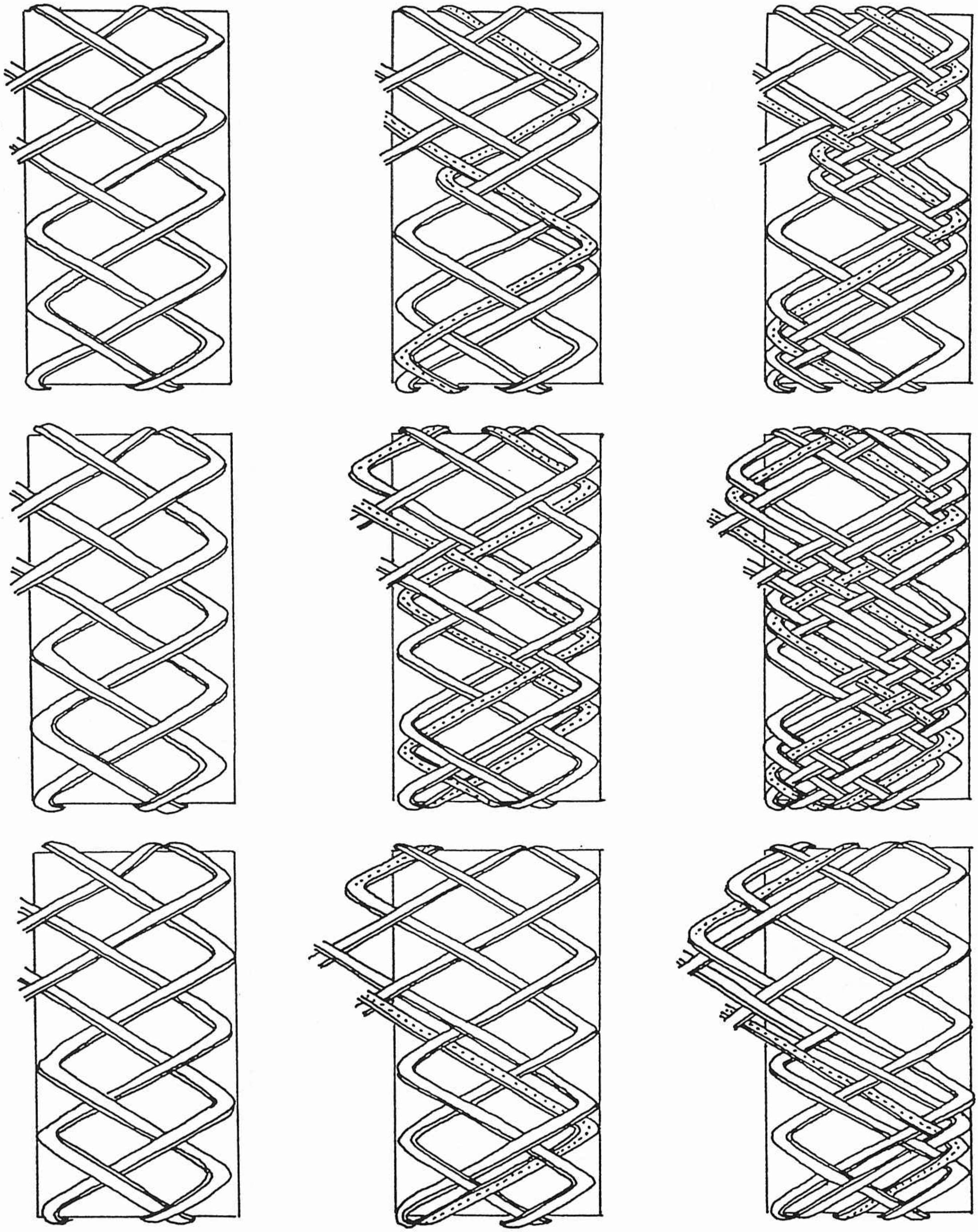


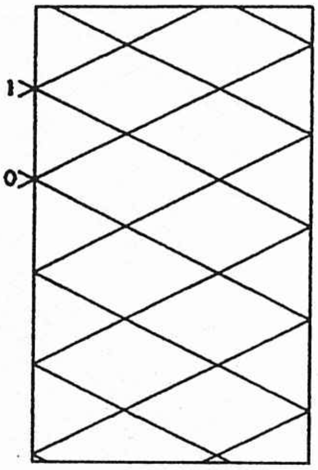
4/6 — METHOD II



5/7 — 2X METHOD II



Pictorial drawings associated with pg. 8.



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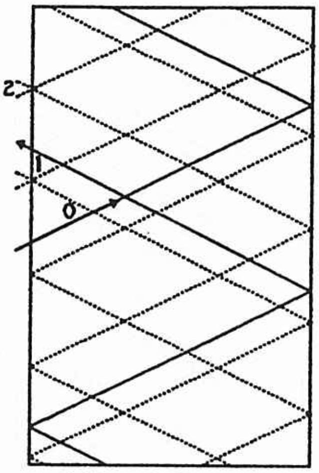
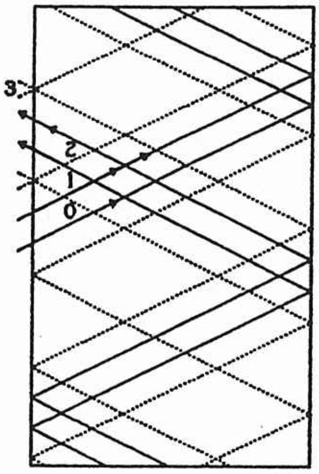


4/7 — TIE I



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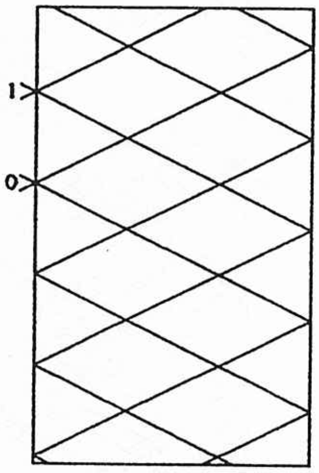

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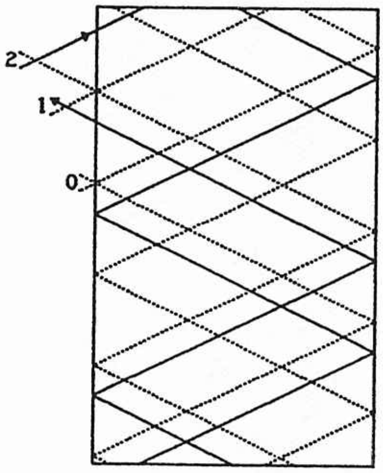

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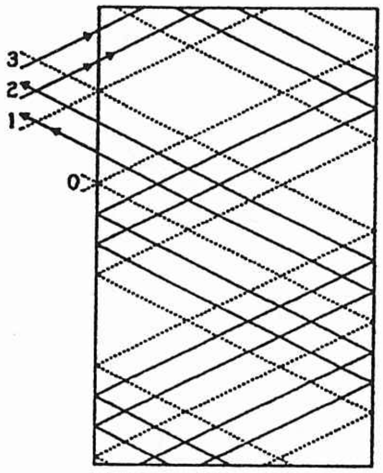

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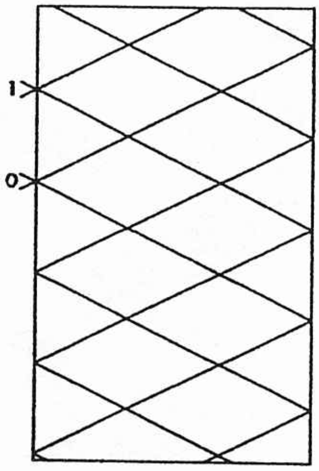

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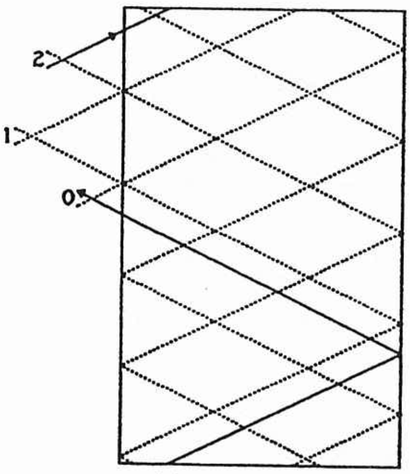

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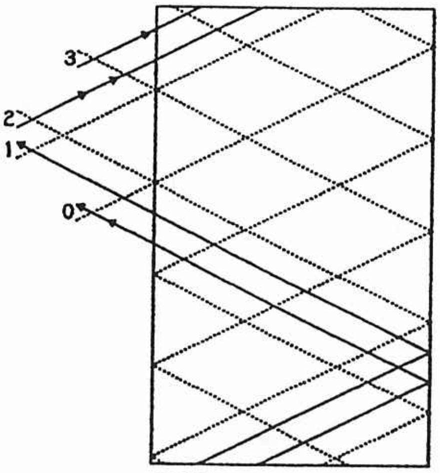

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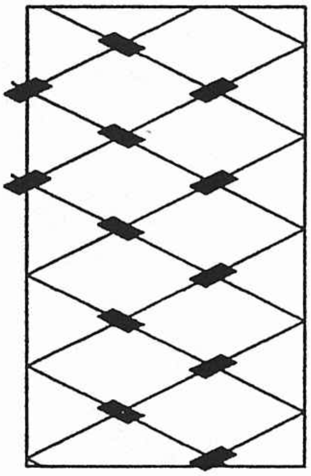

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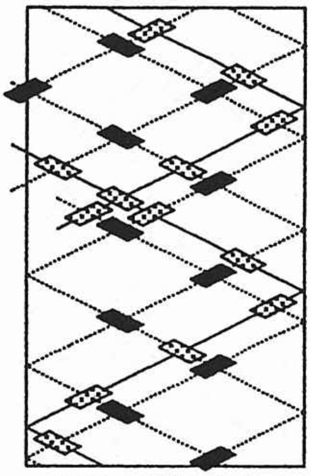

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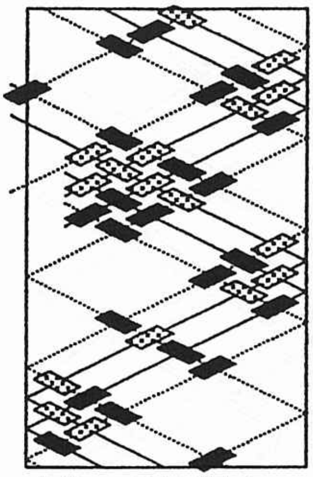

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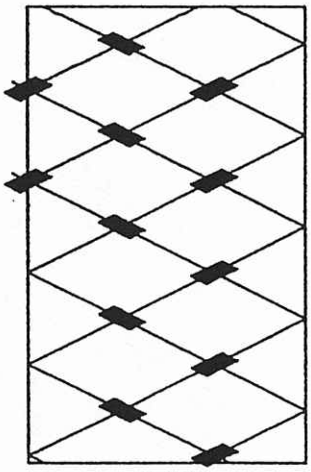

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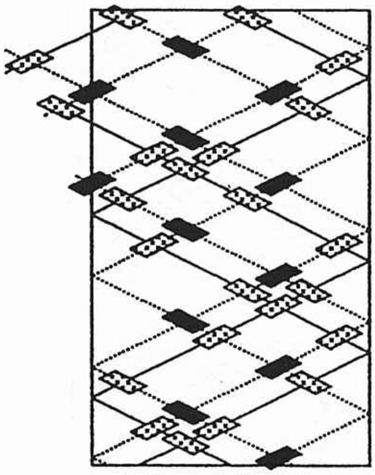

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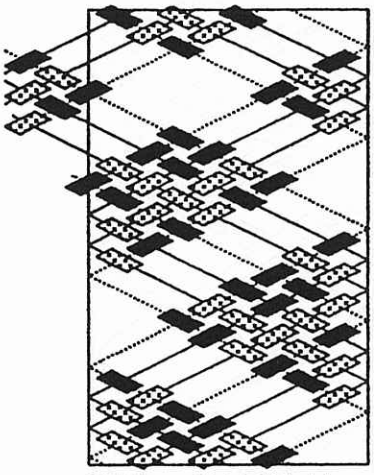

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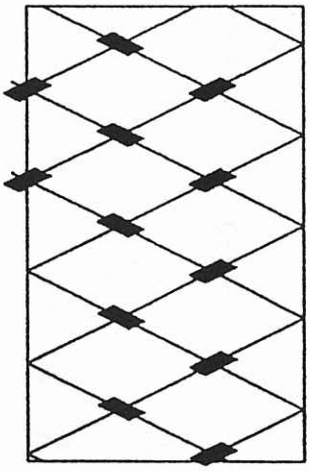

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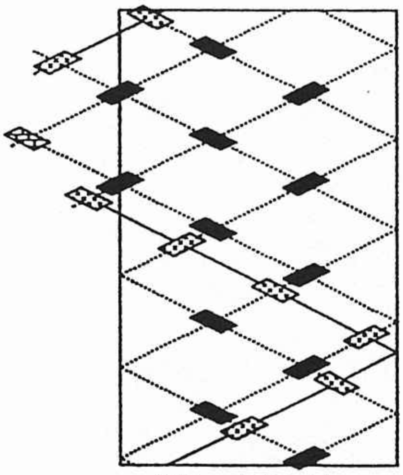

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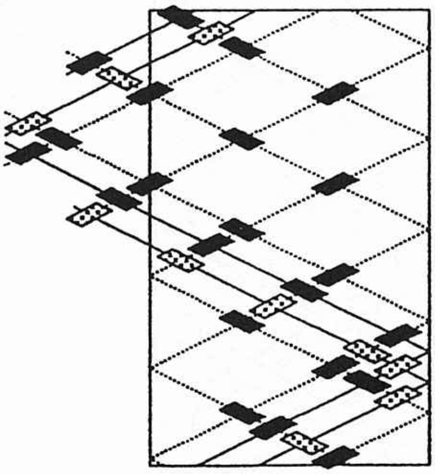

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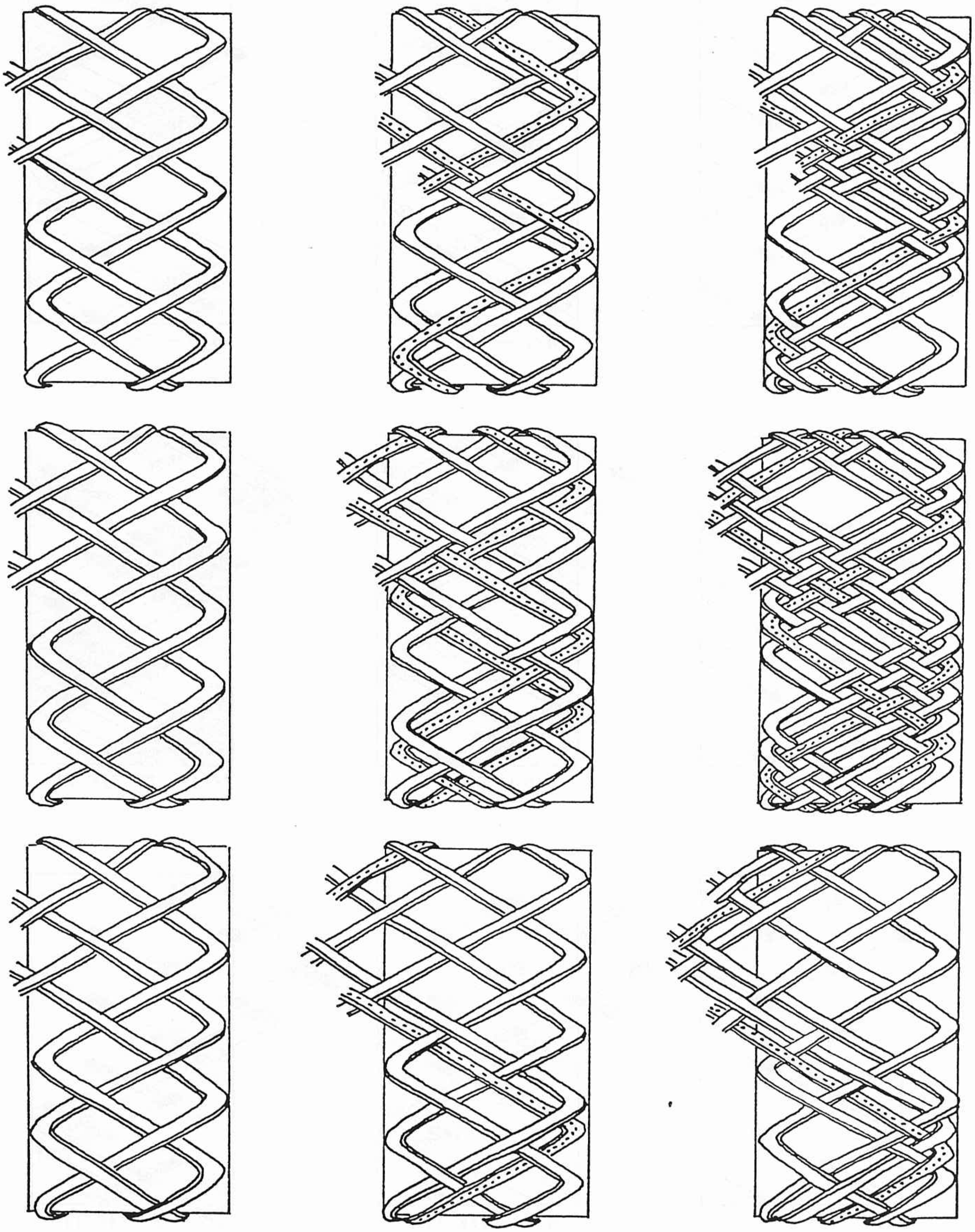
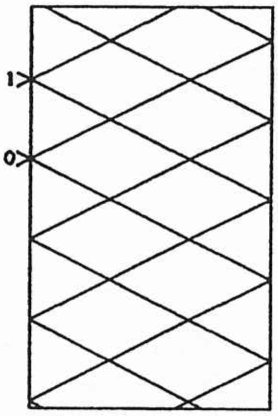


Pictorial drawings associated with pg. 11.



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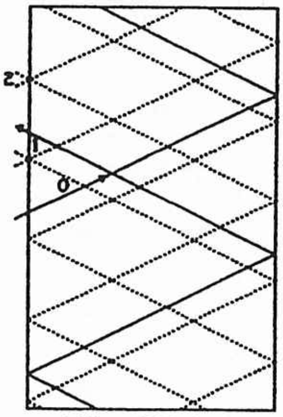
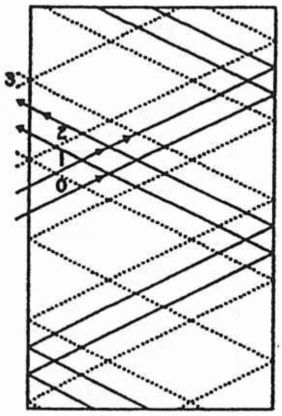


4/7 — TIF 1/2



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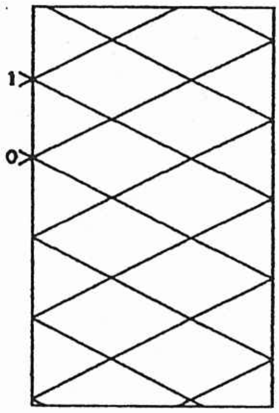

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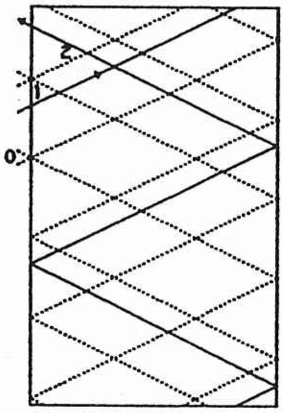

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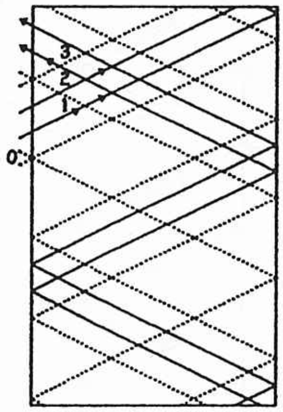

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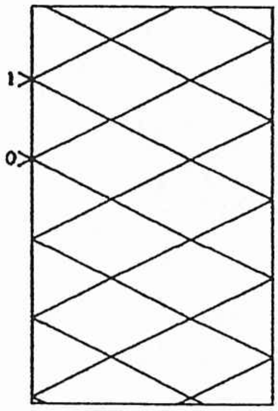

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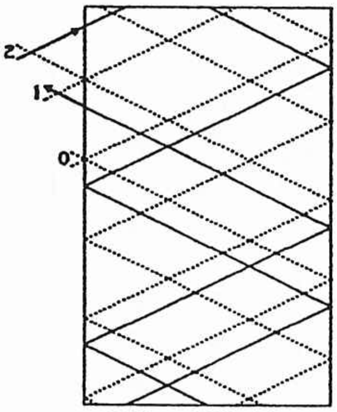

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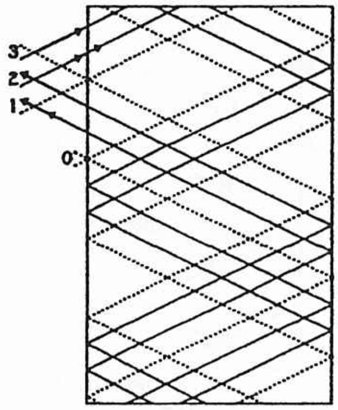

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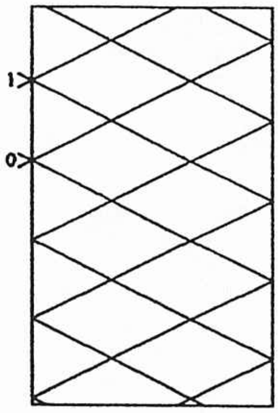

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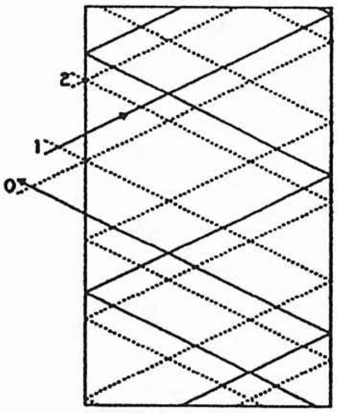

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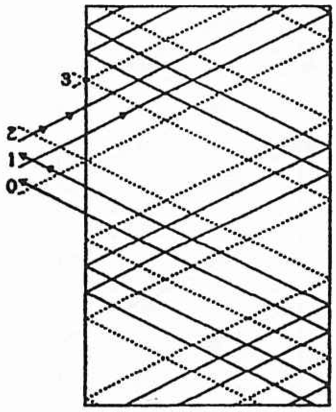

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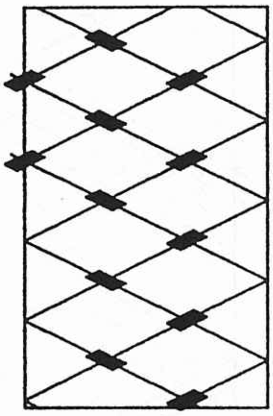
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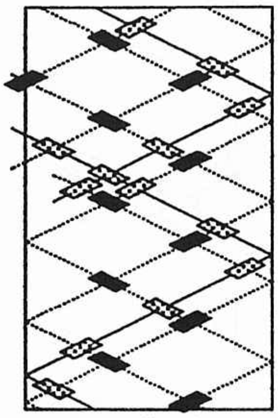
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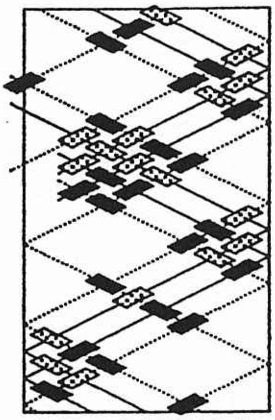
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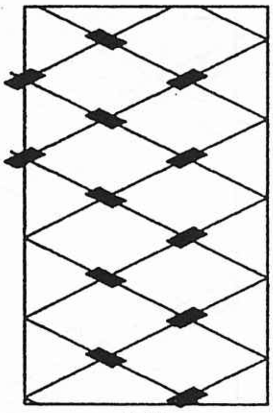
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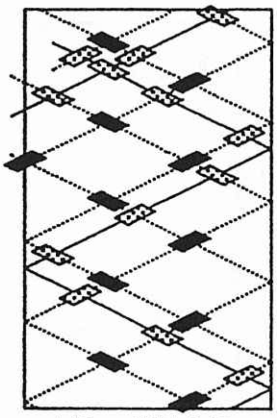
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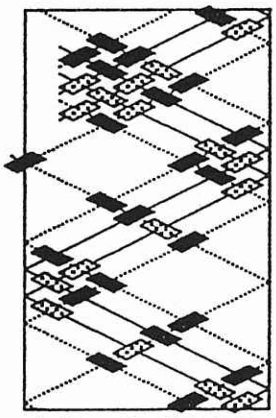
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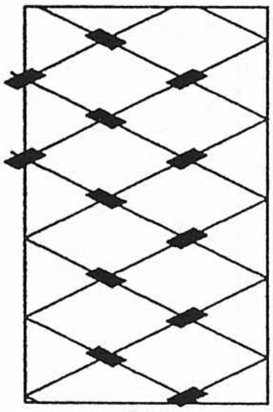
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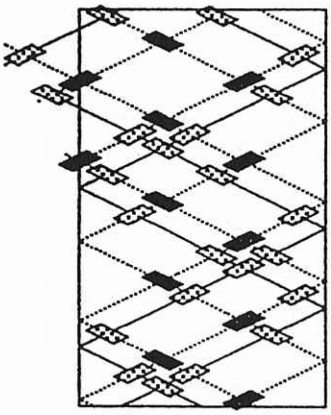
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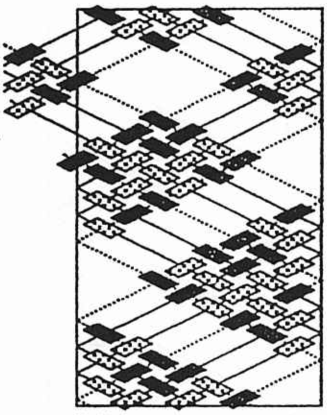
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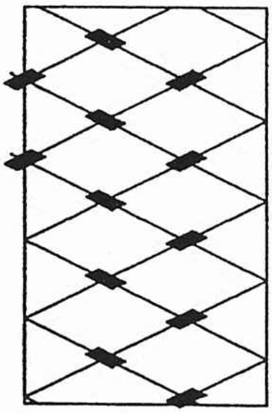
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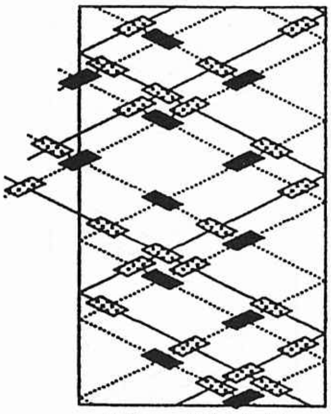
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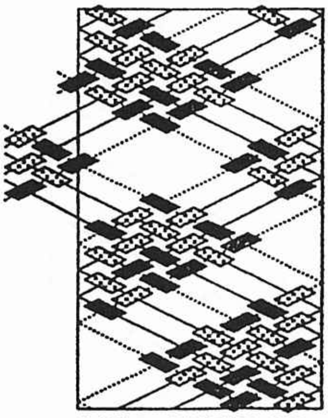
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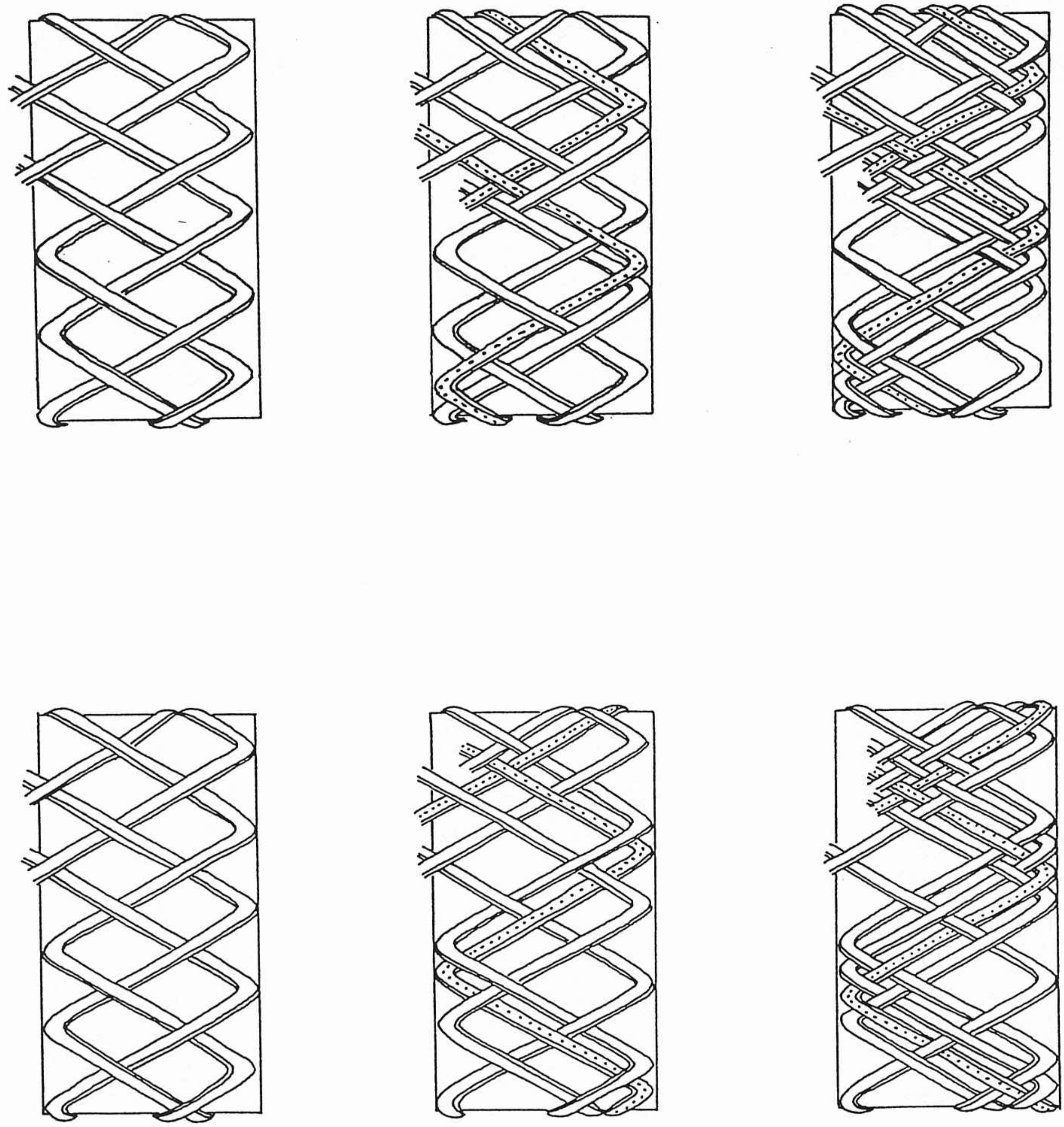
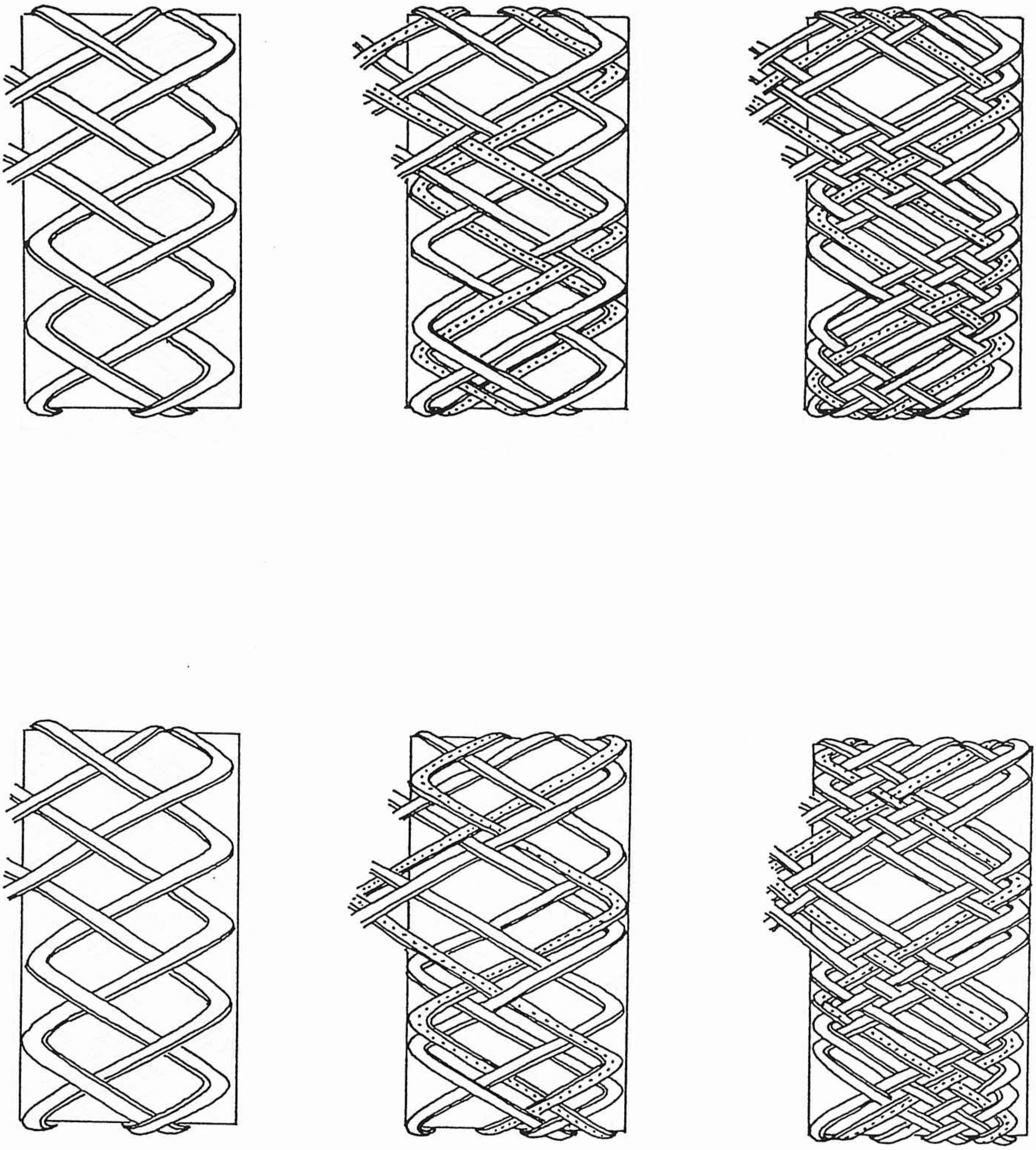


Pictorial drawings associated with pg. 14.



Pictorial drawings associated with pg. 14.

It will be self-evident that the first and third set of diagrams on pgs. 13 and 14 are identical to the first respectively the second set of diagrams on pgs. 10 and 11.

We defined above an *enlargement* to be a raising process whereby the number of strings, and hence the number of transition points between the Regular Cylindrical Braid and the Flat Braid, does not increase.

A Method I enlargement applied to an end-keeper is always associated with that row in its component-table which ends with the transition index-number 0.

A Method II enlargement applied to an end-keeper is always associated with that row in its component-table which ends with the highest transition index-number.

These two rules are self-evident from the string-run diagrams; furthermore the following important properties are easy to prove (see pgs. 92 — 95).

There are three types of component-tables: one in which all the parts/bights specifiations of the components are the same, one in which there are two different parts/bights specifications, and one in which there are three different parts/bights specifications.

End-keepers with component-tables in which all parts/bights specification are the same or in which there are three different parts/bights specifications have two different enlargements: one Method I and one Method II enlargement.

End-keepers with component-tables in which there are two different parts/bights specifications have three different enlargements: two different Method I and one Method II enlargements when the first set of parts/bights specifications in the component table are at a higher level in the RKT (Regular Knot Tree), or one Method I and two different Method II enlargements when the second set of parts/bights specifications in the component table are at a higher level in the RKT.

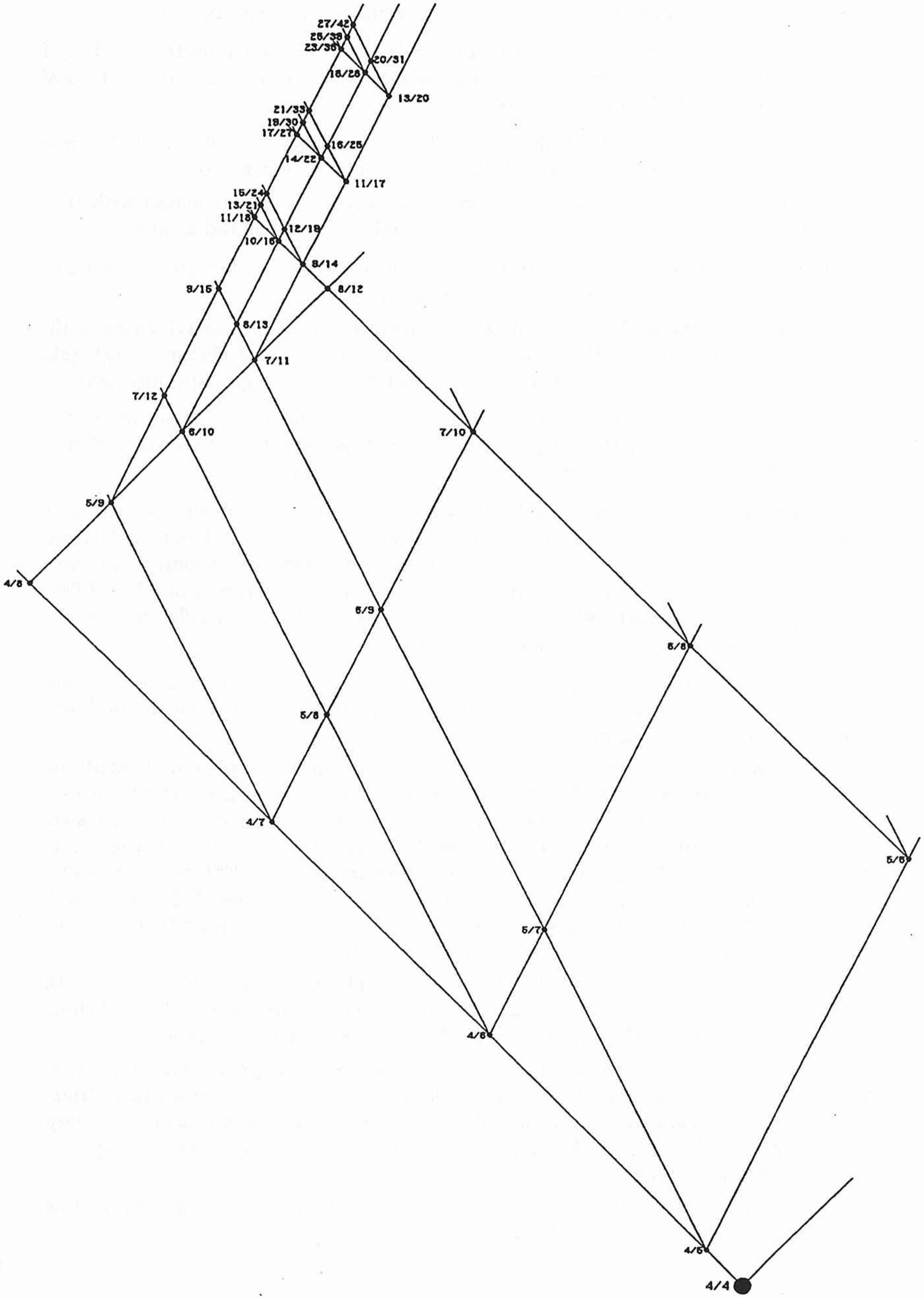
It will now be evident that by means of an enlargement (hence no increase in transition points) an end-keeper can either be produced by two different enlargement methods, or by only one enlargement method.

The enlargement paths are graphically presented in the **Enlargement Graph** associated with a given number of transition-points (x). An Enlargement Graph consists of a left-hand part ($p < b$) and a right-hand part ($p > b$). The left-hand part consists of the **major left-hand Enlargement Graph** (where $p \geq x$), and the **minor left-hand Enlargement Graph** (where $p < x$). The *trunk* of the Enlargement Graph is the node $p/b = x/x$, while its roots are 0/1 and 1/0. Sections of the *major left-hand Enlargement Graph* belonging to end-keepers with four transition-points (hence $p/b = x/x = 4/4$ is its trunk) are shown on pgs. 18 — 20.

A section of the *minor left-hand Enlargement Graph* belonging to end-keepers with four transition-points is shown on pg. 21. Although the nodes with $p = 4$ do not belong to the *minor left-hand Enlargement Graph*, they are shown there for clarity.

It should be noted that the RKT is the limit case of an Enlargement Graph because then we have only one transition-point. Note also that then the *minor left-hand Enlargement Graph* has consequently vanished. Furthermore the RKT is represented in every Enlargement Graph by the nodes xp/xb, where x is the number of transition-points and p and b are coprime.

The right-hand part of the Enlargement Graph is constructed in the same way as the major left-hand Enlargement Graph.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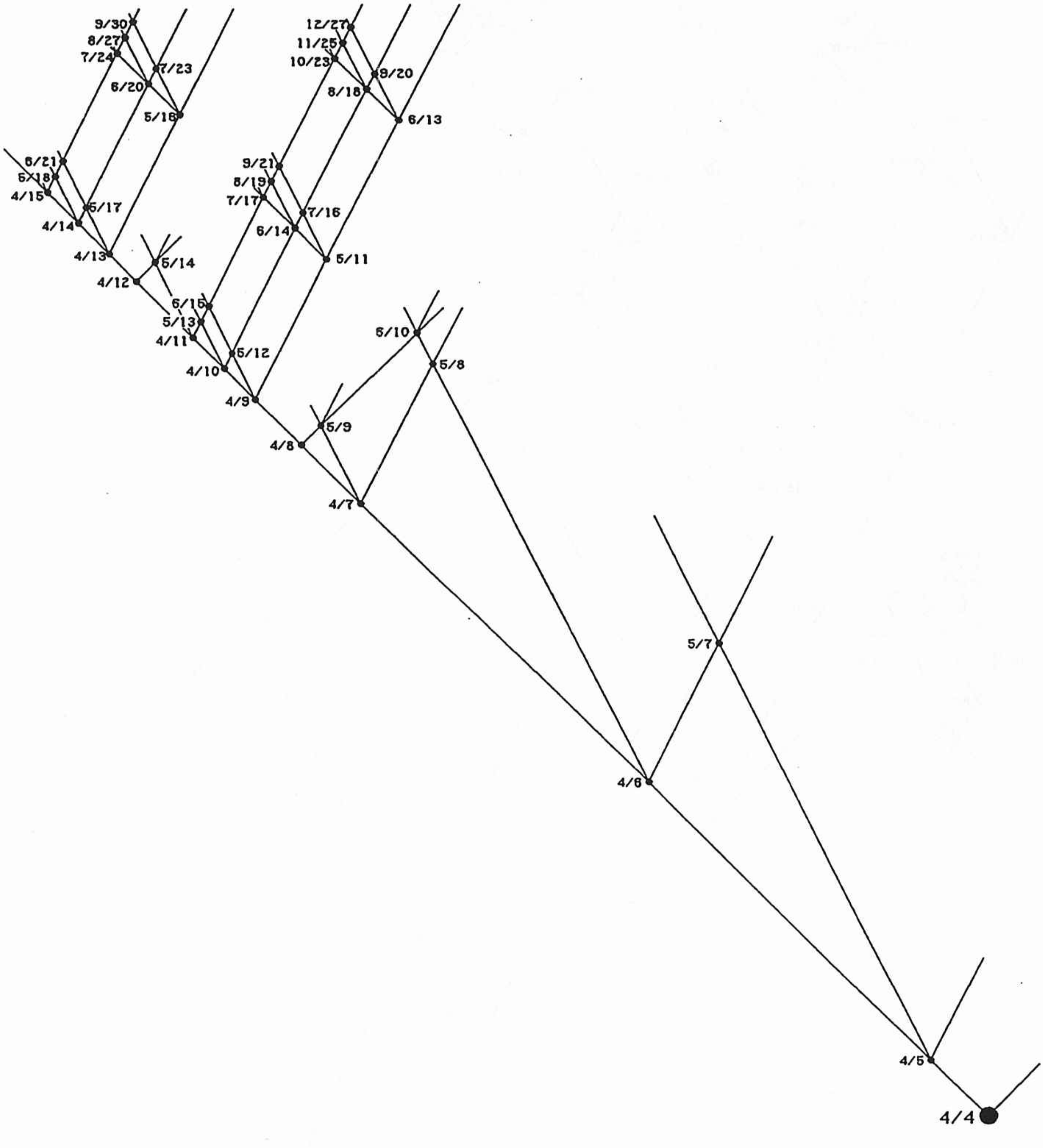
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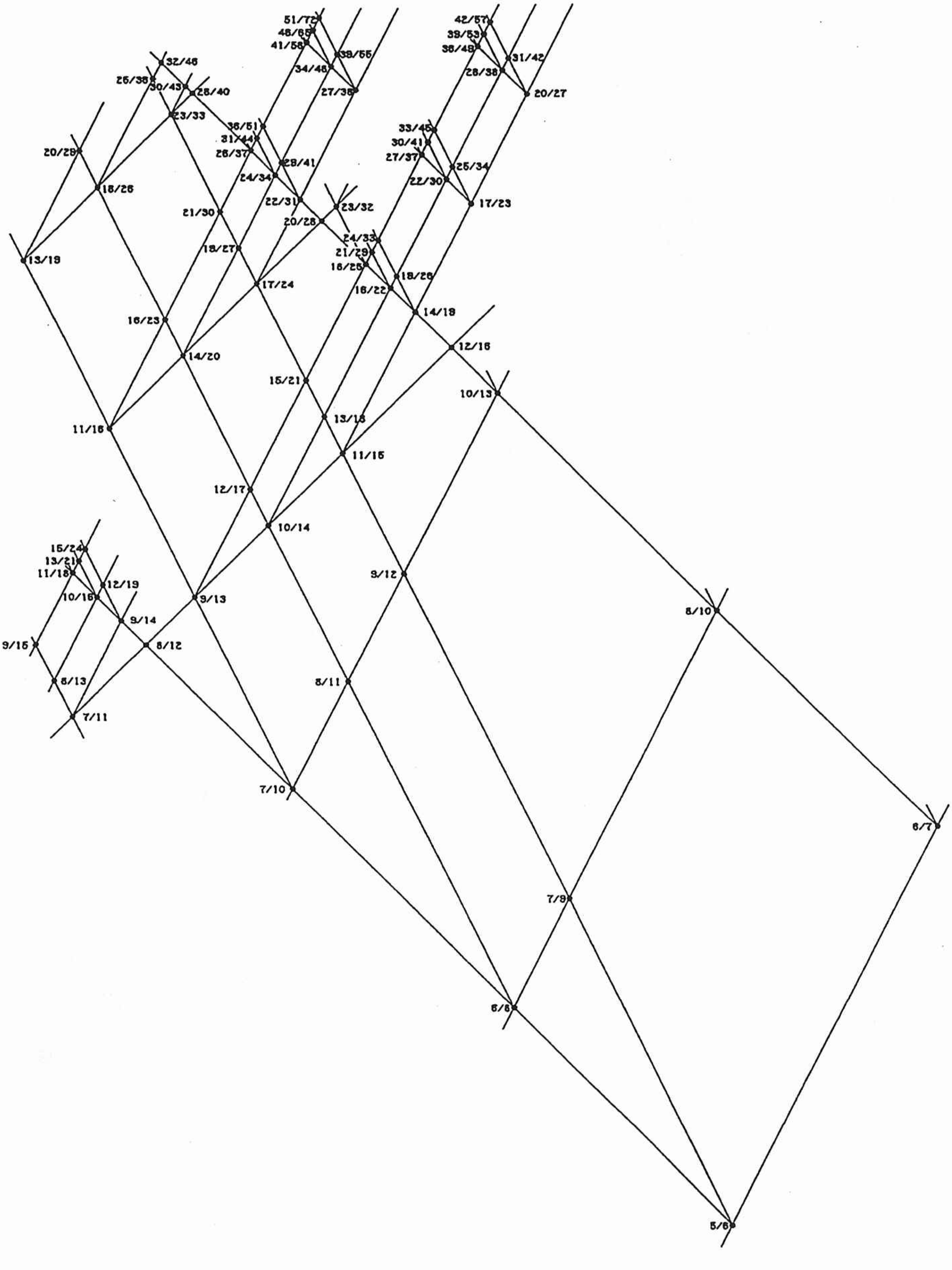
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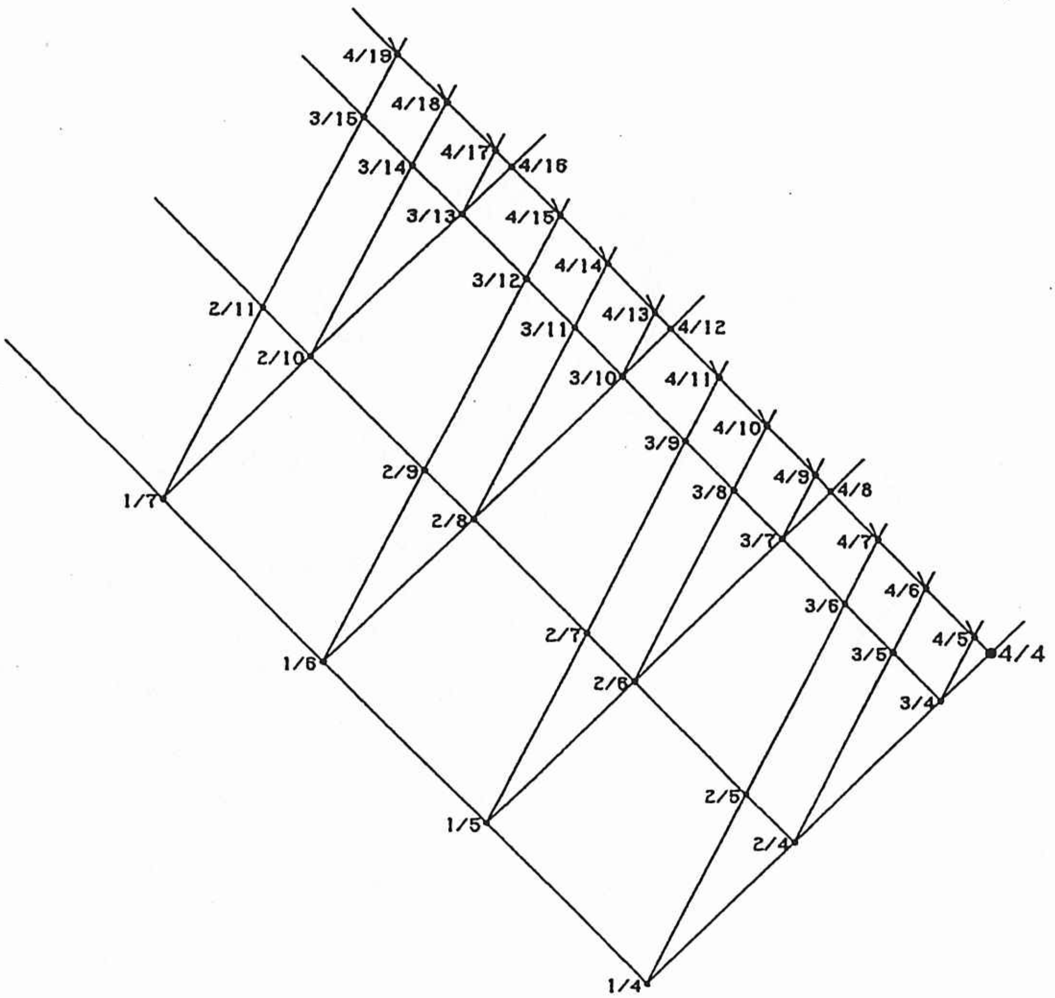


Part of the minor left-hand Enlargement Graph for $x = 4$.

On pg. 22 is shown the general layout of the *left-hand Enlargement Graph* with the magnitudes of the increases $\Delta p/\Delta b$ between consecutive nodes.

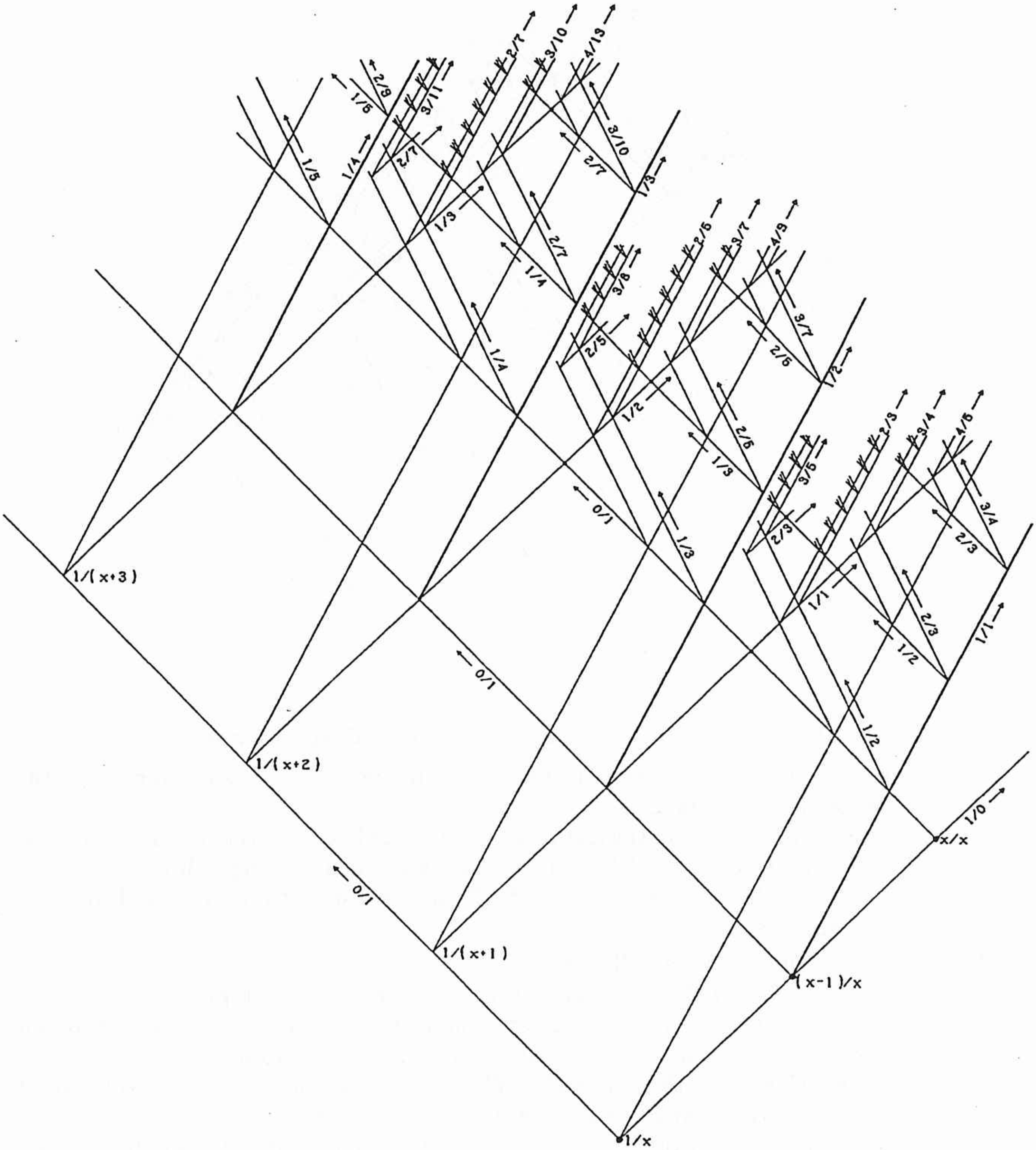
There are four line types in the right-hand and major left-hand Enlargement Graphs:

(1). Lines which slant to the left at 45°. These are called **principal-lines.**
(2). Lines which slant to the right at 45°. These are also called **principal-lines.**
(3). Lines which slant to the left at 60°.
(4). Lines which slant to the right at 60°.

In the minor left-hand Enlargement Graph we have three line types:

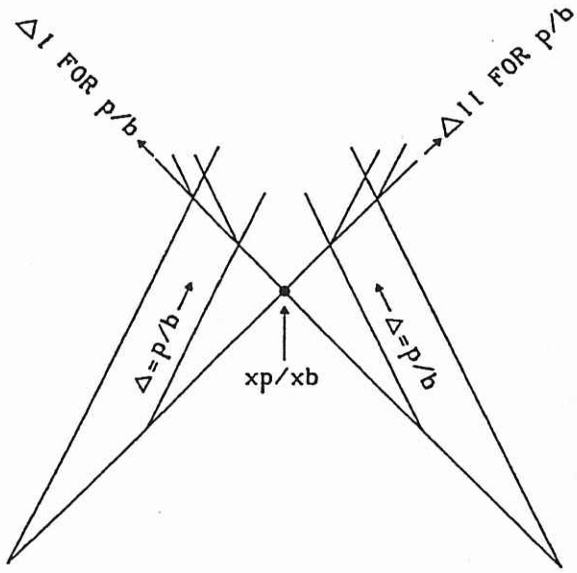
(1). Lines which slant to the right at 45°. These are continuations of the lines which slant at 45° to the right in the major left-hand Enlargement Graph.
(2). Lines which slant to the left at 45°. These are equivalent to the lines which slant at 60° to the left in the major left-hand Enlargement Graph.
(3). Lines which slant to the right at 60°. These are continuations of the lines which slant at 60° to the right in the major left-hand Enlargement Graph.

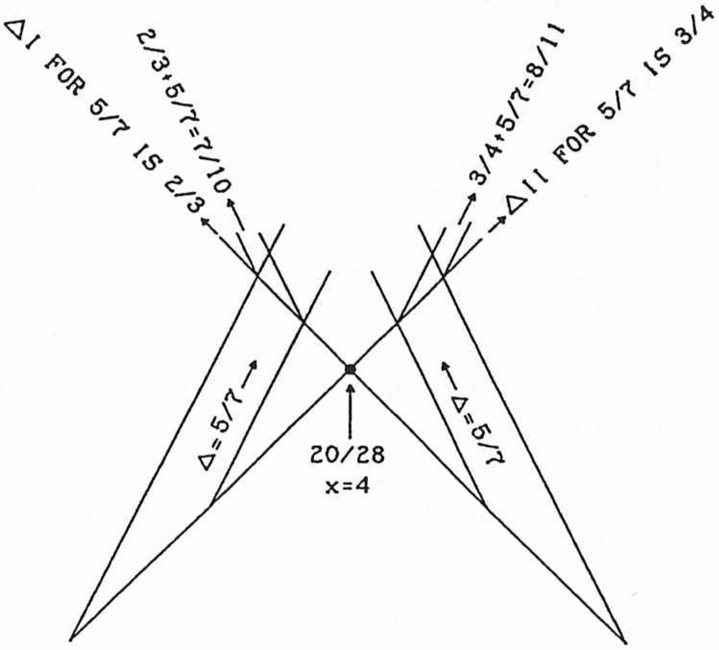
The 45°-sloping lines in the right-hand and major left-hand Enlargement Graphs represent, between their intersection-points together with these points, the RKT with its corresponding node-values multiplied by x. For example: when $x = 4$, the node $5/7$ in the RKT corresponds with the node $20/28$ in the Enlargement Graph ($x = 4$). Such nodes are called the **principal-nodes** of the Enlargement Graph. With the aid of the principal-nodes we can draw the complete Enlargement graph (see also pgs. 96 — 107).



General layout of the left-hand Enlargement Graph.

The 60° sloping lines in the Enlargement Graph form bands, with $(x-1)$ lines per band. The Δ-values of these bands are determined by their principal-node (see diagrams on pg. 23).





The diagram on the left gives the general relationships; the one on the right gives a numerical example. In these diagrams Δ stands for $\Delta p/\Delta b$; ΔI stands for $\Delta p/\Delta b$ of p/b when p/b is enlarged by a 1^{st}-order Method I process; ΔII stands for $\Delta p/\Delta b$ of p/b when p/b is enlarged by a 1^{st}-order Method II process. With these general rules, we are able to trace the enlargement paths of a given p/b-endkeeper with x transition-points from its node p/b **back** to the trunk-node x/x. The position of the node p/b in the Enlargement Graph can readily be determined with the aid of its component specifications. The following Example illustrates the process (see also pgs. 96—107).

Example: An end-keeper with $p/b = 61/86$ and $x = 4$.

First we calculate its component-table:

61/86-ENDKEEPER — x=4

0-61-36-11-72-47-22-83-58-33-8-69-44-19-80-55-30-5-66-41-16-77-52-27-2	17/24
1-62-37-12-73-48-23-84-59-34-9-70-45-20-81-56-31-6-67-42-17-78-53-28-3	17/24
2-63-38-13-74-49-24-85-60-35-10-71-46-21-82-57-32-7-68-43-18-79-54-29-4-65-40-15-76-51-26-1	22/31
3-64-39-14-75-50-25-0	5/7

The largest parts/bights specification is 22/31, and hence the principal-node associated with this parts/bights specification is $22x/31x = 88/124$. Calculate the path-formula (=evolution-vector, see Pamphlet No.4) of the node 22/31 in the RKT. This gives the path-formula $[0; 1, 2, 2, 4] = [0; 1, 2, 2, 3, 1]$. With this path-formula we can draw the evolution-path in the RKT from the trunk-node 1/1 to the node 22/31.

An evolution-path in the RKT is represented by a corresponding **principal-path** in an Enlargement Graph, where the value of the **principal-nodes** on this path are $x\times$ the values of the corresponding nodes on the evolution-path in the RKT.

The actual enlargement paths from the trunk-node x/x to the end-keeper node p/b follow the adjacent bands **below** the principal path or parts of the principal path.

The component-table of our Example has three different parts/bights specifications:

22/31 — 1 component,
17/24 — 2 components, and
5/7 — 1 component.

In the principal node 88/124 we got four components, each with a parts/bights specification of 22/31. From the path in the RKT we see that for the node 22/31, a basic (1^{st}-order) Method I enlargement increases the node-value by 17/24, and that a basic (1^{st}-order) Method II enlargement increases the node-value by 5/7. These respective enlargement increments also apply to the principal-node 88/124 (see pg. 25). In the same way we determine the basic (1^{st}-order) enlargement increments for the next lower component specification and hence its associated principal-node (thus in our case for the RKT-node 17/24 and its associated principal-node 68/96). Since the path in the RKT from the node 5/7 via the node 17/24 to the node 22/31 is a **right** hocky-stick, we also determine the increment-value along the paths in the **right** band adjacent to the principal-node 68/96, which increment-value is equal to the value of its corresponding node in the RKT, hence equal to 17/24.

The position of the end-keeper node 61/86 relative to the principal-node 88/124 can now readily be determined as follows:

Two components of 22/31 each, decreased by 5/7 each, gives two components of 17/22 each. This is represented in the Enlargement Graph by two steps (nodes) down from the node 88/124 along the 5/7 principal increment line (see pg. 25).

One component of 22/31 decreased by 17/24 gives one component of 5/7. This is represented in the Enlargement Graph by a further one step (node) down along the appropriate line (in the right band adjacent to the principal-node 68/96).

This brings us to the node 61/86 in the Enlargement Graph (see pg. 25).

We can also arrive in the node 61/86 via an alternative route from the node 88/124 by first going one step down along the principal increment line 17/24, followed by two steps down the appropriate band-line (5/7). A further route is provided by first going one step down from the node 88/124 along the principal increment line 5/7, followed by one step down along the appropriate band line 17/24, followed by one step down along the appropriate band line 5/7 (see pg. 25).

The Enlargement Graph from which the various enlargement-paths to the node 61/86 can be traced has for clarity been split into two parts (see pgs. 26 & 27).

With the theory presented so far, we shall discuss in some detail two Examples.

Example 1 : End-keeper with $p/b = 6/10$; 8-part flat braid, hence 4 transition-points.

The component-table of such an end-keeper was given on pg. 3. The component-table shows two different parts/bights specifications for the components ($p_1/b_1 = 1/2$ and $p_2/b_2 = 2/3$). Since p_2/b_2 is at a higher level in the RKT than p_1/b_1, it follows that the string-run of this end-keeper can be enlarged by means of one Enlargement Method I process and two different Enlargement Method II processes (see pgs. 92—95). The resulting enlargements are as follows:

7/11 END-KEEPER

0 - 7 - 3	1/2
1 - 8 - 4 - 0	2/3
2 - 9 - 5 - 1	2/3
3 - 10 - 6 - 2	2/3

METHOD II

7/12 END-KEEPER

0 - 7 - 2	1/2
1 - 8 - 3	1/2
2 - 9 - 4 - 11 - 6 - 1	3/5
3 - 10 - 5 - 0	2/3

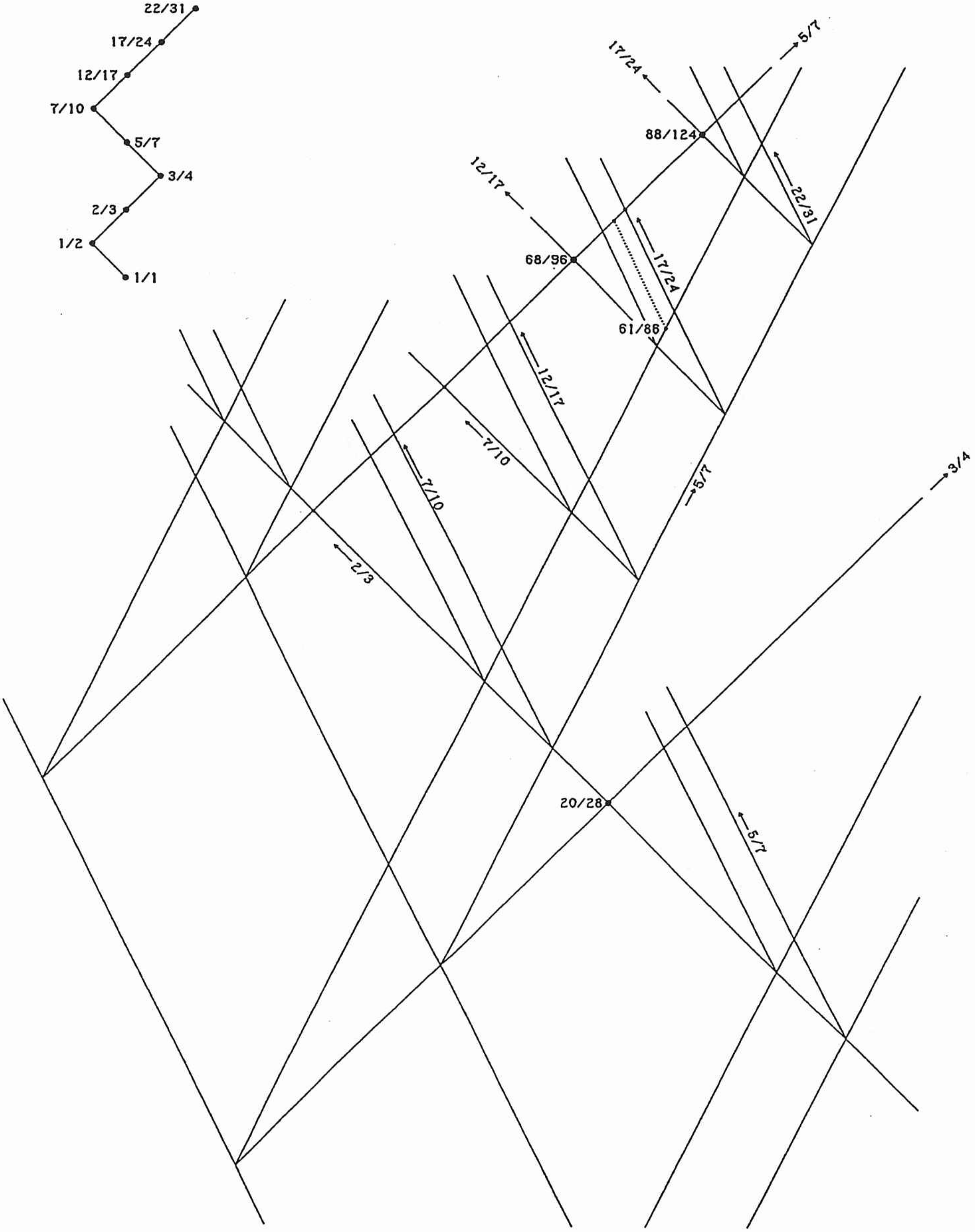
METHOD I

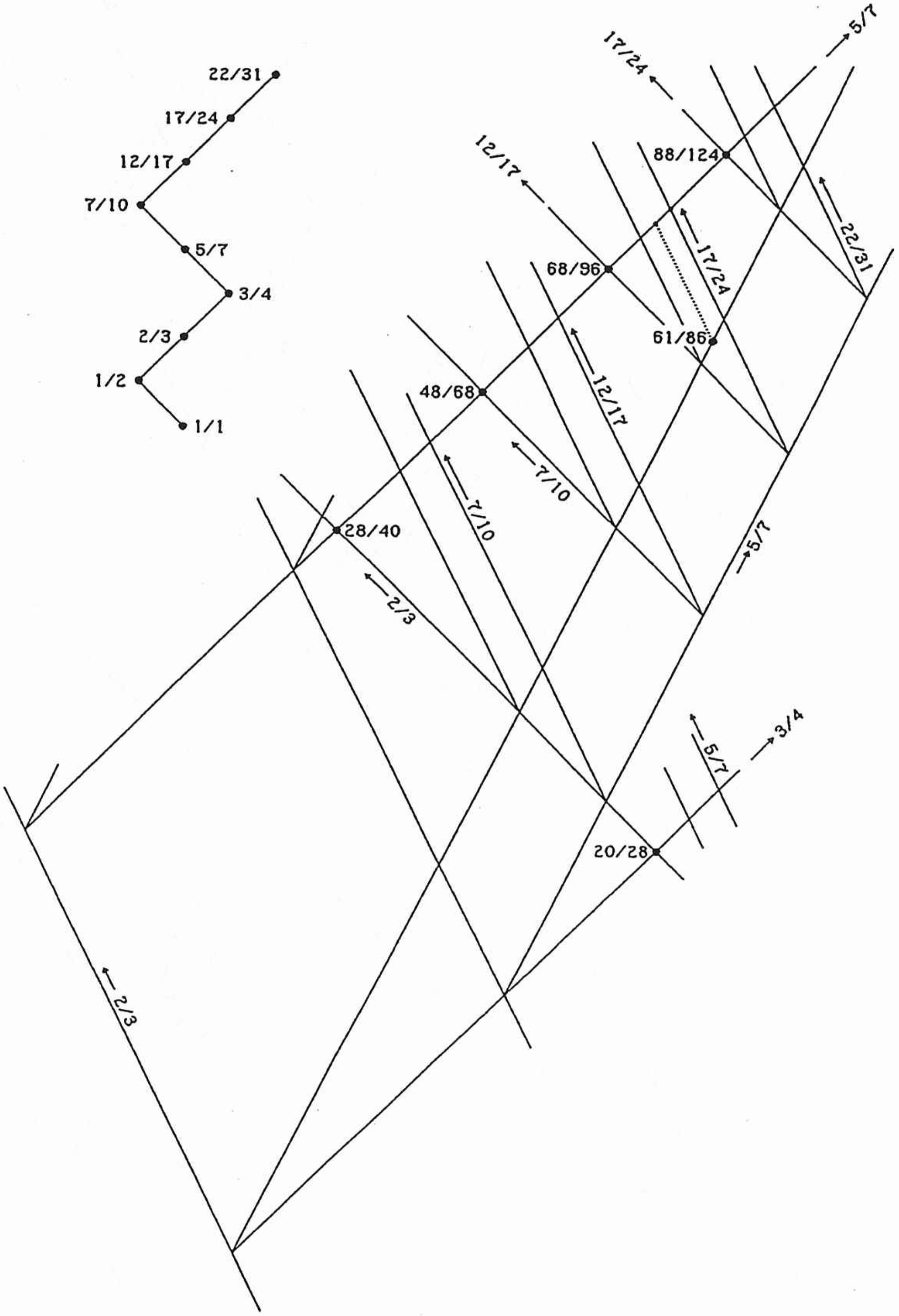
8/13 END-KEEPER

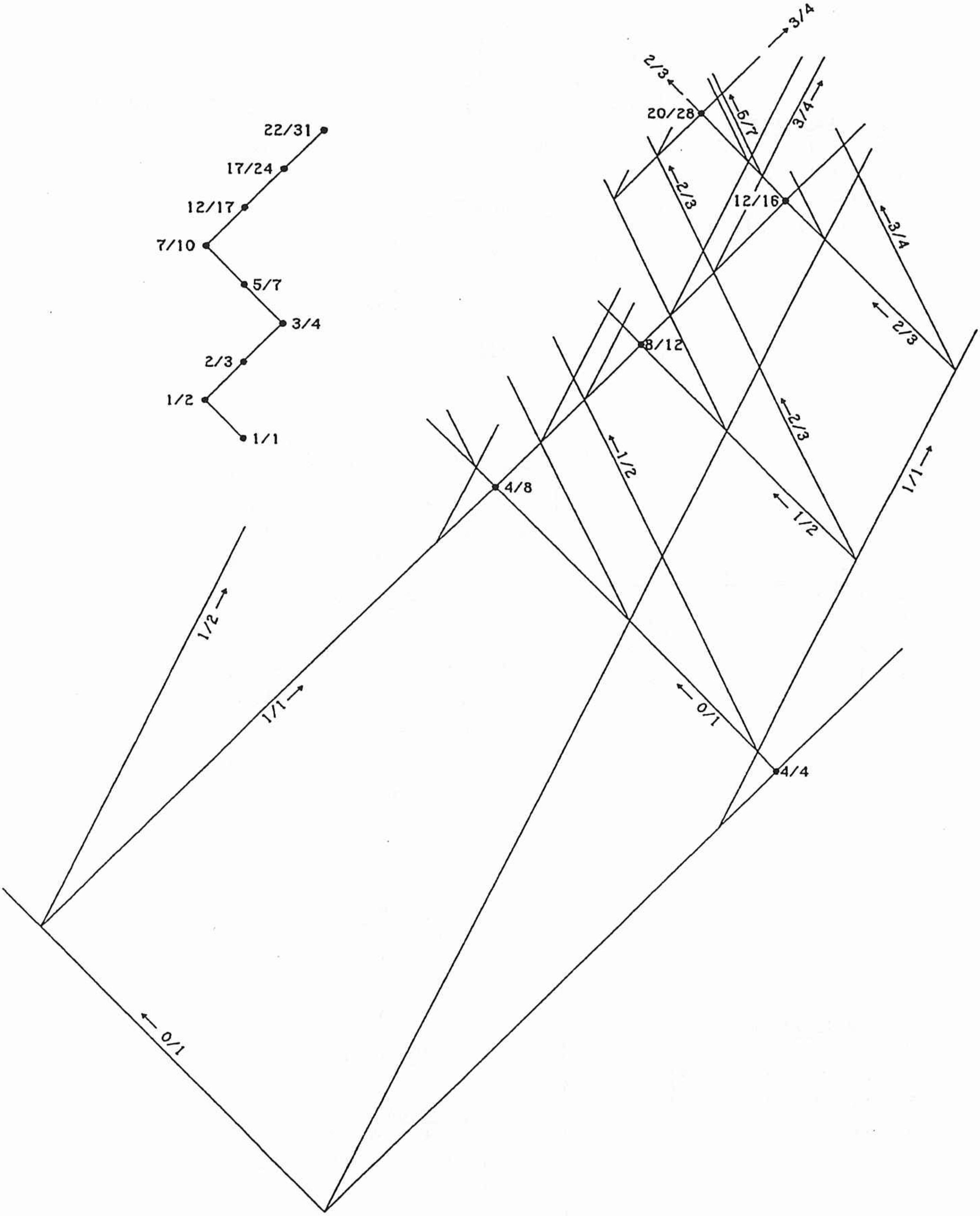
0 - 8 - 3	1/2
1 - 9 - 4 - 12 - 7 - 2	3/5
2 - 10 - 5 - 0	2/3
3 - 11 - 6 - 1	2/3

METHOD II

Further enlargements can be applied to these enlarged end-keepers as shown on pg. 28.


22/31
17/24
12/17
7/10
5/7
3/4
2/3
1/2
1/1
17/24
5/7
88/124
12/17
22/31
68/96
17/24
61/86
12/17
7/10
7/10
5/7
3/4
2/3
20/28
5/7


22/31
17/24
12/17
7/10
5/7
3/4
2/3
1/2
1/1
17/24
5/7
12/17
88/124
22/31
17/24
68/96
61/86
12/17
48/68
7/10
7/10
28/40
5/7
2/3
5/7
3/4
20/28
2/3


22/31
17/24
12/17
7/10
5/7
3/4
2/3
1/2
1/1
→3/4
2/3←
20/28
←5/7
3/4→
←2/3
12/16
←3/4
8/12
← 2/3
←2/3
←1/2
4/8
1/1→
← 1/2
1/2→
1/1→
← 0/1
4/4
← 0/1

7/11 END-KEEPER

0 - 7 - 3	1/2
1 - 8 - 4 - 0	2/3
2 - 9 - 5 - 1	2/3
3 - 10 - 6 - 2	2/3

8/12 END-KEEPER

0 - 8 - 4 - 0	2/3
1 - 9 - 5 - 1	2/3
2 - 10 - 6 - 2	2/3
3 - 11 - 7 - 3	2/3

METHOD II

8/13 END-KEEPER

0 - 8 - 3	1/2
1 - 9 - 4 - 12 - 7 - 2	3/5
2 - 10 - 5 - 0	2/3
3 - 11 - 6 - 1	2/3

METHOD I

9/14 END-KEEPER

0 - 9 - 4 - 13 - 8 - 3	3/5
1 - 10 - 5 - 0	2/3
2 - 11 - 6 - 1	2/3
3 - 12 - 7 - 2	2/3

METHOD II

7/12 END-KEEPER

0 - 7 - 2	1/2
1 - 8 - 3	1/2
2 - 9 - 4 - 11 - 6 - 1	3/5
3 - 10 - 5 - 0	2/3

8/14 END-KEEPER

0 - 8 - 2	1/2
1 - 9 - 3	1/2
2 - 10 - 4 - 12 - 6 - 0	3/5
3 - 11 - 5 - 13 - 7 - 1	3/5

METHOD I

9/15 END-KEEPER

0 - 9 - 3	1/2
1 - 10 - 4 - 13 - 7 - 1	3/5
2 - 11 - 5 - 14 - 8 - 2	3/5
3 - 12 - 6 - 0	2/3

METHOD II

8/13 END-KEEPER

0 - 8 - 3	1/2
1 - 9 - 4 - 12 - 7 - 2	3/5
2 - 10 - 5 - 0	2/3
3 - 11 - 6 - 1	2/3

9/15 END-KEEPER

0 - 9 - 3	1/2
1 - 10 - 4 - 13 - 7 - 1	3/5
2 - 11 - 5 - 14 - 8 - 2	3/5
3 - 12 - 6 - 0	2/3

METHOD I

10/16 END-KEEPER

0 - 10 - 4 - 14 - 8 - 2	3/5
1 - 11 - 5 - 15 - 9 - 3	3/5
2 - 12 - 6 - 0	2/3
3 - 13 - 7 - 1	2/3

METHOD II

In order to enable the reader to obtain a good understanding of these enlargement processes and their path representations in the Enlargement Graph, further enlargements are applied to the 8/12 and 10/16 end-keepers; the results are shown below.

8/12 END-KEEPER	
0 - 8 - 4 - 0	2/3
1 - 9 - 5 - 1	2/3
2 - 10 - 6 - 2	2/3
3 - 11 - 7 - 3	2/3

9/13 END-KEEPER	
0 - 9 - 5 - 1	2/3
1 - 10 - 6 - 2	2/3
2 - 11 - 7 - 3	2/3
3 - 12 - 8 - 4 - 0	3/4

METHOD II

9/14 END-KEEPER	
0 - 9 - 4 - 13 - 8 - 3	3/5
1 - 10 - 5 - 0	2/3
2 - 11 - 6 - 1	2/3
3 - 12 - 7 - 2	2/3

METHOD I

10/16 END-KEEPER	
0 - 10 - 4 - 14 - 8 - 2	3/5
1 - 11 - 5 - 15 - 9 - 3	3/5
2 - 12 - 6 - 0	2/3
3 - 13 - 7 - 1	2/3

11/18 END-KEEPER	
0 - 11 - 4 - 15 - 8 - 1	3/5
1 - 12 - 5 - 16 - 9 - 2	3/5
2 - 13 - 6 - 17 - 10 - 3	3/5
3 - 14 - 7 - 0	2/3

METHOD I

12/19 END-KEEPER	
0 - 12 - 5 - 17 - 10 - 3	3/5
1 - 13 - 6 - 18 - 11 - 4 - 16 - 9 - 2	5/8
2 - 14 - 7 - 0	2/3
3 - 15 - 8 - 1	2/3

METHOD II

13/21 END-KEEPER	
0 - 13 - 5 - 18 - 10 - 2	3/5
1 - 14 - 6 - 19 - 11 - 3	3/5
2 - 15 - 7 - 20 - 12 - 4 - 17 - 9 - 1	5/8
3 - 16 - 8 - 0	2/3

METHOD I

The consecutive string-runs in the braiding of an end-keeper with $p/b = 6/10$ via the enlargement paths:

$$4/4 \longrightarrow 4/5 \longrightarrow 4/6 \longrightarrow 4/7 \longrightarrow 4/8 \longrightarrow 5/9 \longrightarrow 6/10$$

are presented on pg. 30. For a Casa-coded end-keeper with $p/b = 6/10$ the appropriate coding has to be superimposed on these string-runs; this has been done on pg. 31. The associated pictorial braiding sequences are shown on pg. 32.

6/10 END-KEEPER	
0 - 6 - 2	1/2
1 - 7 - 3	1/2
2 - 8 - 4 - 0	2/3
3 - 9 - 5 - 1	2/3

5/9 END-KEEPER	
0 - 5 - 1	1/2
1 - 6 - 2	1/2
2 - 7 - 3	1/2
3 - 8 - 4 - 0	2/3

4/8 END-KEEPER	
0 - 4 - 0	1/2
1 - 5 - 1	1/2
2 - 6 - 2	1/2
3 - 7 - 3	1/2

4/7 END-KEEPER	
0 - 4 - 1	1/2
1 - 5 - 2	1/2
2 - 6 - 3	1/2
3 - 0	1/1

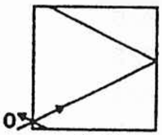
← **METHOD II** ← **METHOD II** ← **METHOD I**

4/6 END-KEEPER	
0 - 4 - 2	1/2
1 - 5 - 3	1/2
2 - 0	1/1
3 - 1	1/1

4/5 END-KEEPER	
0 - 4 - 3	1/2
1 - 0	1/1
2 - 1	1/1
3 - 2	1/1

4/4 END-KEEPER	
0 - 0	1/1
1 - 1	1/1
2 - 2	1/1
3 - 3	1/1

← **METHOD I** ← **METHOD I** ← **METHOD I**



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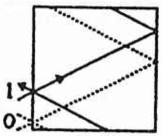

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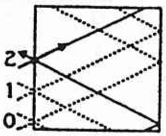

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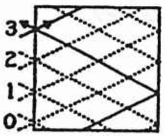

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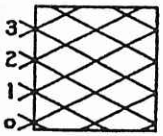

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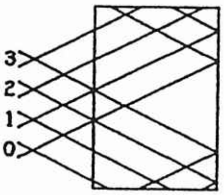

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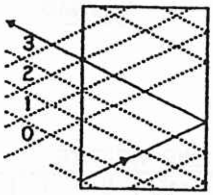

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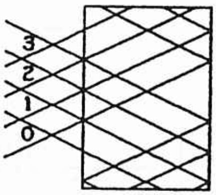

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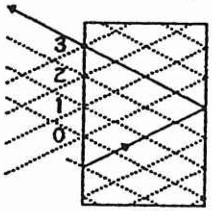

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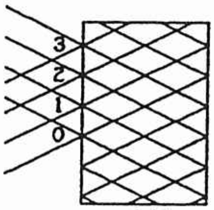

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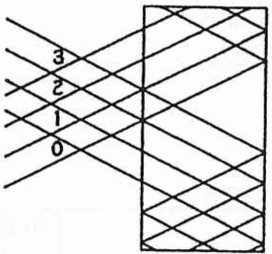

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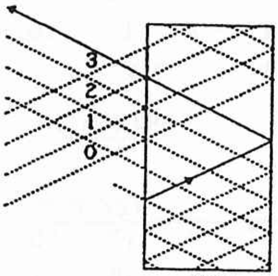

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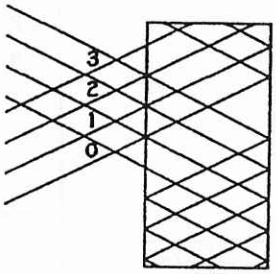

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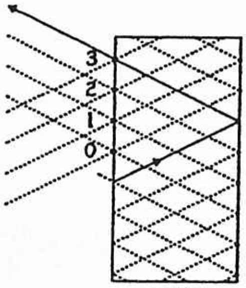

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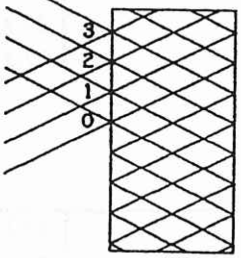

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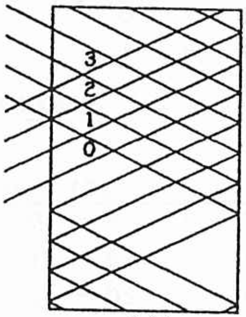

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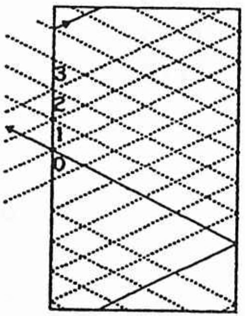

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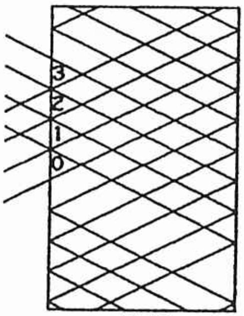

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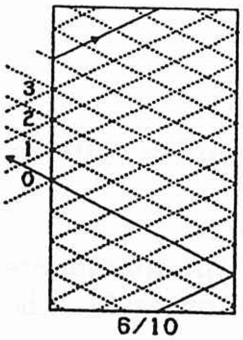
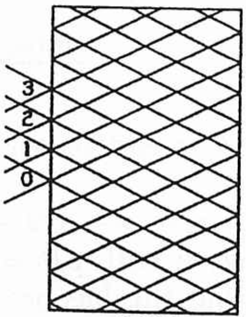


6/10



6/10

6/10 END-KEEPER	
0 - 6 - 2	1/2
1 - 7 - 3	1/2
2 - 8 - 4 - 0	2/3
3 - 9 - 5 - 1	2/3

← METHOD II

5/9 END-KEEPER	
0 - 5 - 1	1/2
1 - 6 - 2	1/2
2 - 7 - 3	1/2
3 - 8 - 4 - 0	2/3

← METHOD II

4/8 END-KEEPER	
0 - 4 - 0	1/2
1 - 5 - 1	1/2
2 - 6 - 2	1/2
3 - 7 - 3	1/2

← METHOD I

4/7 END-KEEPER	
0 - 4 - 1	1/2
1 - 5 - 2	1/2
2 - 6 - 3	1/2
3 - 0	1/1

4/6 END-KEEPER	
0 - 4 - 2	1/2
1 - 5 - 3	1/2
2 - 0	1/1
3 - 1	1/1

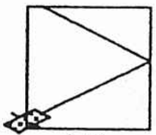
← METHOD I

4/5 END-KEEPER	
0 - 4 - 3	1/2
1 - 0	1/1
2 - 1	1/1
3 - 2	1/1

← METHOD I

4/4 END-KEEPER	
0 - 0	1/1
1 - 1	1/1
2 - 2	1/1
3 - 3	1/1

← METHOD I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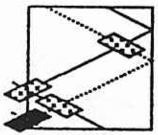
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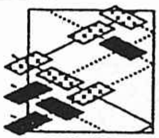
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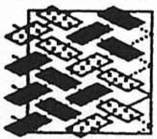
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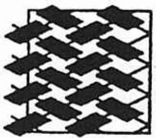
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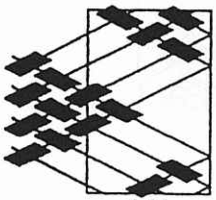
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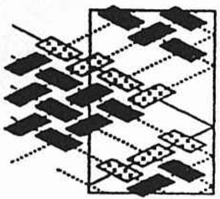
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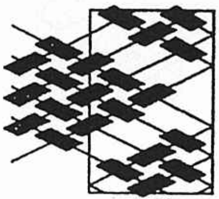
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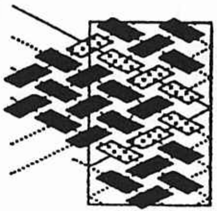
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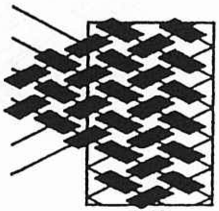
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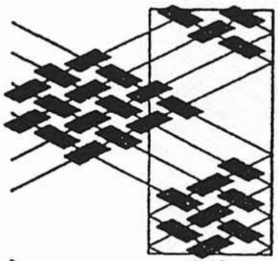
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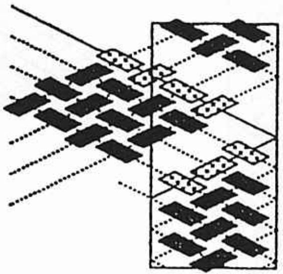
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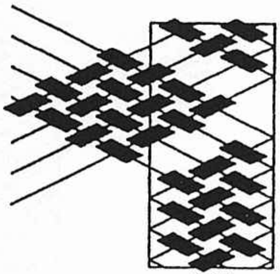
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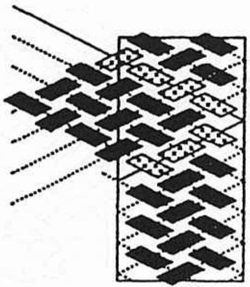
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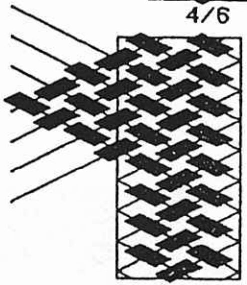
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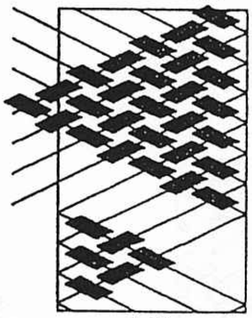
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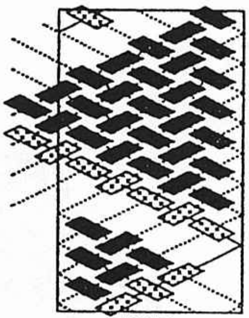
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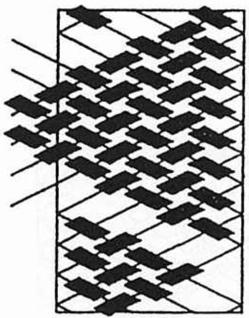
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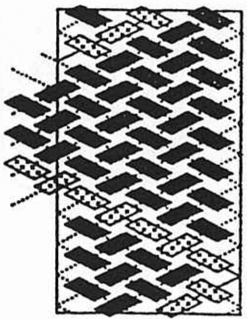
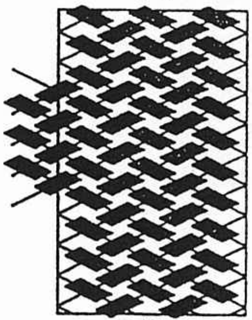
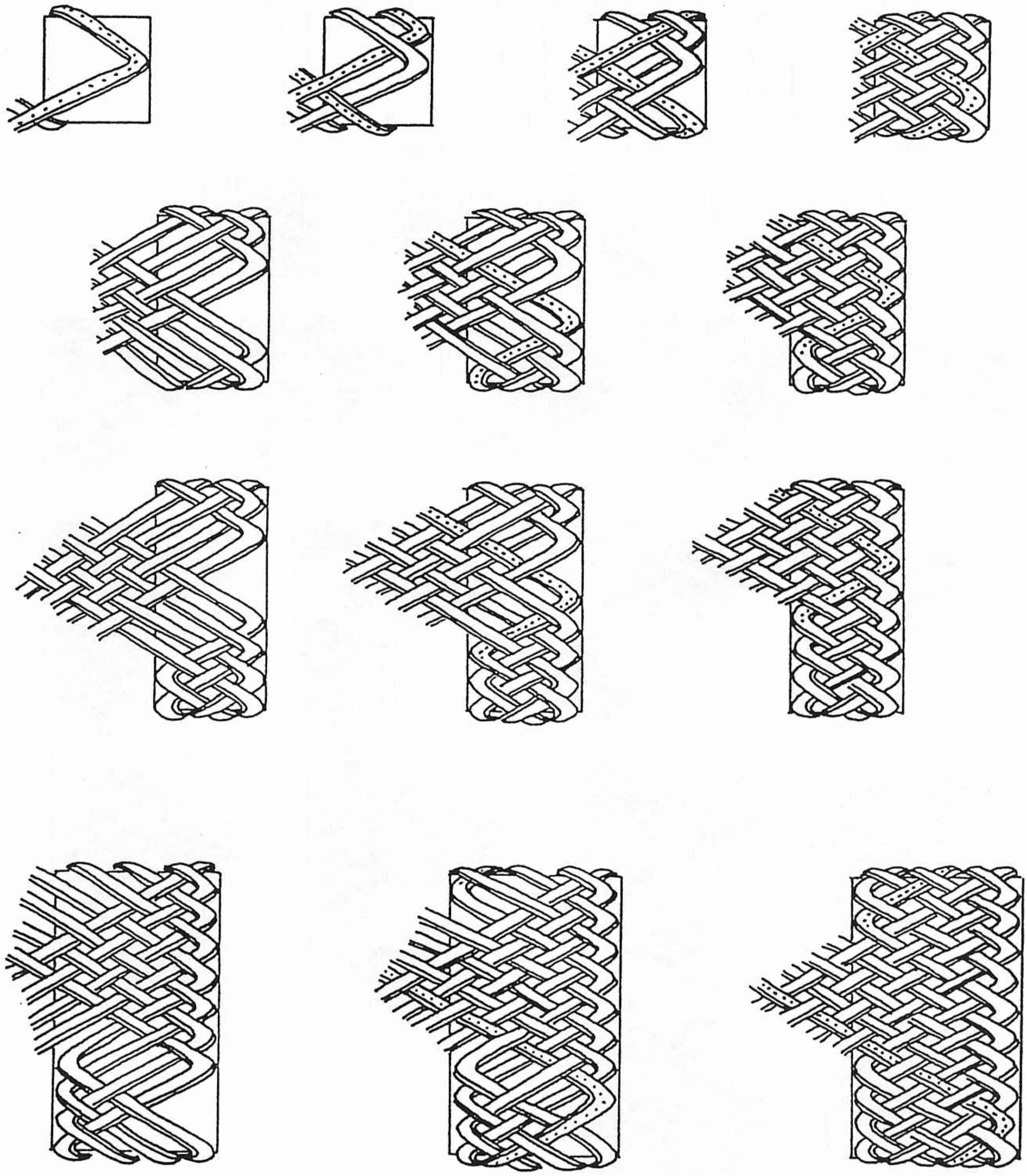

6/10


6/10



Pictorial drawings associated with pg. 31.

The end-keeper with $p/b = 6/10$ can also be constructed via the enlargement paths:

$$4/4 \longrightarrow 4/5 \longrightarrow 4/6 \longrightarrow 5/8 \longrightarrow 6/10$$

The associated consecutive string-run diagrams are presented on pg. 34, while pg. 35 shows the appropriate superimposed coding for a Casa-coded 6/10-endkeeper. The corresponding pictorial braiding sequences are depicted on pg. 36. Note that the enlargement Method I, employed in raising the 4/6-endkeeper to the 5/8-endkeeper, enlarges the component starting at transition index-number 2 from its parts/bights specification 1/1 to 2/3. This enlargement is a 2^{nd}-order Method I enlargement, which consists of a 1^{st}-order Method I enlargement followed by a 1^{st}-order Method II enlargement (see Pamphlet No. 4 and its Supplement). Similarly the enlargement Method I employed in raising the 5/8-endkeeper to the 6/10-endkeeper is a 2^{nd}-order Method I enlargement.

The 4/6-endkeeper (with $x = 4$), which forms an intermediate stage in the enlargement processes, can also be obtained from a 2/2-endkeeper with $x = 2$ via two Method I enlargement processes, followed by two TIE II processes as set out on pg. 37 (the corresponding pictorial braiding sequences are shown on pg. 38).

The TIE II raising processes used here are 1^{st}-order TIE II processes. If instead 2^{nd}-order TIE II processes are used, the final result will be a 6/10-endkeeper as detailed on pg. 39 (the corresponding pictorial braiding sequences are shown on pg. 40). A 2^{nd}-order TIE II process is a raising-process in which the string-run belonging to the added transition-point is the *increase* in string-run of a 2^{nd}-order Method II enlargement process. A 2^{nd}-order Method II enlargement process consists of a 1^{st}-order Method II enlargement followed by a 1^{st}-order Method I enlargement. The 2^{nd}-order TIE II process used on the 2/4-endkeeper creates a new component with a parts/bights specification which is equal to the *increase* in the parts/bights specification of the component starting at transition index-number 1 when subjected to a 2^{nd}-order Method II enlargement. Hence the parts/bights specification of this new component is 2/3 (2^{nd}-order Method II enlargement on 1/2 gives an *increase* of $1/1 + 1/2 = 2/3$). Since this is also $(1/2 + 1/1)$, it will be evident that that the new component can be created by following the component starting at transition index-number 1 and then enlarging it with a 1^{st}-order Method II enlargement (TIF 1/2 – Method II).

The 6/10-endkeeper ($x = 4$) can also be constructed by raising the 2/4-endkeeper ($x = 2$) to a 3/5-endkeeper by a Method II enlargement of the 1^{st}-order, then to a 4/6-enkeeper ($x = 2$) by a Method II enlargement of the 1^{st}-order, then to a 5/8-endkeeper ($x = 3$) by a 1^{st}-order TIE I process and finally by again applying a 1^{st}-order TIE I process to the 5/8-endkeeper. This construction process has been outlined on pg. 41, while the corresponding pictorial braiding sequences are shown on pg. 42.

This particular construction process of the 6/10-endkeeper ($x = 4$) can be speeded up by constructing the 4/6-endkeeper ($x = 2$) from the 2/2-endkeeper ($x = 2$) via two 2^{nd}-order method I enlargement steps as shown on pg. 43 (pg. 44 shows the associated pictorial braiding sequences).

In the construction procedures outlined on pgs. 41, 42, 43 and 44 the 4/6-endkeeper ($x = 2$) formed an intermediate stage. Since this end-keeper is an interweave of two identical components (each with a parts/bights specification of 2/3), we can braid the 4/6-endkeeper quickly by interbraiding two identical 2/3-Matthew Walker knots. Hence we first braid the 2/3-endkeeper ($x = 1$), then raise it to the 4/6-endkeeper ($x = 2$) by employing the TIF 2/3 process. The construction steps are outlined on pg. 45, while the associated pictorial braiding sequences are depicted on pg. 46.

6/10 END-KEEPER	
0 - 6 - 2	1/2
1 - 7 - 3	1/2
2 - 8 - 4 - 0	2/3
3 - 9 - 5 - 1	2/3

← METHOD I

5/8 END-KEEPER	
0 - 5 - 2	1/2
1 - 6 - 3	1/2
2 - 7 - 4 - 1	2/3
3 - 0	1/1

← METHOD I

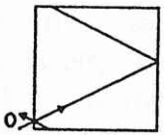
4/6 END-KEEPER	
0 - 4 - 2	1/2
1 - 5 - 3	1/2
2 - 0	1/1
3 - 1	1/1

4/5 END-KEEPER	
0 - 4 - 3	1/2
1 - 0	1/1
2 - 1	1/1
3 - 2	1/1

← METHOD I

4/4 END-KEEPER	
0 - 0	1/1
1 - 1	1/1
2 - 2	1/1
3 - 3	1/1

← METHOD I



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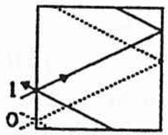

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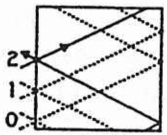

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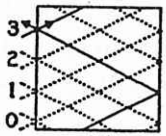

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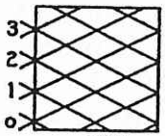

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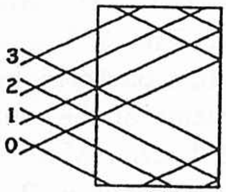

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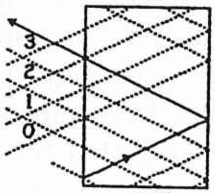
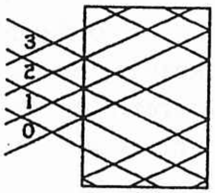


4/5



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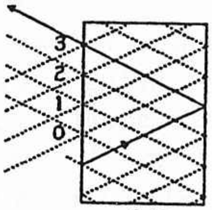

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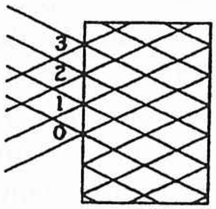

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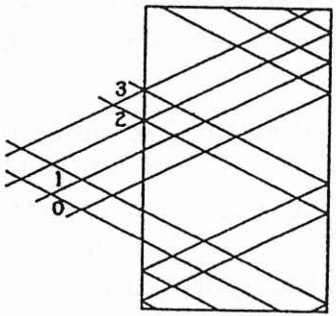

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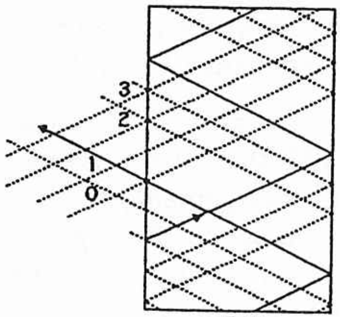
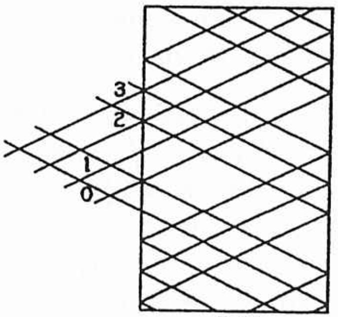


5/8



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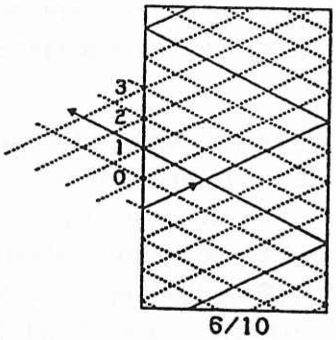

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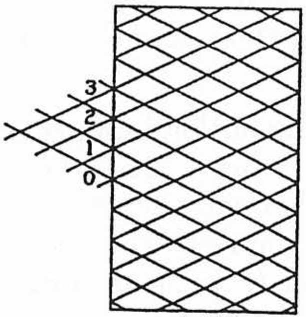


6/10

6/10 END-KEEPER

0 - 6 - 2	1/2
1 - 7 - 3	1/2
2 - 8 - 4 - 0	2/3
3 - 9 - 5 - 1	2/3

← METHOD I

5/8 END-KEEPER

0 - 5 - 2	1/2
1 - 6 - 3	1/2
2 - 7 - 4 - 1	2/3
3 - 0	1/1

← METHOD I

4/6 END-KEEPER

0 - 4 - 2	1/2
1 - 5 - 3	1/2
2 - 0	1/1
3 - 1	1/1

4/5 END-KEEPER

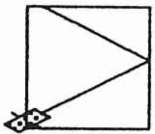
0 - 4 - 3	1/2
1 - 0	1/1
2 - 1	1/1
3 - 2	1/1

← METHOD I

4/4 END-KEEPER

0 - 0	1/1
1 - 1	1/1
2 - 2	1/1
3 - 3	1/1

← METHOD I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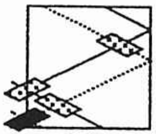
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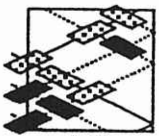
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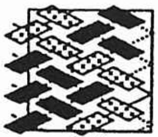
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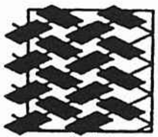
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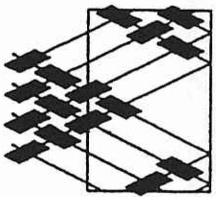
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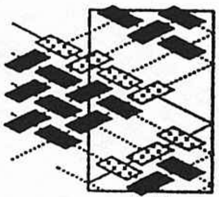
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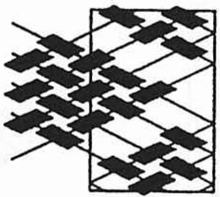
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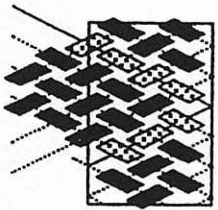
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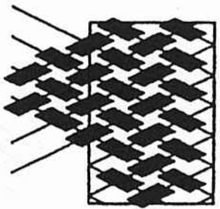
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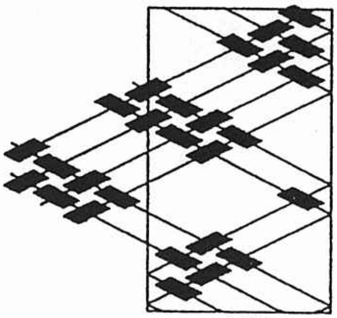
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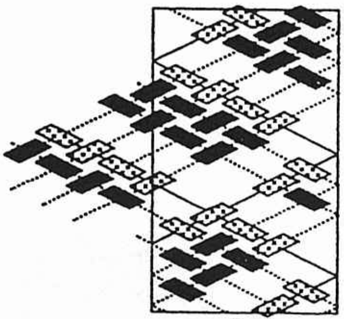
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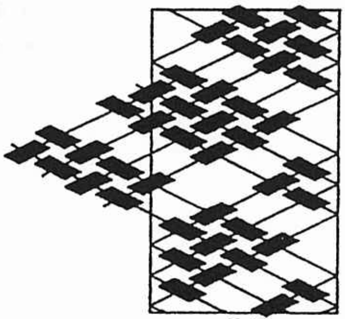
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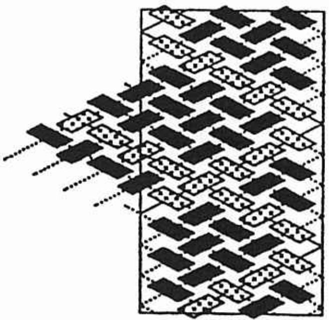
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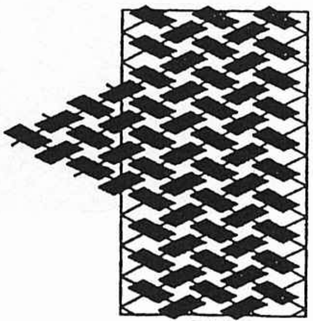
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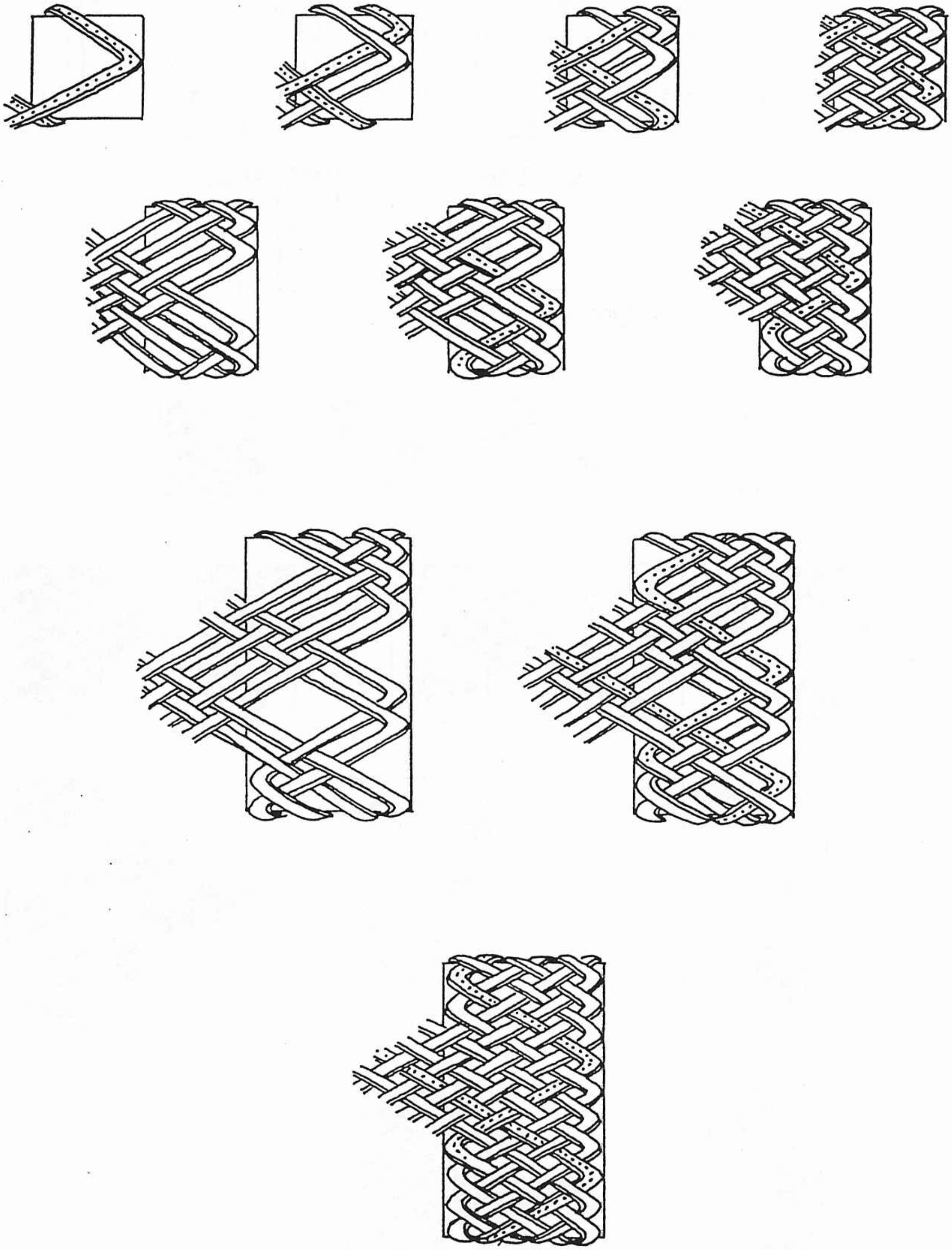


Pictorial drawings associated with pg. 35.

4/6 END-KEEPER

0 - 4 - 2	1/2
1 - 5 - 3	1/2
2 - 0	1/1
3 - 1	1/1

← TIE II

3/5 END-KEEPER

0 - 3 - 1	1/2
1 - 4 - 2	1/2
2 - 0	1/1

← TIE II

2/4 END-KEEPER

0 - 2 - 0	1/2
1 - 3 - 1	1/2

2/3 END-KEEPER

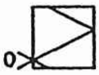
0 - 2 - 1	1/2
1 - 0	1/1

← METHOD I

2/2 END-KEEPER

0 - 0	1/1
1 - 1	1/1

← METHOD I



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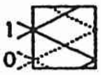
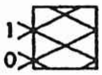


2/2



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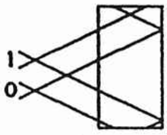

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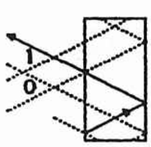

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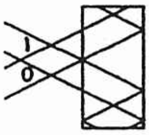

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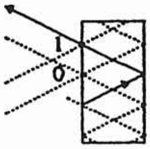

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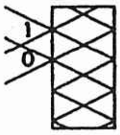

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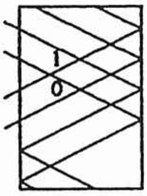

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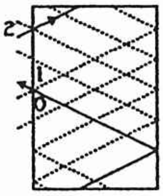

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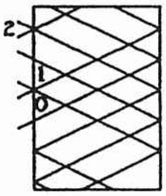

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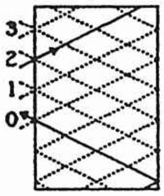

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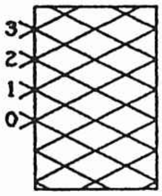

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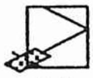
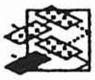


1/1



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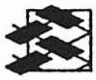

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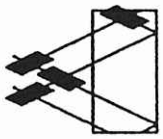

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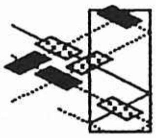

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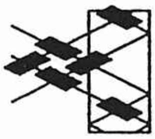

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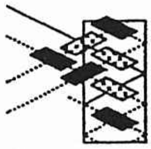

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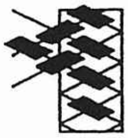

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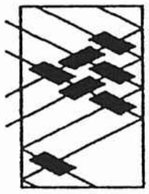

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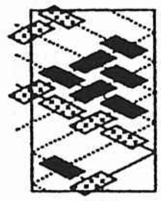
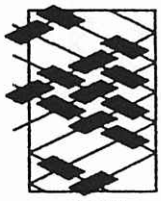


3/5



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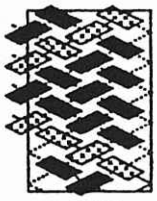
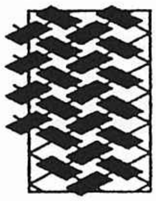
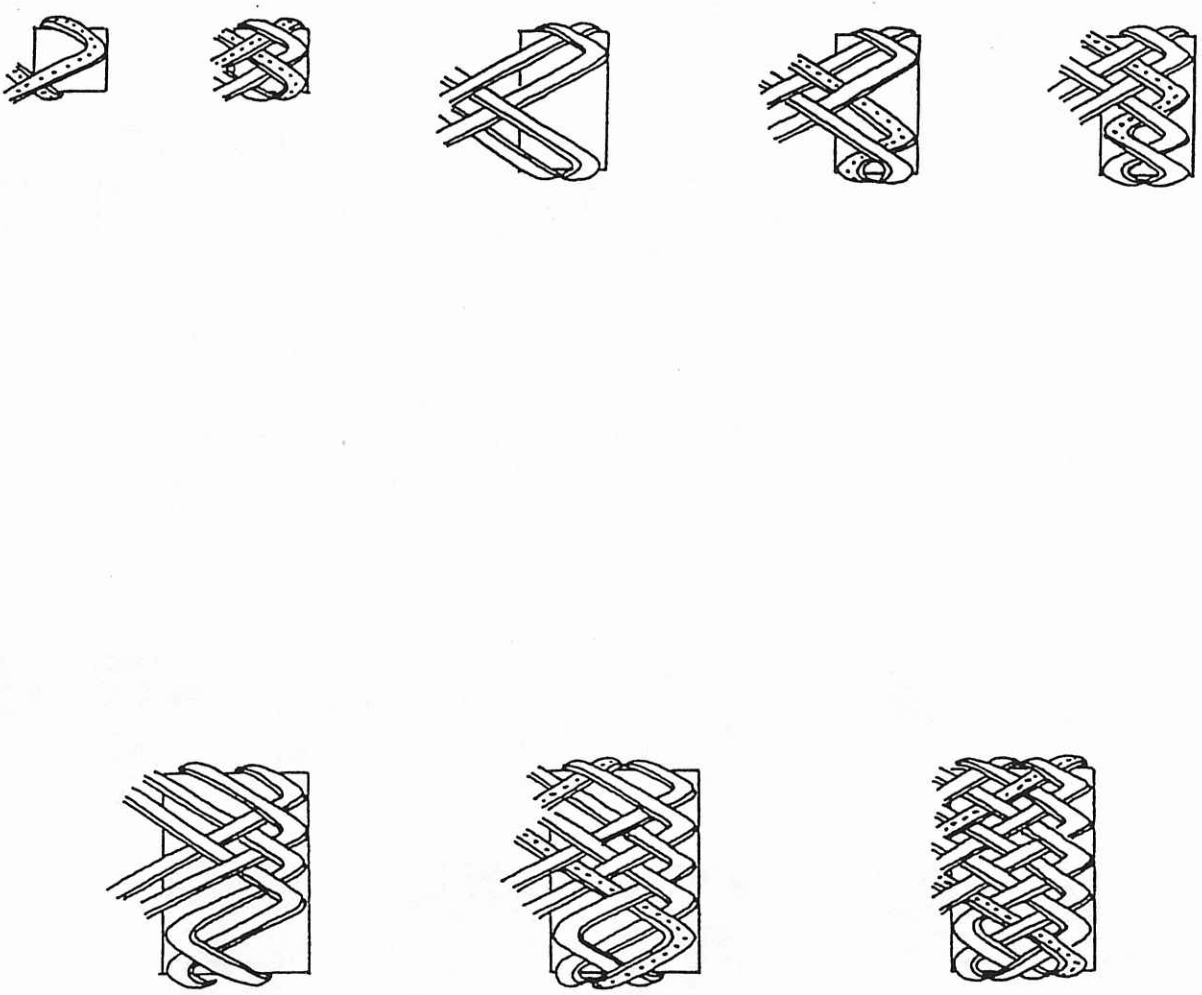


4/6



4/6



Pictorial drawings associated with pg. 37.

6/10 END-KEEPER

0 - 6 - 2	1/2
1 - 7 - 3	1/2
2 - 8 - 4 - 0	2/3
3 - 9 - 5 - 1	2/3

← TIF 1/2 – METHOD II

4/7 END-KEEPER

0 - 4 - 1	1/2
1 - 5 - 2	1/2
2 - 6 - 3 - 0	2/3

← TIF 1/2 – METHOD II

2/4 END-KEEPER

0 - 2 - 0	1/2
1 - 3 - 1	1/2

← METHOD I

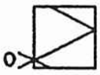
2/3 END-KEEPER

0 - 2 - 1	1/2
1 - 0	1/1

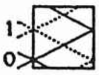
← METHOD I

2/2 END-KEEPER

0 - 0	1/1
1 - 1	1/1



1/1



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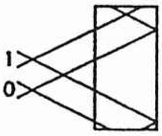

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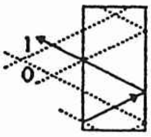
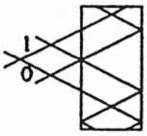


2/3



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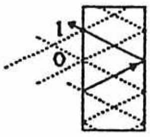

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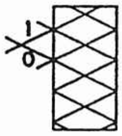

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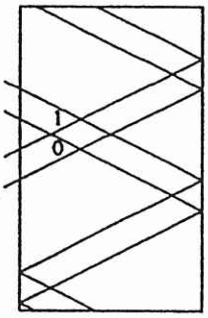

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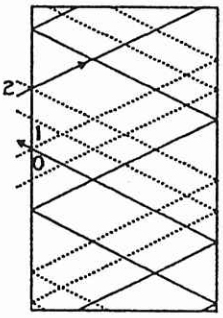

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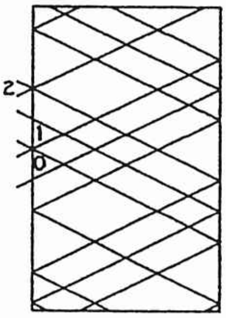

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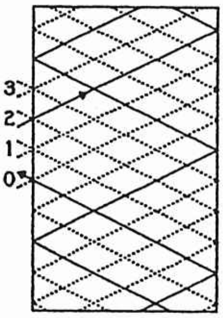
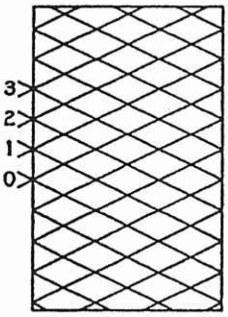
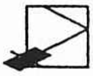
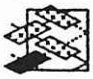


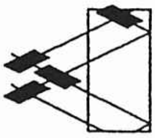
6/10



6/10


1/1


2/2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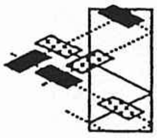
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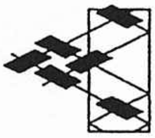
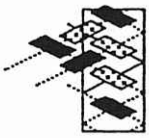
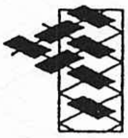
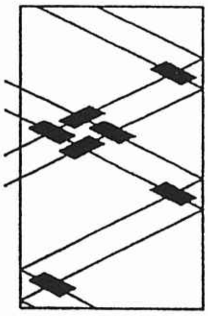
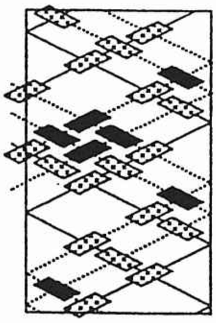
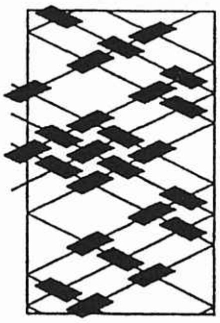

2/3

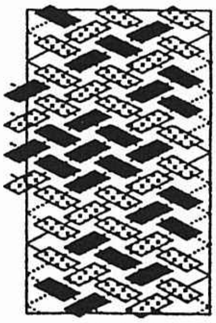

2/4


2/4


2/4


4/7


4/7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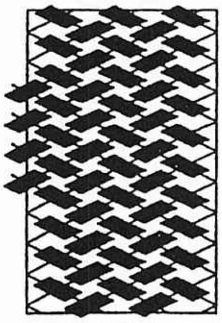
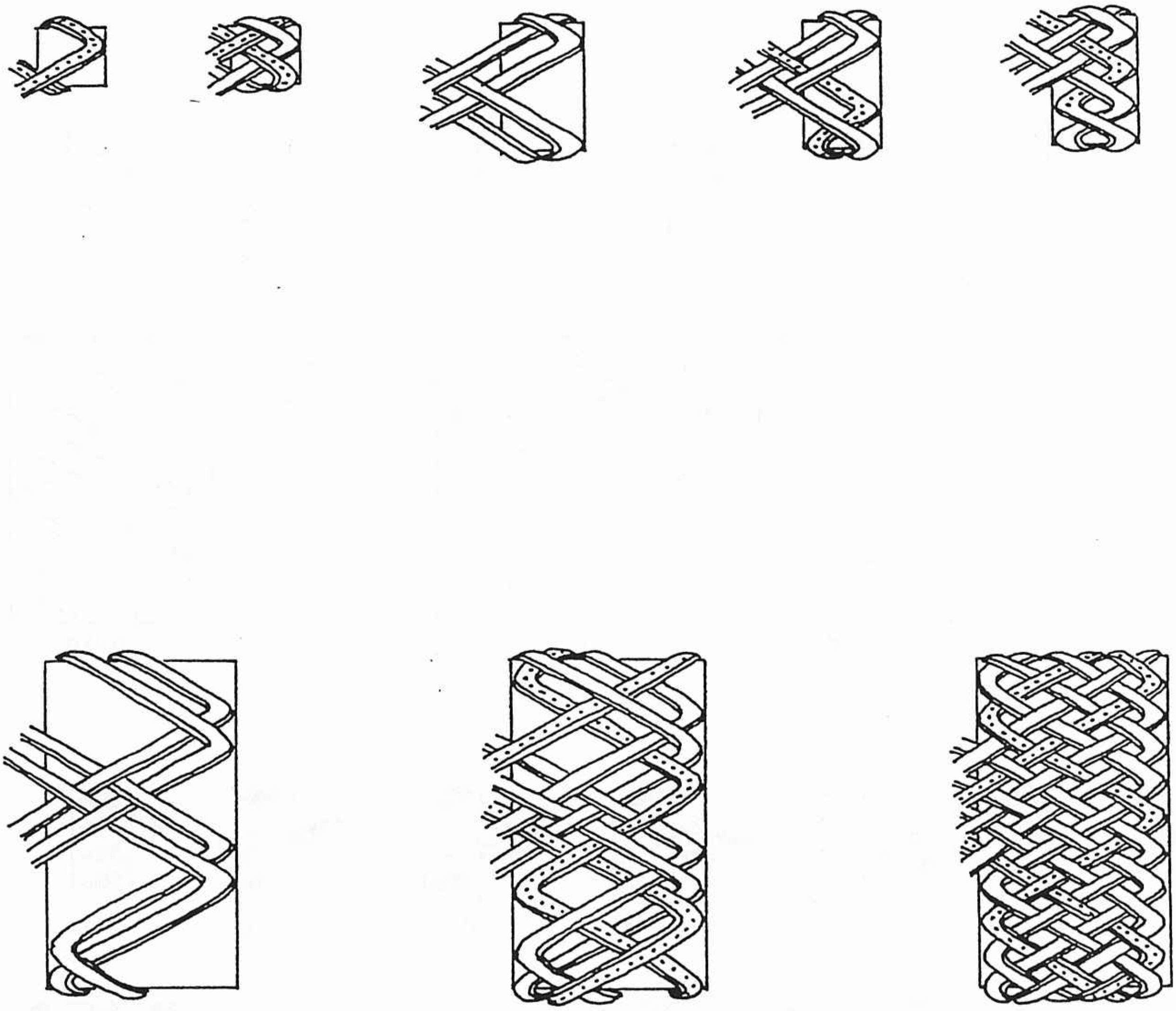

6/10



Pictorial drawings associated with pg. 39.

6/10 END-KEEPER	
0 - 6 - 2	1/2
1 - 7 - 3	1/2
2 - 8 - 4 - 0	2/3
3 - 9 - 5 - 1	2/3

← TIE METHOD I

5/8 END-KEEPER	
0 - 5 - 2	1/2
1 - 6 - 3 - 0	2/3
2 - 7 - 4 - 1	2/3

← TIE METHOD I

4/6 END-KEEPER	
0 - 4 - 2 - 0	2/3
1 - 5 - 3 - 1	2/3

← METHOD II

3/5 END-KEEPER	
0 - 3 - 1	1/2
1 - 4 - 2 - 0	2/3

2/4 END-KEEPER	
0 - 2 - 0	1/2
1 - 3 - 1	1/2

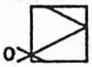
← METHOD II

2/3 END-KEEPER	
0 - 2 - 1	1/2
1 - 0	1/1

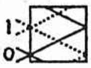
← METHOD I

2/2 END-KEEPER	
0 - 0	1/1
1 - 1	1/1

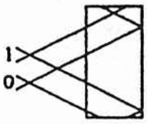
← METHOD I



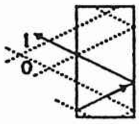
1/1



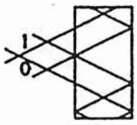
2/2



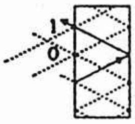
2/2



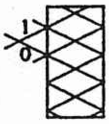
2/3



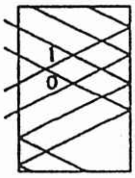
2/3



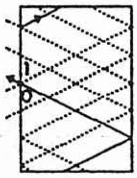
2/4



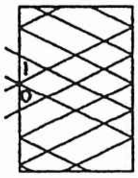
2/4



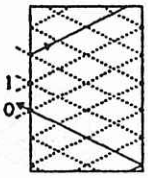
2/4



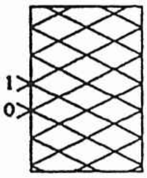
3/5



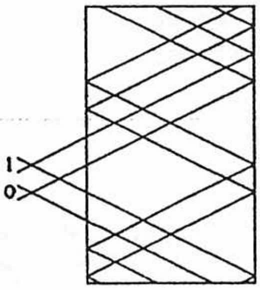
3/5



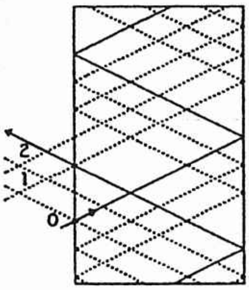
4/6



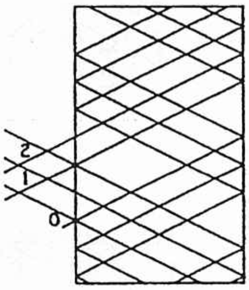
4/6



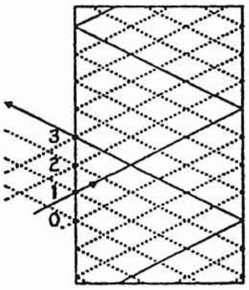
4/6



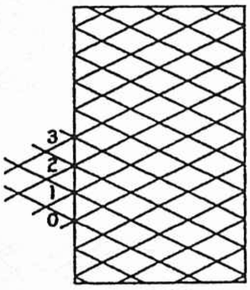
5/8



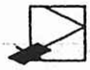
5/8



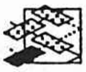
6/10



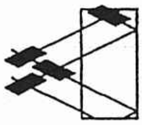
6/10



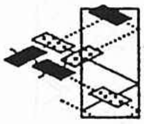
1/1



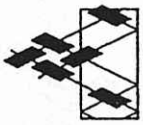
2/2



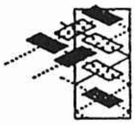
2/2



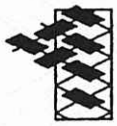
2/3



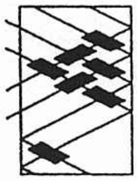
2/3



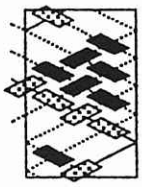
2/4



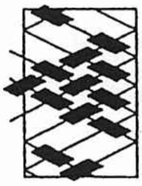
2/4



2/4



3/5



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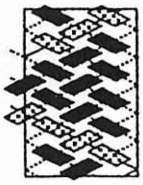

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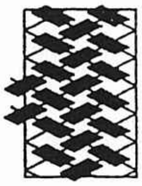

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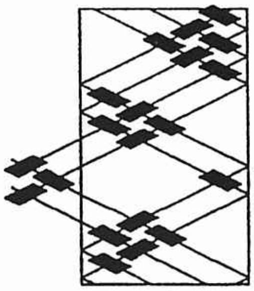
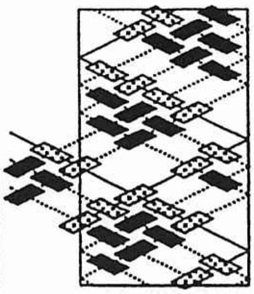
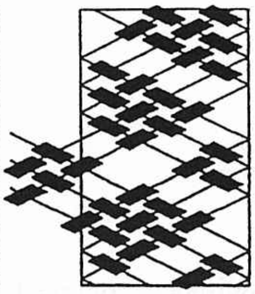


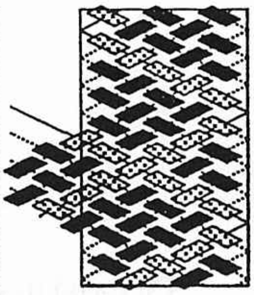
4/6



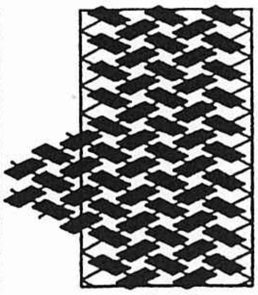
5/8



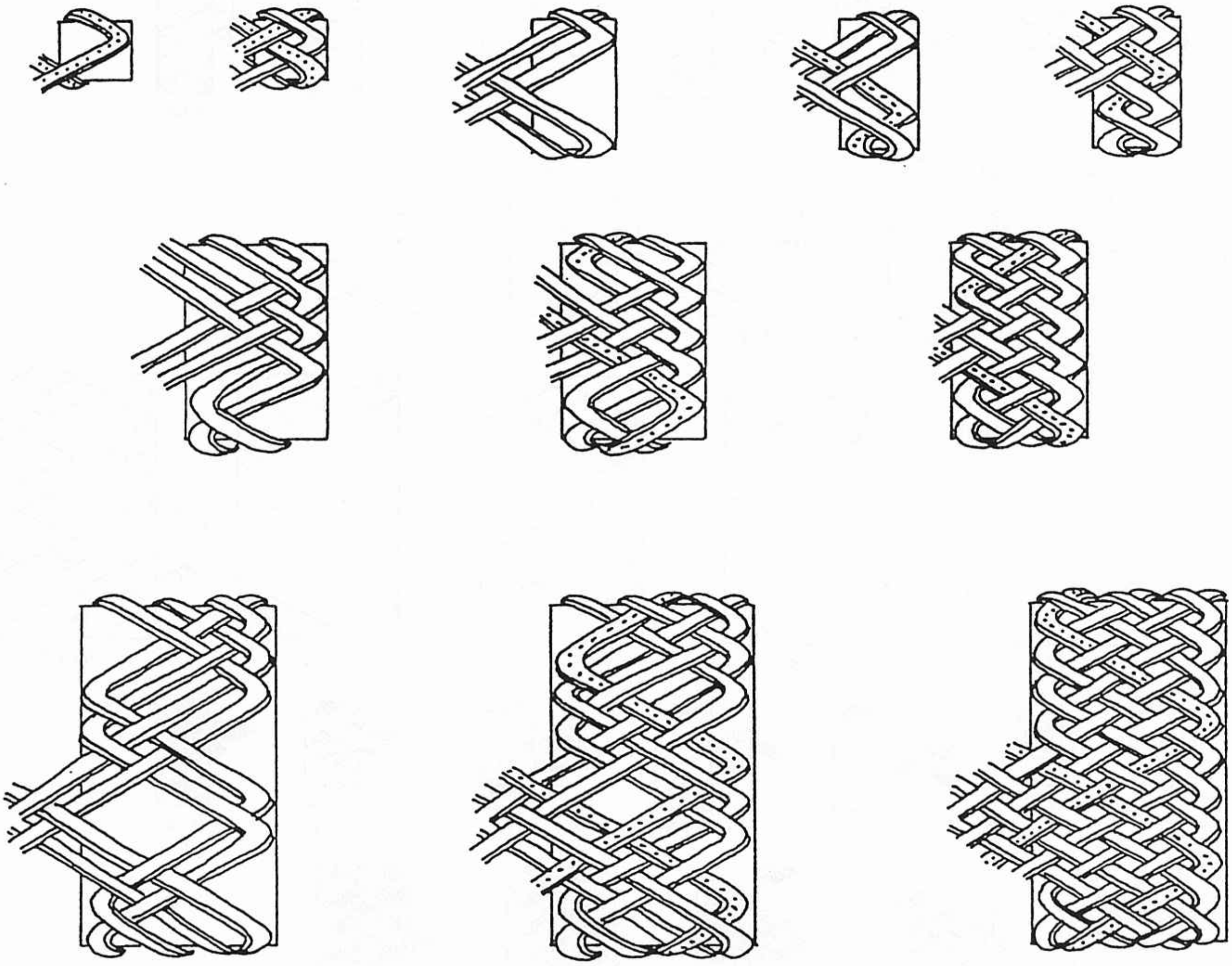
5/8



6/10



6/10



Pictorial drawings associated with pg. 41.

6/10 END-KEEPER	
0 - 6 - 2	1/2
1 - 7 - 3	1/2
2 - 8 - 4 - 0	2/3
3 - 9 - 5 - 1	2/3

5/8 END-KEEPER	
0 - 5 - 2	1/2
1 - 6 - 3 - 0	2/3
2 - 7 - 4 - 1	2/3

4/6 END-KEEPER	
0 - 4 - 2 - 0	2/3
1 - 5 - 3 - 1	2/3

← TIE METHOD I

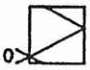
← TIE METHOD I

3/4 END-KEEPER	
0 - 3 - 2 - 1	2/3
1 - 0	1/1

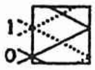
2/2 END-KEEPER	
0 - 0	1/1
1 - 1	1/1

← METHOD I

← METHOD I



1/1



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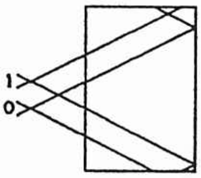
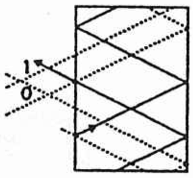
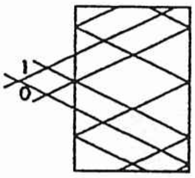


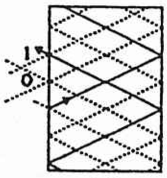
2/2



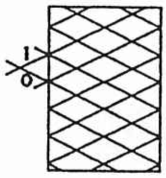
3/4



3/4



4/6



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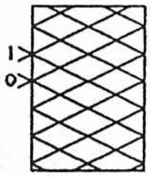

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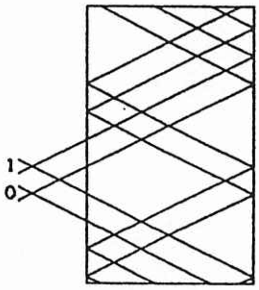

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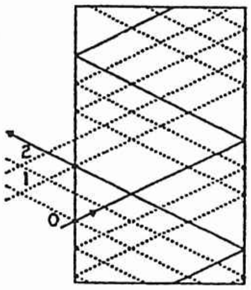
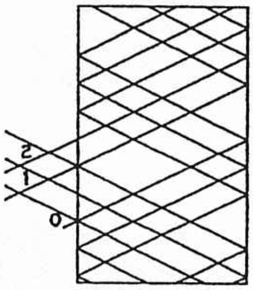


5/8



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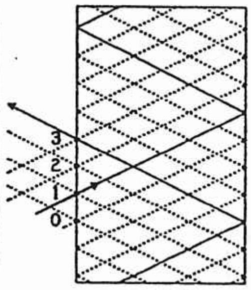
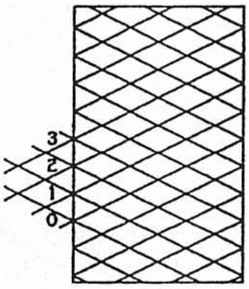


6/10



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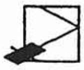
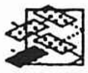
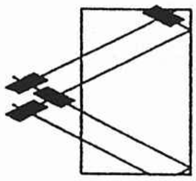


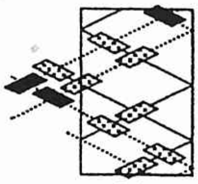
1/1



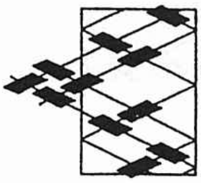
2/2



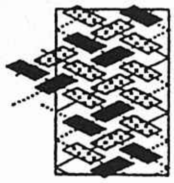
2/2



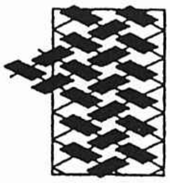
3/4



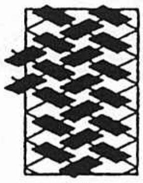
3/4



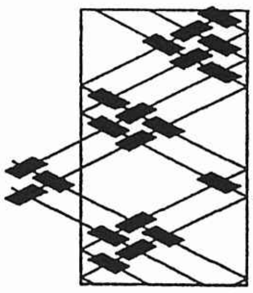
4/6



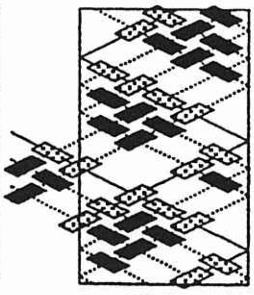
4/6



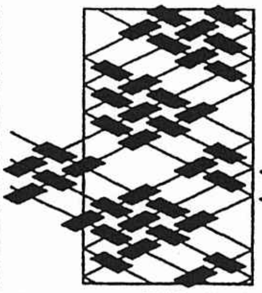
4/6



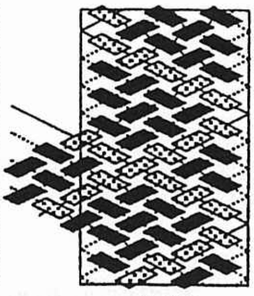
4/6



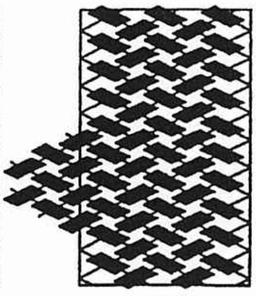
5/8



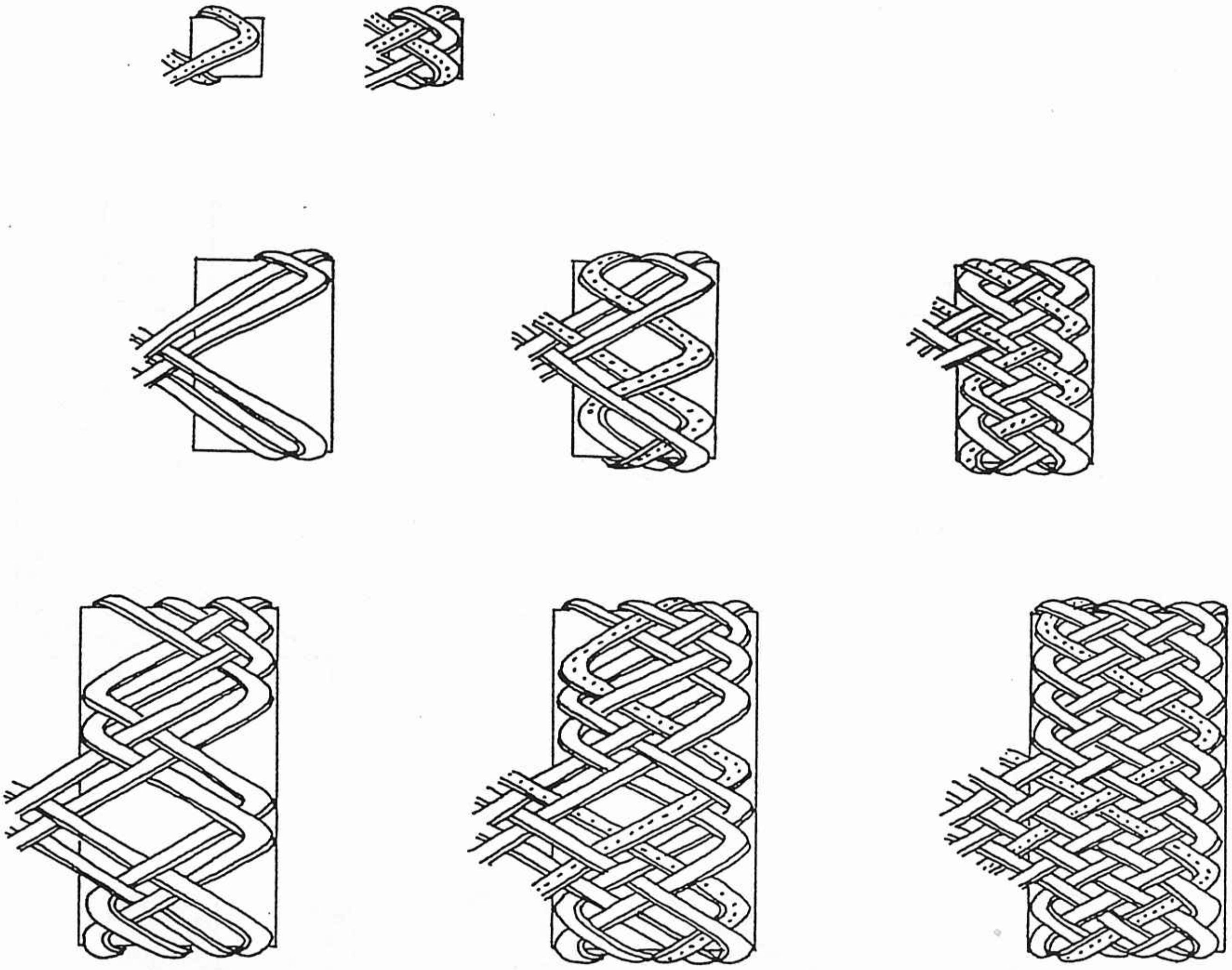
5/8



6/10



6/10



Pictorial drawings associated with pg. 43.

6/10 END-KEEPER	
0 - 6 - 2	1/2
1 - 7 - 3	1/2
2 - 8 - 4 - 0	2/3
3 - 9 - 5 - 1	2/3

← TIE METHOD I

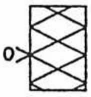
5/8 END-KEEPER	
0 - 5 - 2	1/2
1 - 6 - 3 - 0	2/3
2 - 7 - 4 - 1	2/3

← TIE METHOD I

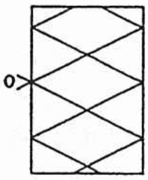
4/6 END-KEEPER	
0 - 4 - 2 - 0	2/3
1 - 5 - 3 - 1	2/3

← TIF 2/3

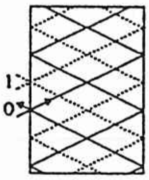
2/3 END-KEEPER	
0 - 2 - 1 - 0	2/3



2/3



2/3



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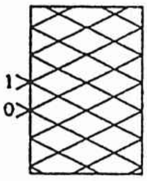
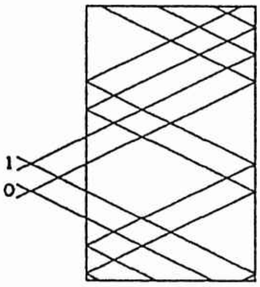
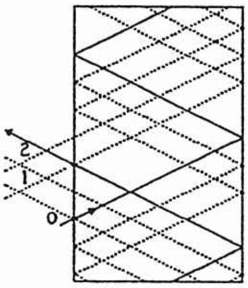


4/6



4/6



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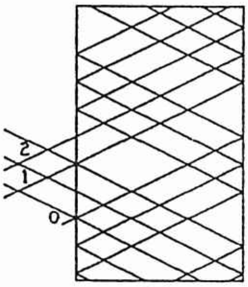
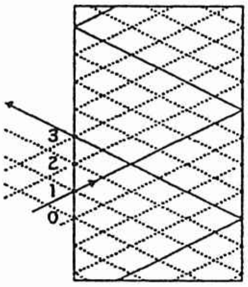
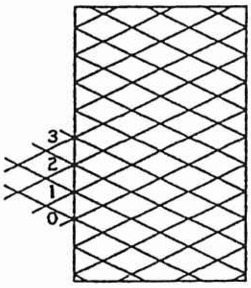


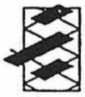
5/8



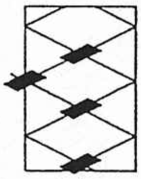
6/10



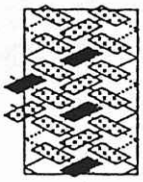
6/10



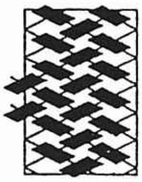
2/3



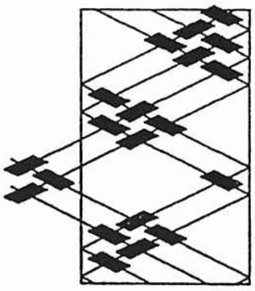
2/3



4/6



4/6



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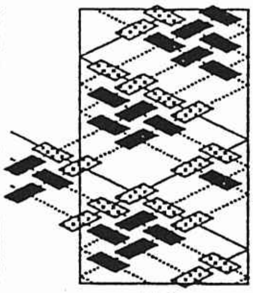
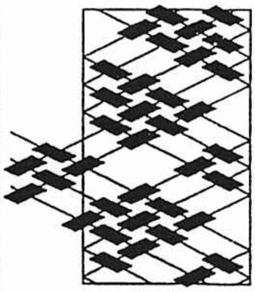
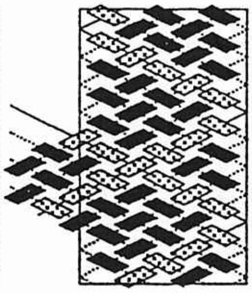


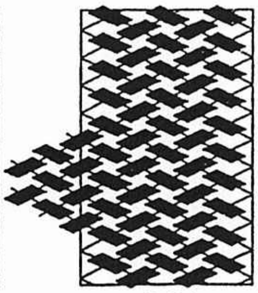
5/8



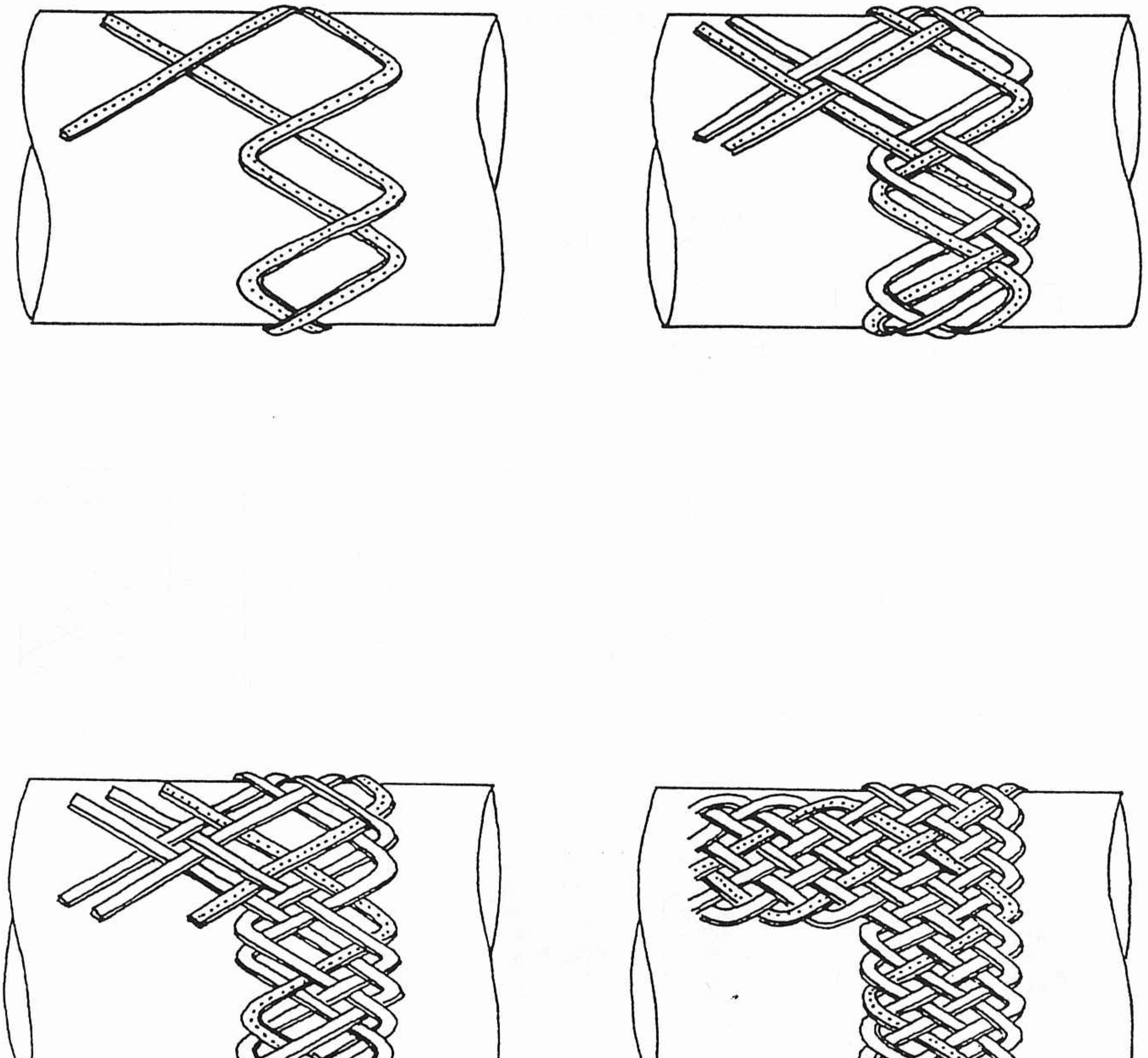
5/8



6/10



6/10



Pictorial drawings associated with pg. 45.

6/8 END-KEEPER	
0 - 6 - 4 - 2	2/3
1 - 7 - 5 - 3	2/3
2 - 0	1/1
3 - 1	1/1

← TIE METHOD II

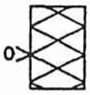
5/7 END-KEEPER	
0 - 5 - 3 - 1	2/3
1 - 6 - 4 - 2	2/3
2 - 0	1/1

← TIE METHOD II

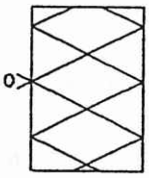
4/6 END-KEEPER	
0 - 4 - 2 - 0	2/3
1 - 5 - 3 - 1	2/3

← TIF 2/3

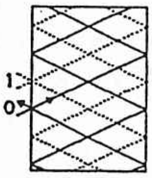
2/3 END-KEEPER	
0 - 2 - 1 - 0	2/3



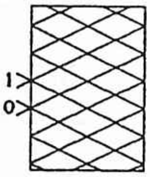
2/3



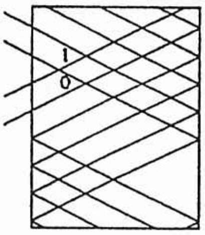
2/3



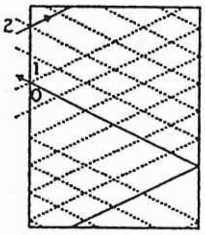
4/6



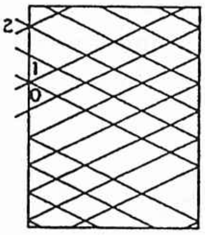
4/6



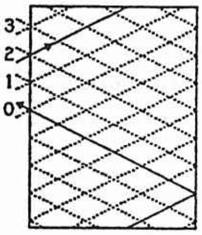
4/6



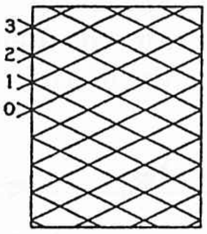
5/7



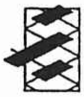
5/7



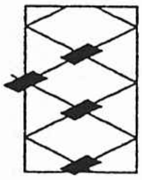
6/8



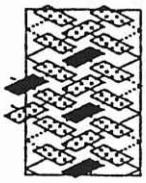
6/8



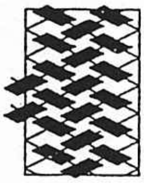
2/3



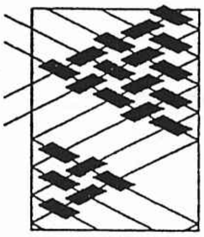
2/3



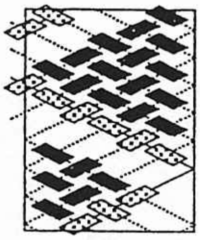
4/6



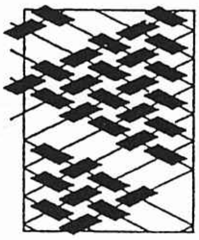
4/6



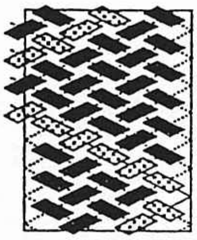
4/6



5/7



5/7



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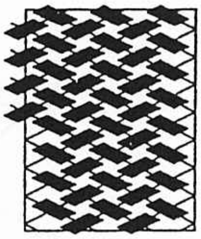
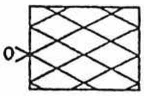


6/8

8/6 END-KEEPER	
0 - 2 - 4 - 0	4/3
1 - 3 - 5 - 1	4/3

← TIF 4/3

4/3 END-KEEPER	
0 - 1 - 2 - 0	4/3



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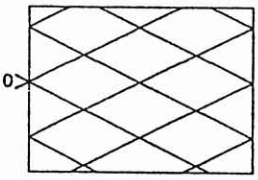

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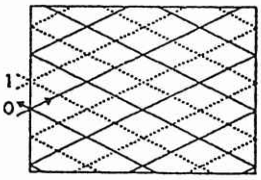

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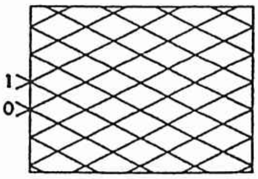

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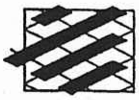

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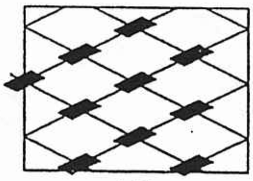

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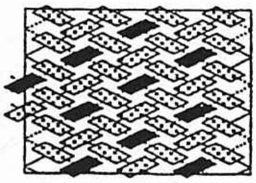

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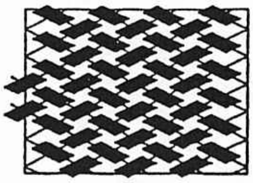
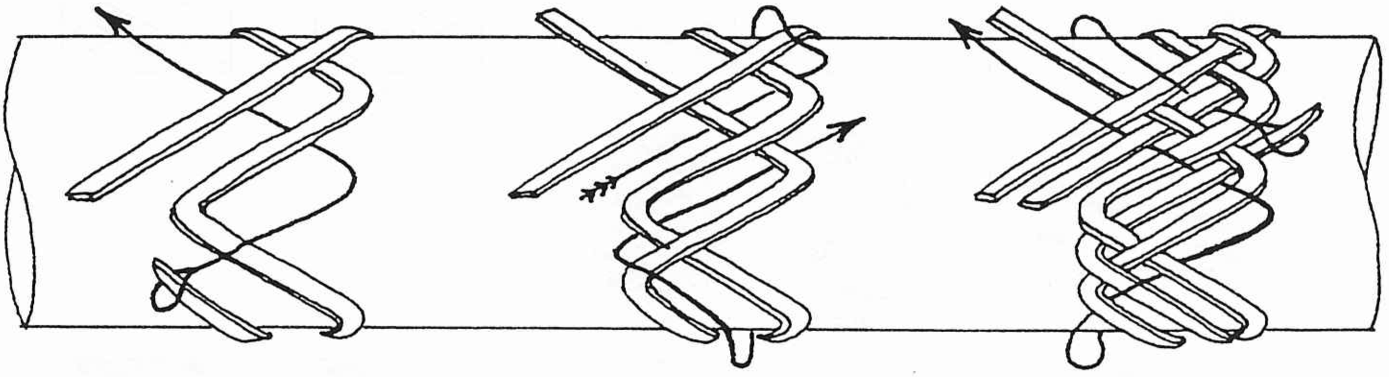


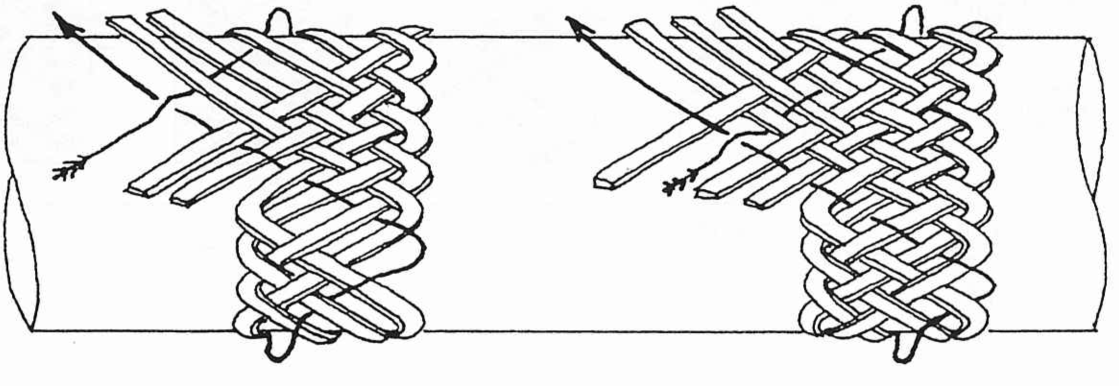
8/6

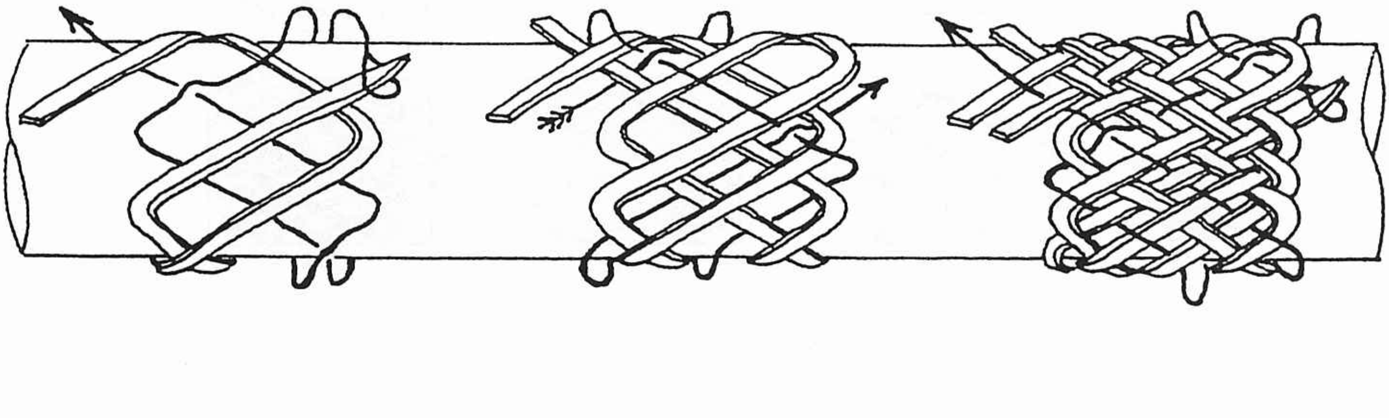
If the TIE II raising process is applied to the 4/6-endkeeper ($x = 2$) instead of the TIE I raising process, we obtain a 5/7-endkeeper ($x = 3$), and by applying once more the TIE II process to this 5/7-endkeeper we obtain a 6/8-endkeeper ($x = 4$). The sequential diagrams are shown on pg. 47 (upper part).

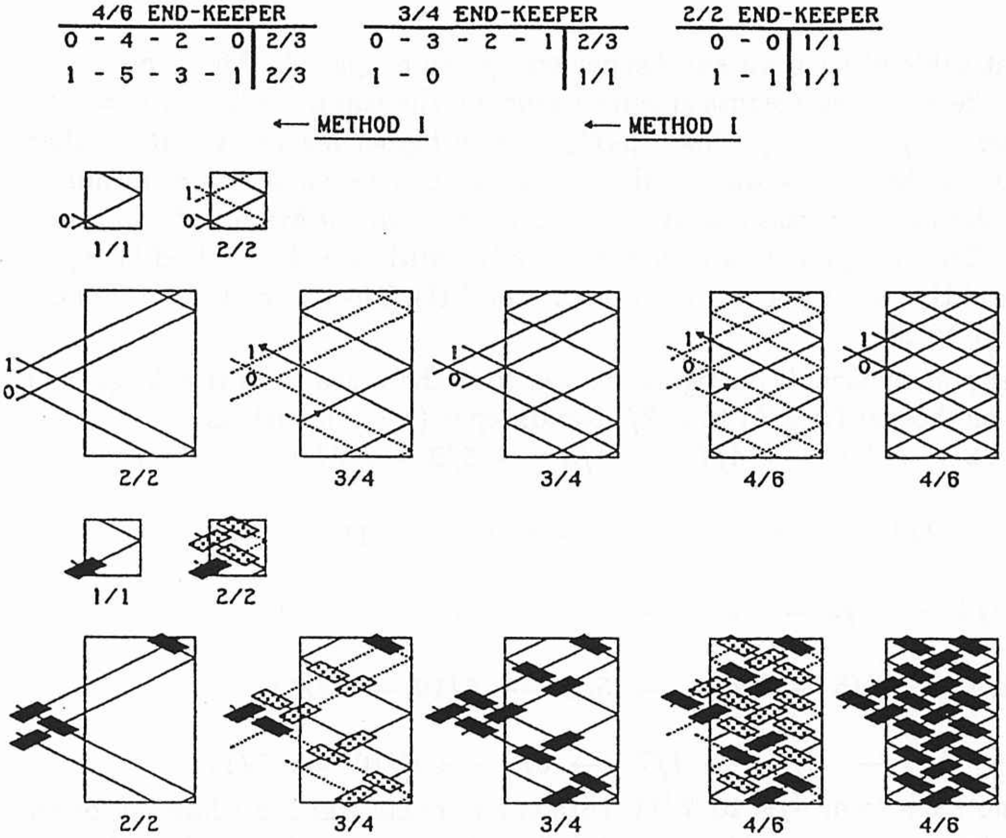
The lower part of pg. 47 shows the sequential diagrams for the construction of a Casa-coded 8/6-endkeeper ($x = 2$) from a Matthew Walker-coded 4/3-endkeeper ($x = 1$) via a TIF 4/3 raising process. The pictorial braiding sequences associated with the construction procedures presented on pg. 47 are shown below.

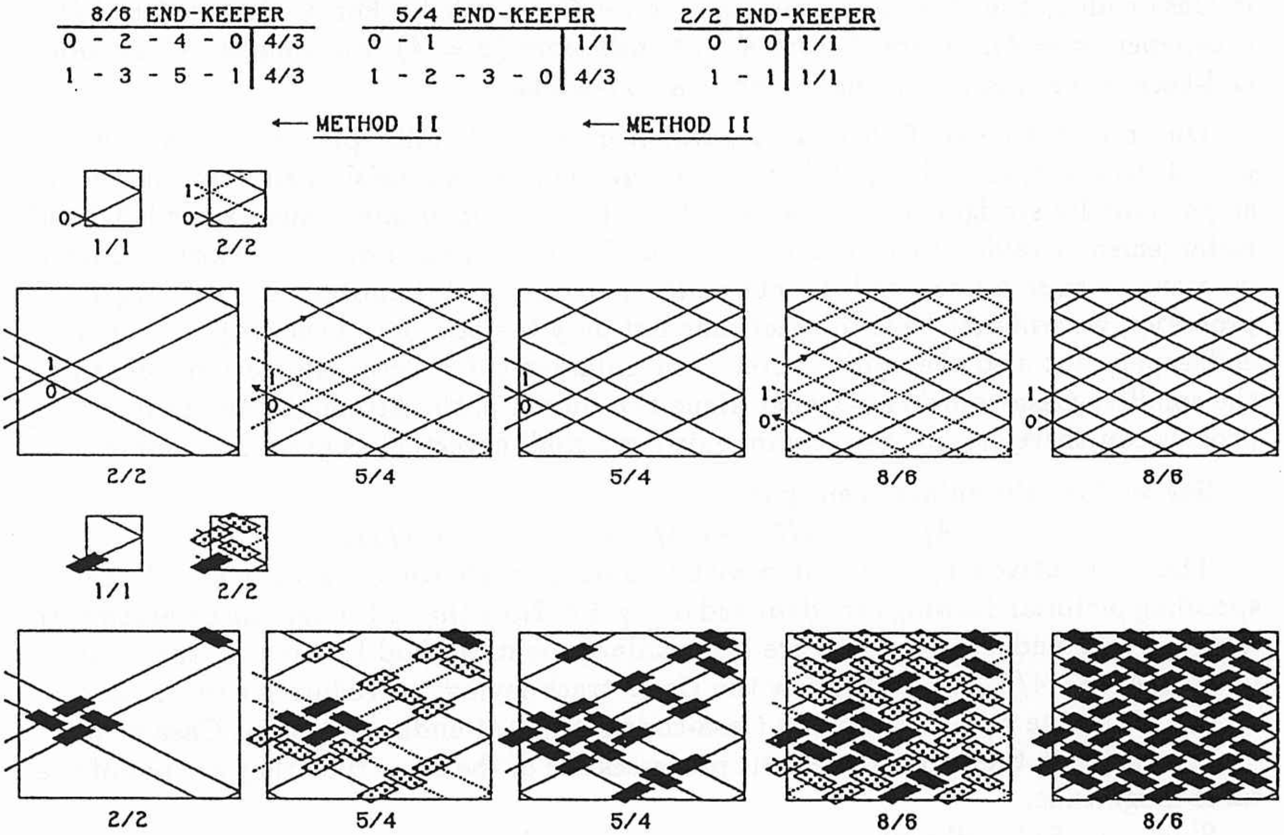
Although such raising processes from Matthew Walker-coded p/b-endkeepers ($x = 1$) to Casa-coded $2p/2b$-endkeepers ($x = 2$) are quick, the beginner may prefer to braid these Casa-coded $2p/2b$-endkeepers ($x = 2$) via two consecutive identical enlargement processes from a 'Casa-coded' 2/2-endkeeper ($x = 2$). For the Casa-coded 4/6 and 8/6-endkeepers ($x = 2$) this construction procedure is shown on pg. 49. Note that the Method I enlargements in the 4/6-endkeeper construction are 2^{nd}-order types, while the Method II enlargements in the 8/6-endkeeper construction are 3^{rd}-order types.








4/6 END-KEEPER
0 - 4 - 2 - 0 | 2/3
1 - 5 - 3 - 1 | 2/3
3/4 END-KEEPER
0 - 3 - 2 - 1 | 2/3
1 - 0 | 1/1
2/2 END-KEEPER
0 - 0 | 1/1
1 - 1 | 1/1
← METHOD I
← METHOD I
1
0
1/1
2/2
2/2
3/4
3/4
4/6
4/6
1/1
2/2
2/2
3/4
3/4
4/6
4/6


8/6 END-KEEPER
0 - 2 - 4 - 0 | 4/3
1 - 3 - 5 - 1 | 4/3
5/4 END-KEEPER
0 - 1 | 1/1
1 - 2 - 3 - 0 | 4/3
2/2 END-KEEPER
0 - 0 | 1/1
1 - 1 | 1/1
← METHOD II
← METHOD II
1
0
1/1
2/2
2/2
5/4
5/4
8/6
8/6
1/1
2/2
2/2
5/4
5/4
8/6
8/6

Example 2 :

An end-keeper with $p/b = 7/11$; 4 transition-points ($x = 4$), hence an 8-part flat braid.

The component-table of such an end-keeper was given on pg. 28. The component-table shows two different parts/bights specifications for the component ($p_1/b_1 = 1/2$ and the components $p_2/b_2 = 2/3$). Since p_2/b_2 is at a higher level in the RKT than p_1/b_1, it follows that the string-run of this end-keeper can be enlarged by means of one Enlargement Method I process and two different Enlargement Method II processes (see pgs. 92—95). The resulting enlargements are $8/13$ with $x = 4$ (Method I); $8/12$ with $x = 4$ (Method II), and $9/14$ with $x = 4$ (Method II). The associated component tables are shown on pg. 28.

From the Enlargement Graph on pg. 18 we see that there are several enlargement paths from a 4/4-endkeeper ($x = 4$) to a 7/11-endkeeper ($x = 4$), such as

$$4/4 \longrightarrow 4/5 \longrightarrow 4/6 \longrightarrow 4/7 \longrightarrow 4/8 \longrightarrow 5/9 \longrightarrow 6/10 \longrightarrow 7/11$$

or

$$4/4 \longrightarrow 4/5 \longrightarrow 5/7 \longrightarrow 6/9 \longrightarrow 7/11$$

or

$$4/4 \longrightarrow 4/5 \longrightarrow 4/6 \longrightarrow 5/7 \longrightarrow 6/9 \longrightarrow 7/11$$

or

$$4/4 \longrightarrow 4/5 \longrightarrow 4/6 \longrightarrow 5/8 \longrightarrow 6/10 \longrightarrow 7/11$$

or

$$4/4 \longrightarrow 4/5 \longrightarrow 4/6 \longrightarrow 4/7 \longrightarrow 5/9 \longrightarrow 6/10 \longrightarrow 7/11.$$

Since there is no path from 4/4 to 7/11 with components which **all** have an even number of steps along their straight lines, it follows that when the 7/11-endkeeper is Casa-coded, the 4/4-endkeeper cannot be Casa-coded. For a Casa-coded 7/11-endkeeper ($x = 4$), enlarged from a 4/4-endkeeper ($x = 4$), the smallest Casa-coded end-keeper is either a 5/9-endkeeper or a 5/7-endkeeper.

One might wonder if there is a path, from a $p^*/4$-endkeeper ($x = 4$ and where $p^* < 4$) to the 7/11-endkeeper, in which **each** component consist of an even number of steps along its straight lines. However, from the combined minor and major left-hand Enlargement Graphs it can readily be seen that such a path does not exist. Hence if we want to raise a Casa-coded 7/11-endkeeper from a 4/4-endkeeper by enlargement processes, we will first have to determine not only the appropriate coding for this 4/4-endkeeper, but also the coding after each enlargement process till we have obtained the smallest Casa-coded end-keeper along the chosen path. Although this construction process can therefore not be recommended, we shall nevertheless give an example.

Say we take the enlargement-path:

$$4/4 \longrightarrow 4/5 \longrightarrow 5/7 \longrightarrow 6/9 \longrightarrow 7/11.$$

The consecutive steps associated with this path are shown on pg. 51, while the corresponding pictorial drawings are depicted on pg. 52. Note that although the enlargements $4/4 \longrightarrow 4/5$ and $4/5 \longrightarrow 5/7$ are both Enlargement Method I processes, the enlargement $4/4 \longrightarrow 4/5$ does not follow the Casa 'track-laying' procedure for two reasons:

(1) While the 5/7-endkeeper is Casa-coded, the 4/4-endkeeper is not Casa-coded.

(2) Although the two enlargement processes are of the same type they are not of the same magnitude.

Since the 5/7-endkeeper is Casa-coded, the enlargement $4/5 \longrightarrow 5/7$ is a Casa 'track-splitting' procedure.

7/11 END-KEEPER	
0 - 7 - 3	1/2
1 - 8 - 4 - 0	2/3
2 - 9 - 5 - 1	2/3
3 - 10 - 6 - 2	2/3

← METHOD I

6/9 END-KEEPER	
0 - 6 - 3	1/2
1 - 7 - 4 - 1	2/3
2 - 8 - 5 - 2	2/3
3 - 0	1/1

← METHOD I

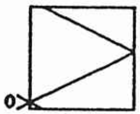
5/7 END-KEEPER	
0 - 5 - 3	1/2
1 - 6 - 4 - 2	2/3
2 - 0	1/1
3 - 1	1/1

← METHOD I

4/5 END-KEEPER	
0 - 4 - 3	1/2
1 - 0	1/1
2 - 1	1/1
3 - 2	1/1

← METHOD I

4/4 END-KEEPER	
0 - 0	1/1
1 - 1	1/1
2 - 2	1/1
3 - 3	1/1



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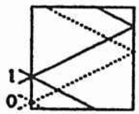
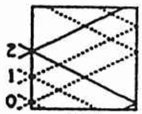


2/2



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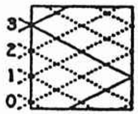
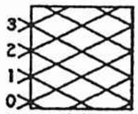
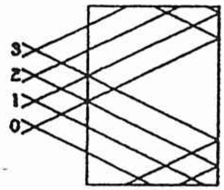


4/4



4/4



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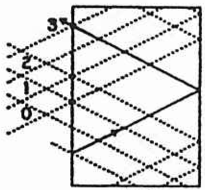
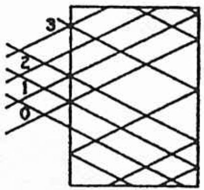
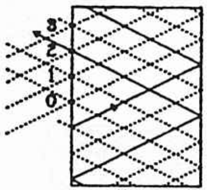


4/5



4/5



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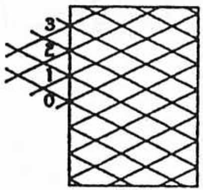

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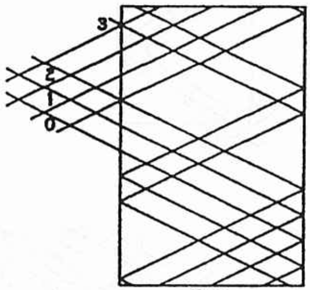

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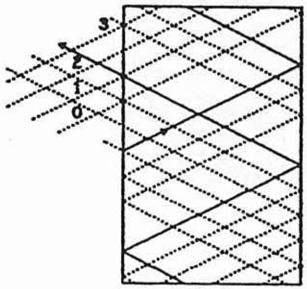
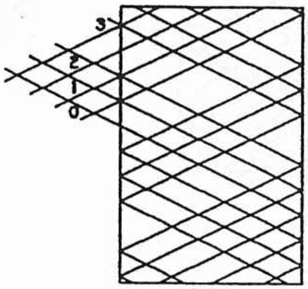


6/9



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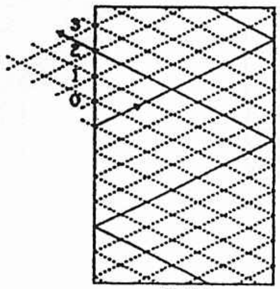

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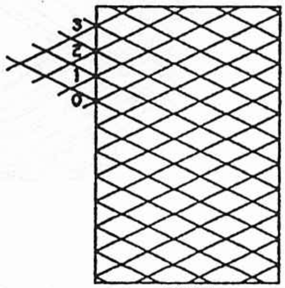

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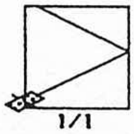
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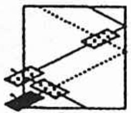
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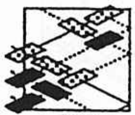
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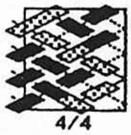
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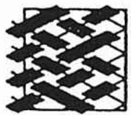
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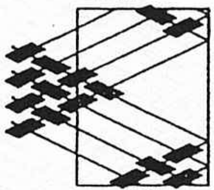
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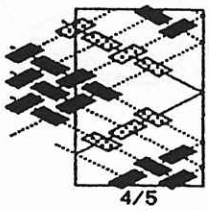
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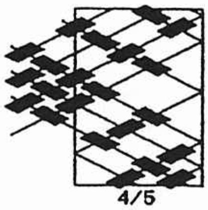
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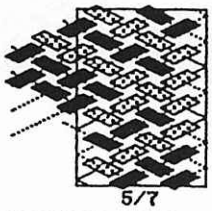
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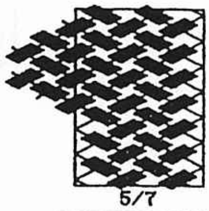
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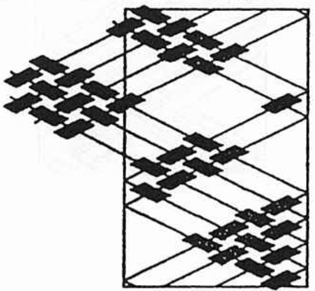
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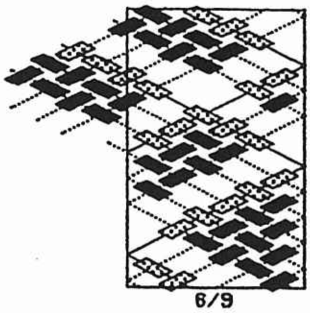
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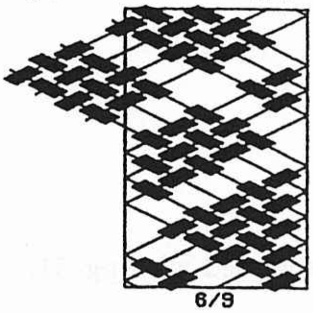
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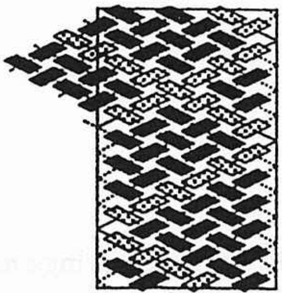
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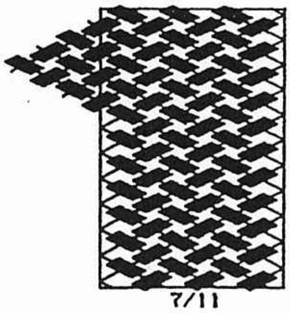
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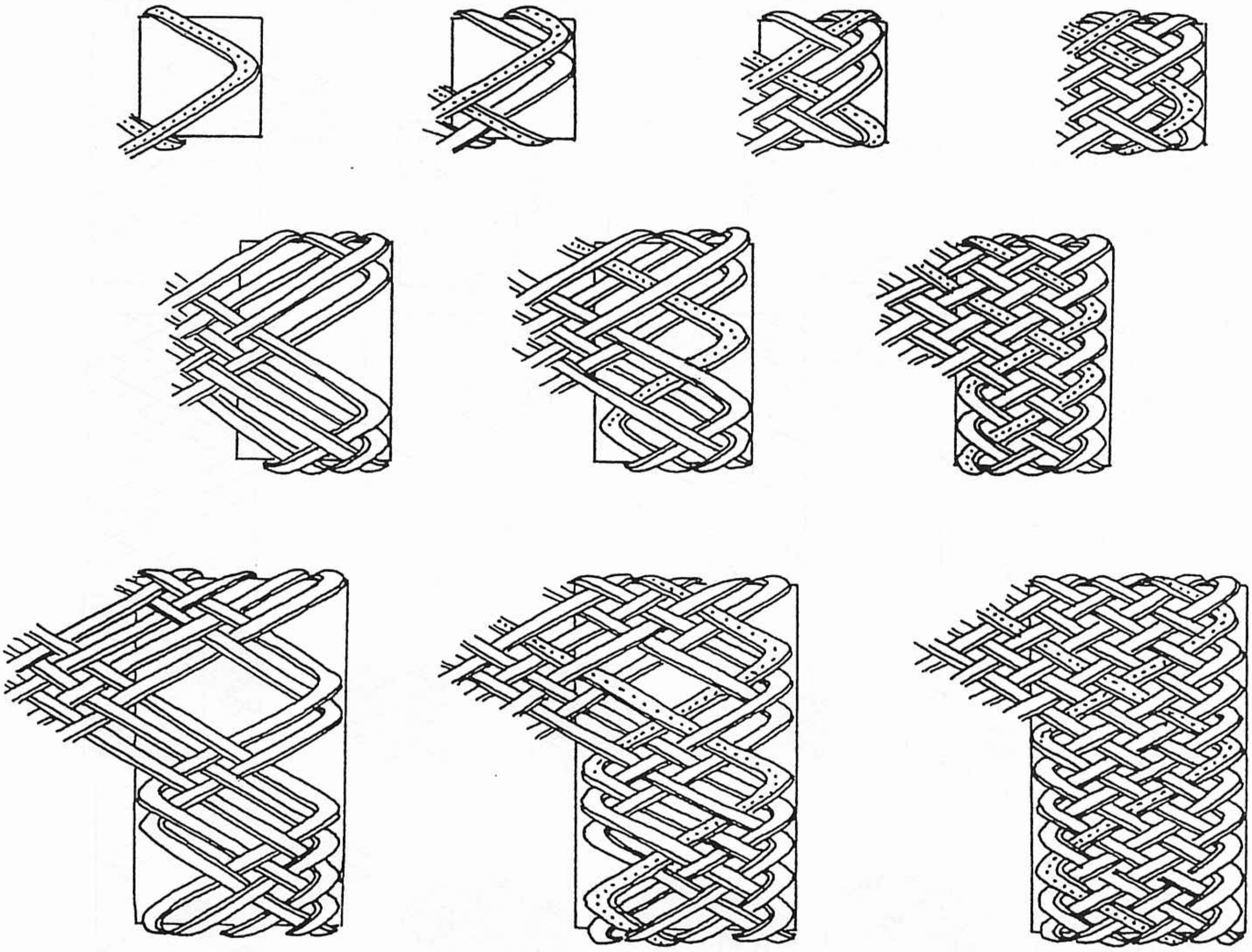


Pictorial drawings associated with pg. 51.

When there is no enlargement-path (from the trunk x/x or node p^*/x where $p^* < x$) to the desired node (p/b with x transition-points) with an even number of steps along each of its straight-line components, then it is much more convenient to use for the construction of such a Casa-coded p/b-endkeeper, at least initially, a 'TIE' or 'TIF' Casa 'track-laying' and 'track-splitting' raising processes with as starting point a Casa-coded end-keeper.

For a Casa-coded 7/11-endkeeper, such a raising process is shown on pgs. 54 and 55 with the corresponding pictorial drawings on pg. 56.

We start with a 1/1-endkeeper, and enlarge it with Enlargement Method I (2^{nd}-order) Casa 'track-laying' procedure. Then we follow with a TIE Method I raising process using the Casa 'track-splitting' procedure. The result is a Casa-coded 3/5-endkeeper ($x = 2$).

Next, we employ TIF 1/2 using the Casa 'track-laying' procedure, followed by a TIF 1/2 Casa 'track-splitting' procedure. The result is a Casa-coded 5/9-endkeeper ($x = 4$). We have already mentioned earlier that this is one of the two smallest $x = 4$ Casa-coded end-keepers which can be raised to a Casa-coded 7/11-endkeeper ($x = 4$) by Enlargement processes.

We have seen earlier on pgs. 13 and 14 that a 'TIF' raising process can be accomplished in two ways. Both these ways are shown on pgs. 54 and 55.

The reader should make sure to be conversant with tying small Casa-coded end-keepers with $x = 2$ on the hand, the same as he is undoubtedly conversant with tying small Casa-coded Regular Knots (single string cylindrical braids) on the hand. When able to tie a Casa-coded 3/5-endkeeper with $x = 2$ on the hand (see pg. 57), he or she can braid the Casa-coded 7/11-endkeeper much quicker, by omitting the $1/1\ (x = 1) \longrightarrow 2/3\ (x = 1) \longrightarrow 3/5\ (x = 2)$ steps in the above outlined braiding process.

We can further shorten this braiding process with raising the 3/5-endkeeper ($x = 2$) by the TIF 2/3 Casa 'track-laying' procedure, followed by the TIF 2/3 Casa 'track-splitting' procedure. These two steps may be found in the construction process $1/1\ (x = 1) \longrightarrow 2/3\ (x = 1) \longrightarrow 3/5\ (x = 2) \longrightarrow 5/8\ (x = 3) \longrightarrow 7/11\ (x = 4)$ on pgs. 58 and 59, with its associated pictorial drawings on pg. 60.

The construction process on pg. 61 (with its associated pictorial drawings on pg. 62) starts with a Casa-coded 3/3-endkeeper ($x = 3$) to which the Casa 'track-laying' Enlargement Method I (1^{st}-order) is applied. This is followed by the Casa 'track-splitting' Enlargement Method I (1^{st}-order). The result is a 3/5-endkeeper ($x = 3$). On this end-keeper we use the Casa 'track-laying' Enlargement Method II (1^{st}-order), followed by the Casa 'track-splitting' TIE II (1^{st}-order). This results in a Casa-coded 5/7-endkeeper ($x = 4$) which can be raised to the Casa-coded 7/11-endkeeper ($x = 4$) by 2^{nd}-order Method I Enlargements as we already have seen on pgs. 51 and 52.

The raising process from the Casa-coded 3/3-endkeeper ($x = 3$) to the Casa-coded 5/7-endkeeper ($x = 4$) can be shortened by using the Casa 'track-laying' Enlargement Method I (2^{nd}-order) on the Casa-coded 3/3-endkeeper, followed by the Casa 'track-splitting' TIE I (2^{nd}-order). This procedure is presented on pg. 63. The associated pictorial drawings for the consecutive construction steps can be found on pg. 64.

7/11 END-KEEPER	
0 - 7 - 3	1/2
1 - 8 - 4 - 0	2/3
2 - 9 - 5 - 1	2/3
3 - 10 - 6 - 2	2/3

6/10 END-KEEPER	
0 - 6 - 2	1/2
1 - 7 - 3	1/2
2 - 8 - 4 - 0	2/3
3 - 9 - 5 - 1	2/3

5/9 END-KEEPER	
0 - 5 - 1	1/2
1 - 6 - 2	1/2
2 - 7 - 3	1/2
3 - 8 - 4 - 0	2/3

4/7 END-KEEPER	
0 - 4 - 1	1/2
1 - 5 - 2	1/2
2 - 6 - 3 - 0	2/3

← METHOD II ← METHOD II ← TIF 1/2

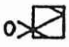
3/5 END-KEEPER	
0 - 3 - 1	1/2
1 - 4 - 2 - 0	2/3

2/3 END-KEEPER	
0 - 2 - 1 - 0	2/3

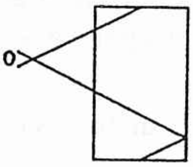
1/1 END-KEEPER	
0 - 0	1/1

← METHOD I

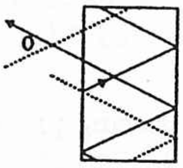
← TIF 1/2 ← TIE METHOD I



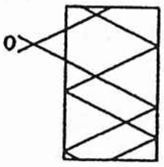
1/1



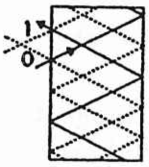
1/1



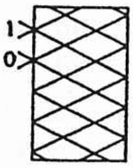
2/3



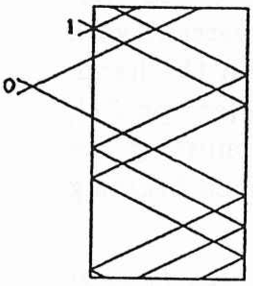
2/3



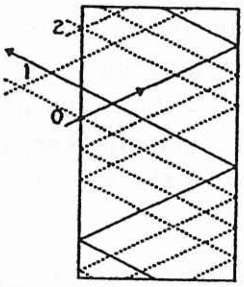
3/5



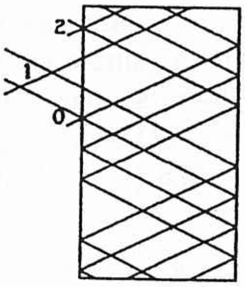
3/5



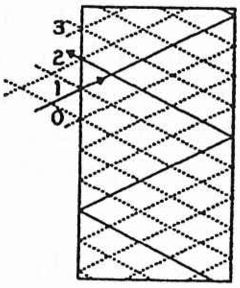
3/5



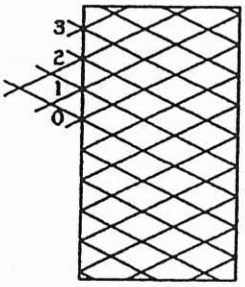
4/7



4/7



5/9



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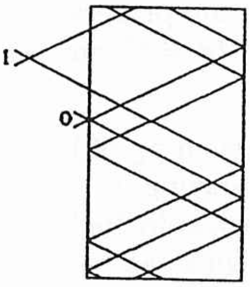

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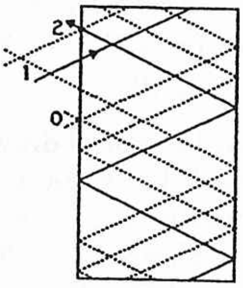

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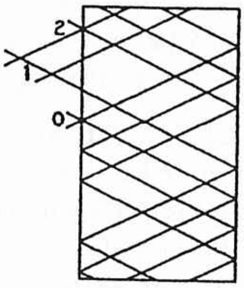

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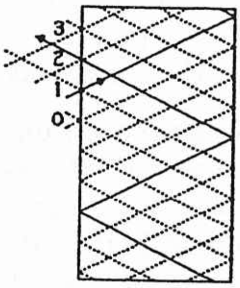

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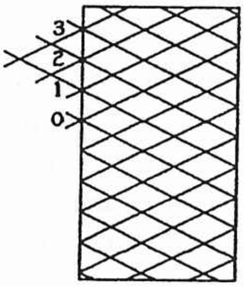

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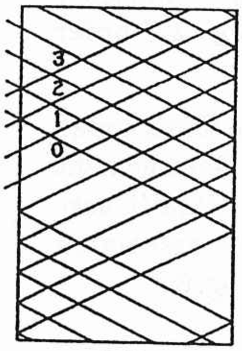

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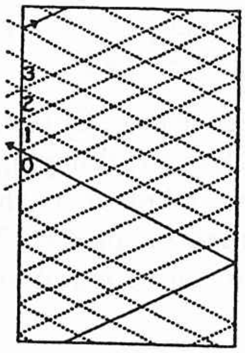

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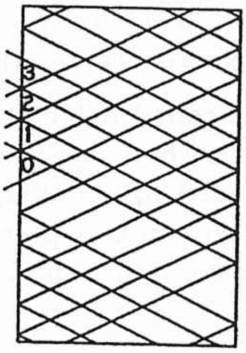
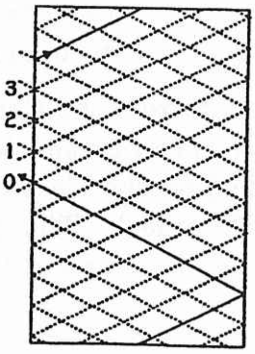
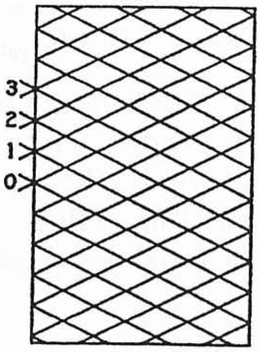


6/10



7/11



7/11

7/11 END-KEEPER

0 - 7 - 3	1/2
1 - 8 - 4 - 0	2/3
2 - 9 - 5 - 1	2/3
3 - 10 - 6 - 2	2/3

← **METHOD II**

6/10 END-KEEPER

0 - 6 - 2	1/2
1 - 7 - 3	1/2
2 - 8 - 4 - 0	2/3
3 - 9 - 5 - 1	2/3

← **METHOD II**

5/9 END-KEEPER

0 - 5 - 1	1/2
1 - 6 - 2	1/2
2 - 7 - 3	1/2
3 - 8 - 4 - 0	2/3

← **TIF 1/2**

4/7 END-KEEPER

0 - 4 - 1	1/2
1 - 5 - 2	1/2
2 - 6 - 3 - 0	2/3

3/5 END-KEEPER

0 - 3 - 1	1/2
1 - 4 - 2 - 0	2/3

2/3 END-KEEPER

0 - 2 - 1 - 0	2/3

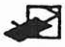
1/1 END-KEEPER

0 - 0	1/1

← **METHOD I**

← **TIF 1/2**

← **TIE METHOD I**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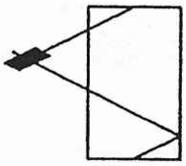
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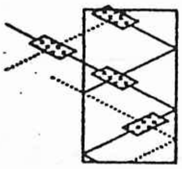
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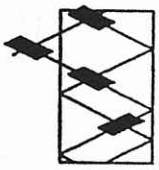
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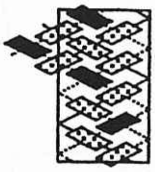
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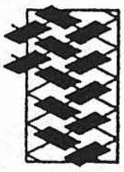
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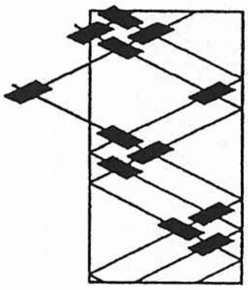
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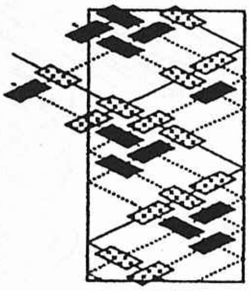
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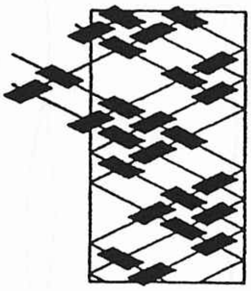
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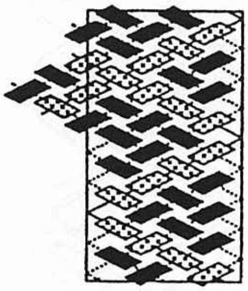
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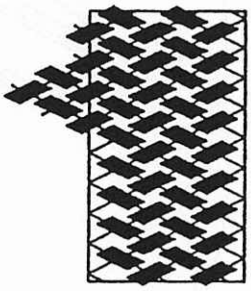
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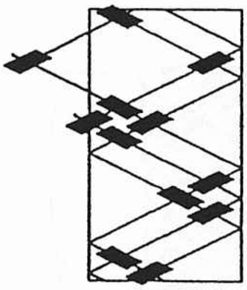
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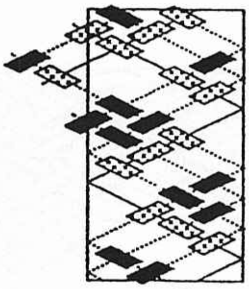
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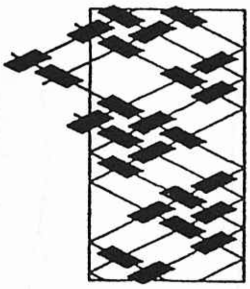
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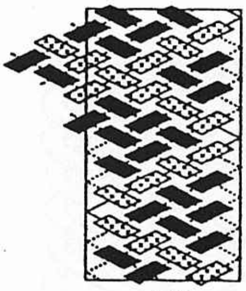
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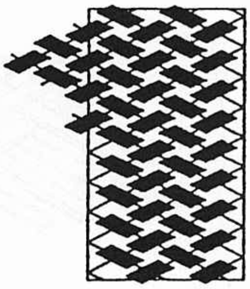
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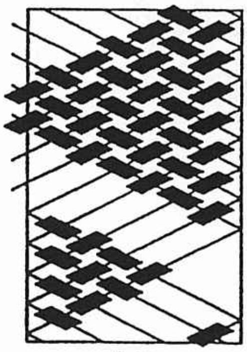
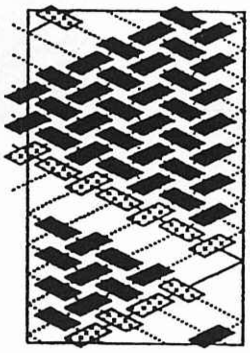
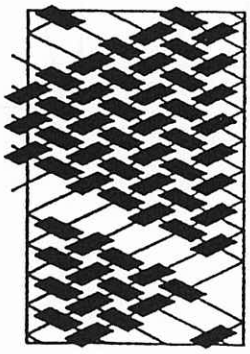

5/9


6/10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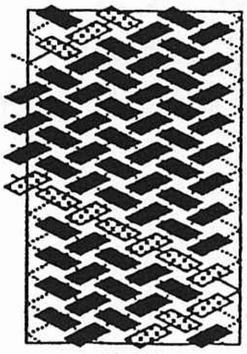
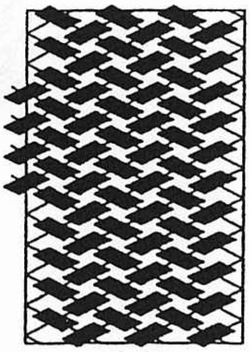
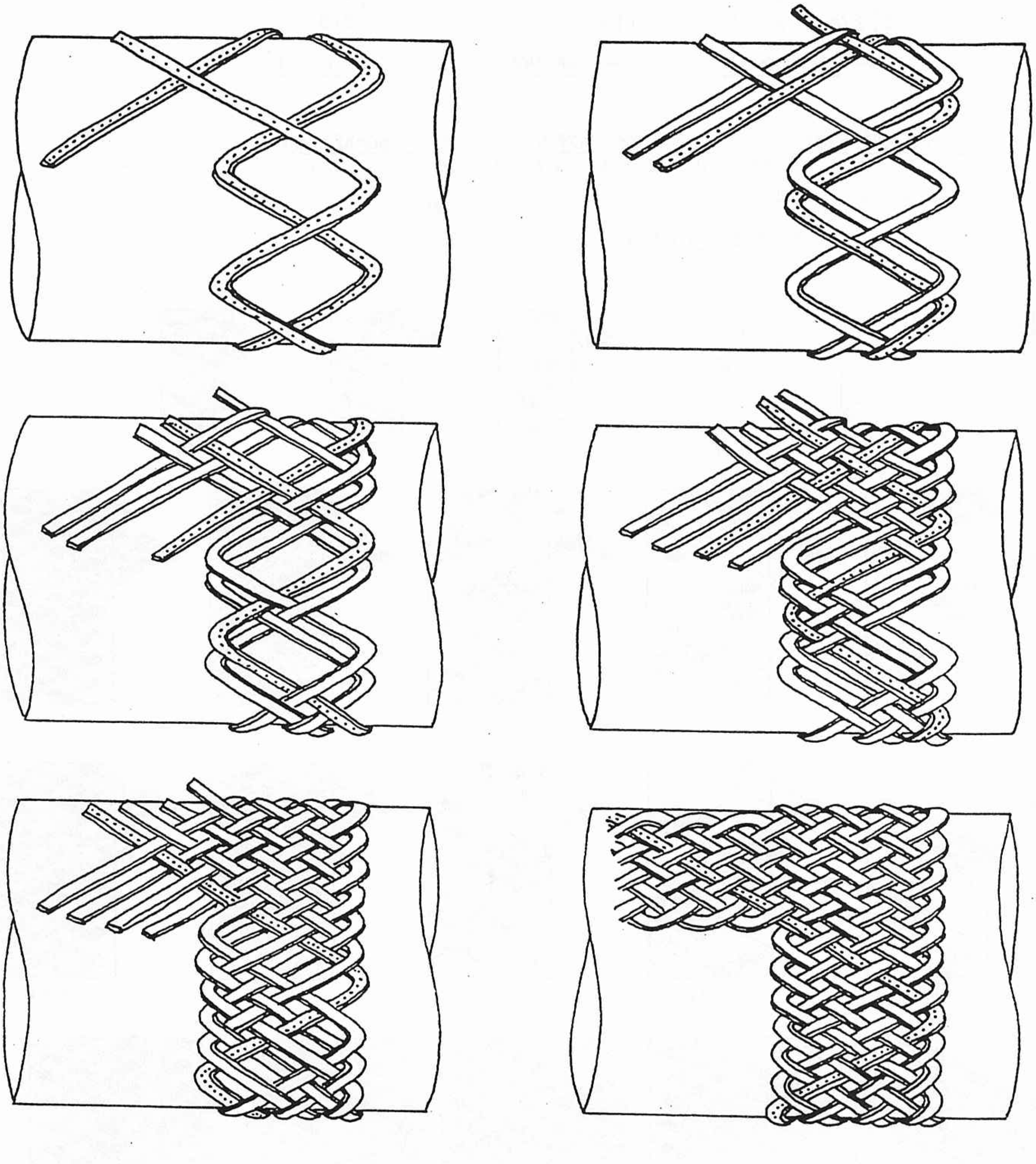
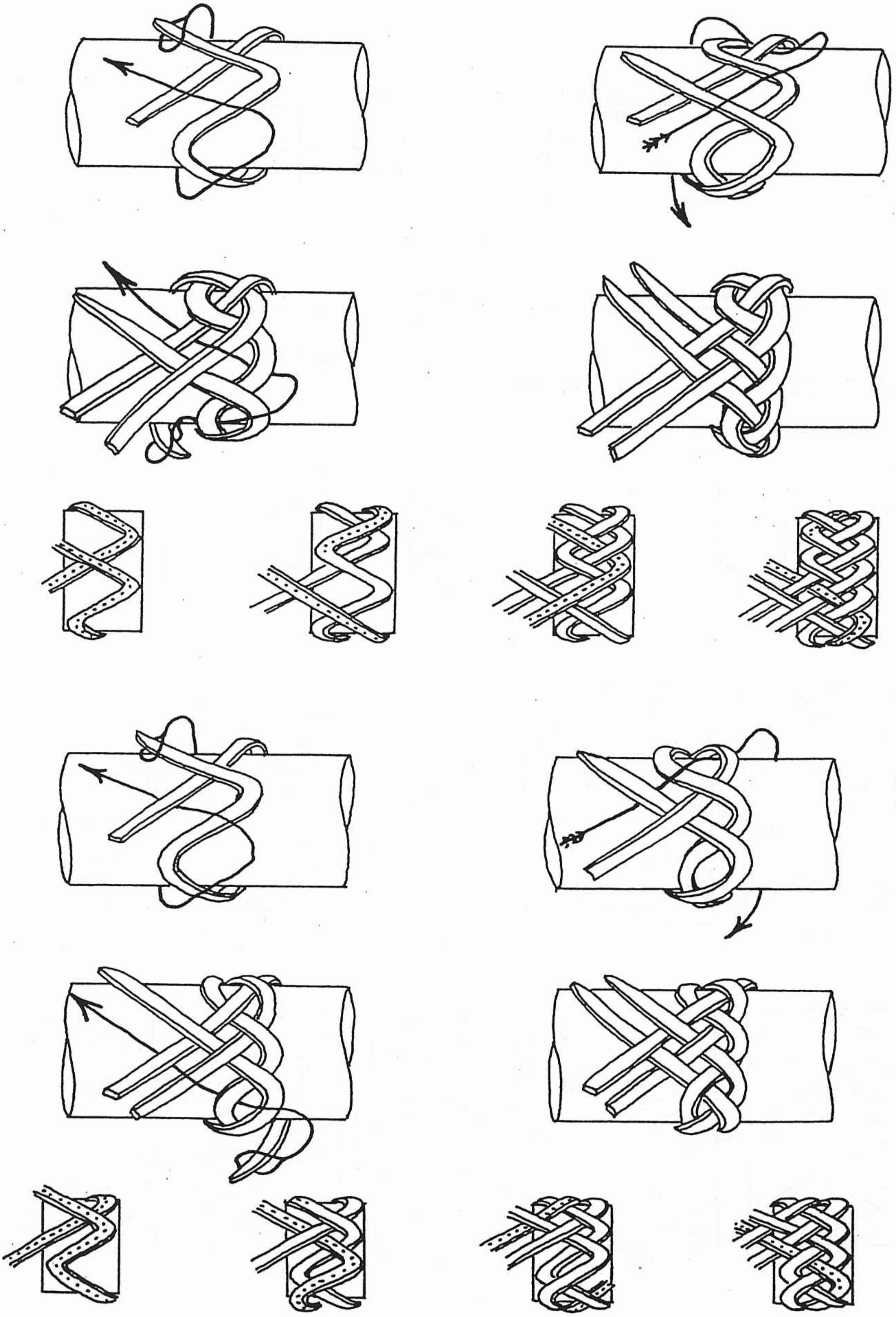

7/11


7/11



Pictorial drawings associated with pg. 55.



Tying the Casa-coded 3/5 end-keeper ($x = 2$) on the hand.

7/11 END-KEEPER

0 - 7 - 3	1/2
1 - 8 - 4 - 0	2/3
2 - 9 - 5 - 1	2/3
3 - 10 - 6 - 2	2/3

5/8 END-KEEPER

0 - 5 - 2	1/2
1 - 6 - 3 - 0	2/3
2 - 7 - 4 - 1	2/3

← TIF 2/3

3/5 END-KEEPER

0 - 3 - 1	1/2
1 - 4 - 2 - 0	2/3

2/3 END-KEEPER

0 - 2 - 1 - 0	2/3

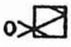
1/1 END-KEEPER

0 - 0	1/1

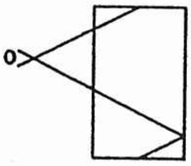
← METHOD I

← TIF 2/3

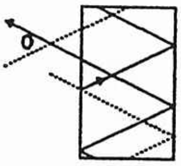
← TIE METHOD I



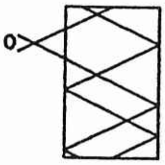
1/1



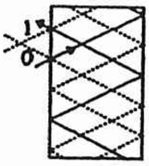
1/1



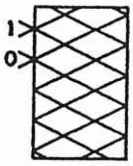
2/3



2/3



3/5



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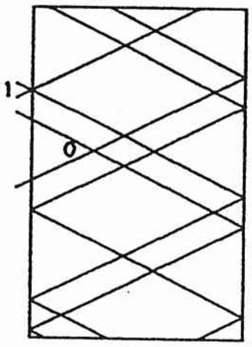

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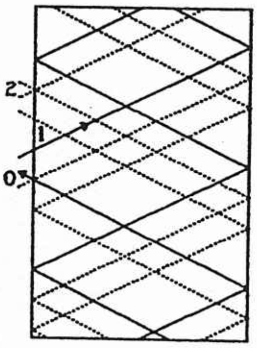

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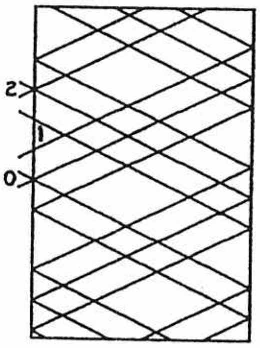

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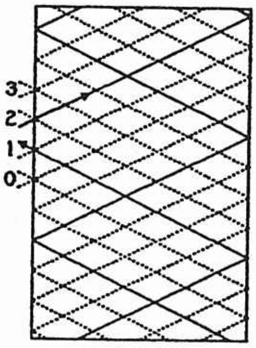

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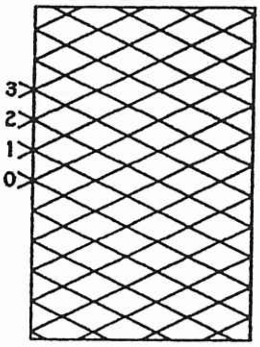

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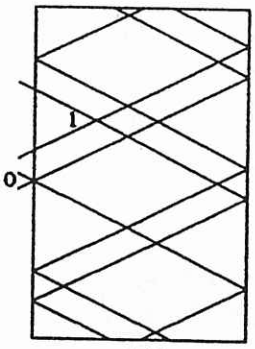

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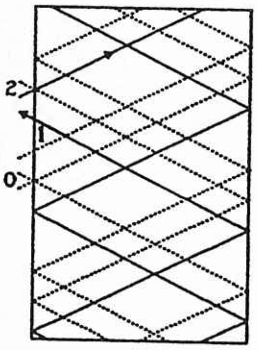
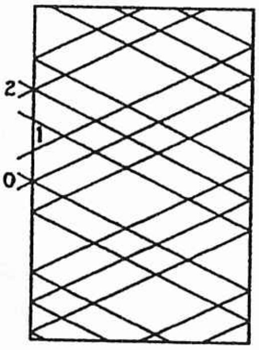
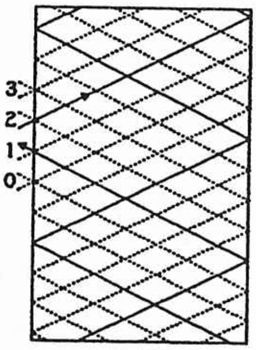


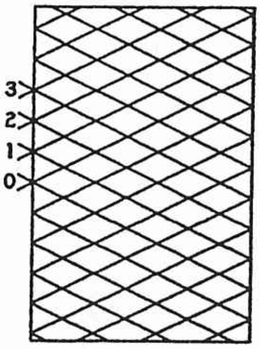
5/8



5/8



7/11



7/11

7/11 END-KEEPER

0 - 7 - 3	1/2
1 - 8 - 4 - 0	2/3
2 - 9 - 5 - 1	2/3
3 - 10 - 6 - 2	2/3

5/8 END-KEEPER

0 - 5 - 2	1/2
1 - 6 - 3 - 0	2/3
2 - 7 - 4 - 1	2/3

← TIF 2/3

3/5 END-KEEPER

0 - 3 - 1	1/2
1 - 4 - 2 - 0	2/3

2/3 END-KEEPER

0 - 2 - 1 - 0	2/3

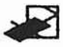
1/1 END-KEEPER

0 - 0	1/1

← METHOD I

← TIF 2/3

← TIE METHOD I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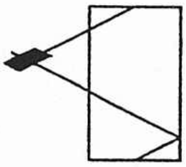
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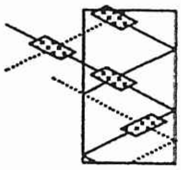
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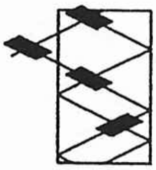
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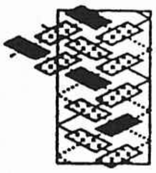
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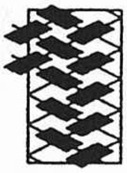
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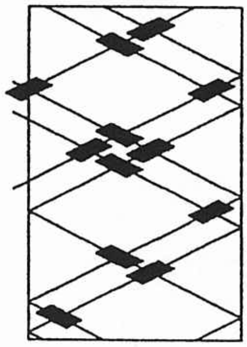
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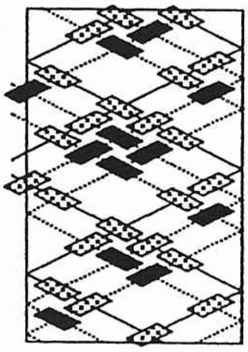
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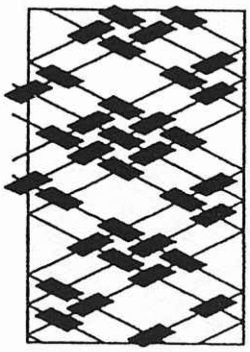
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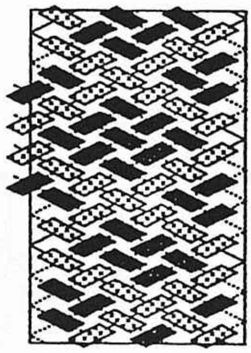
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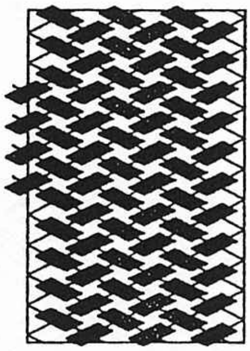
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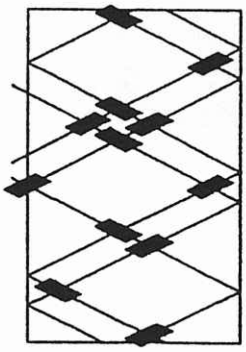
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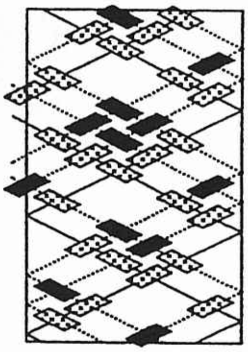
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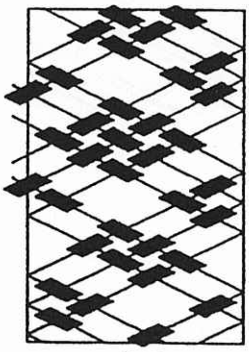
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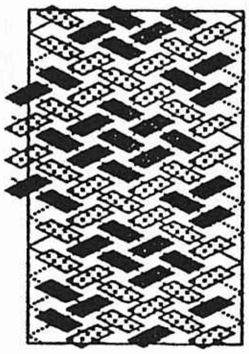
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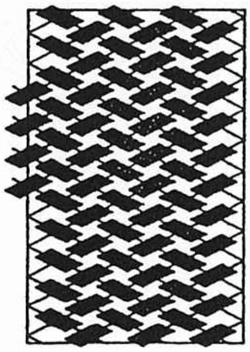
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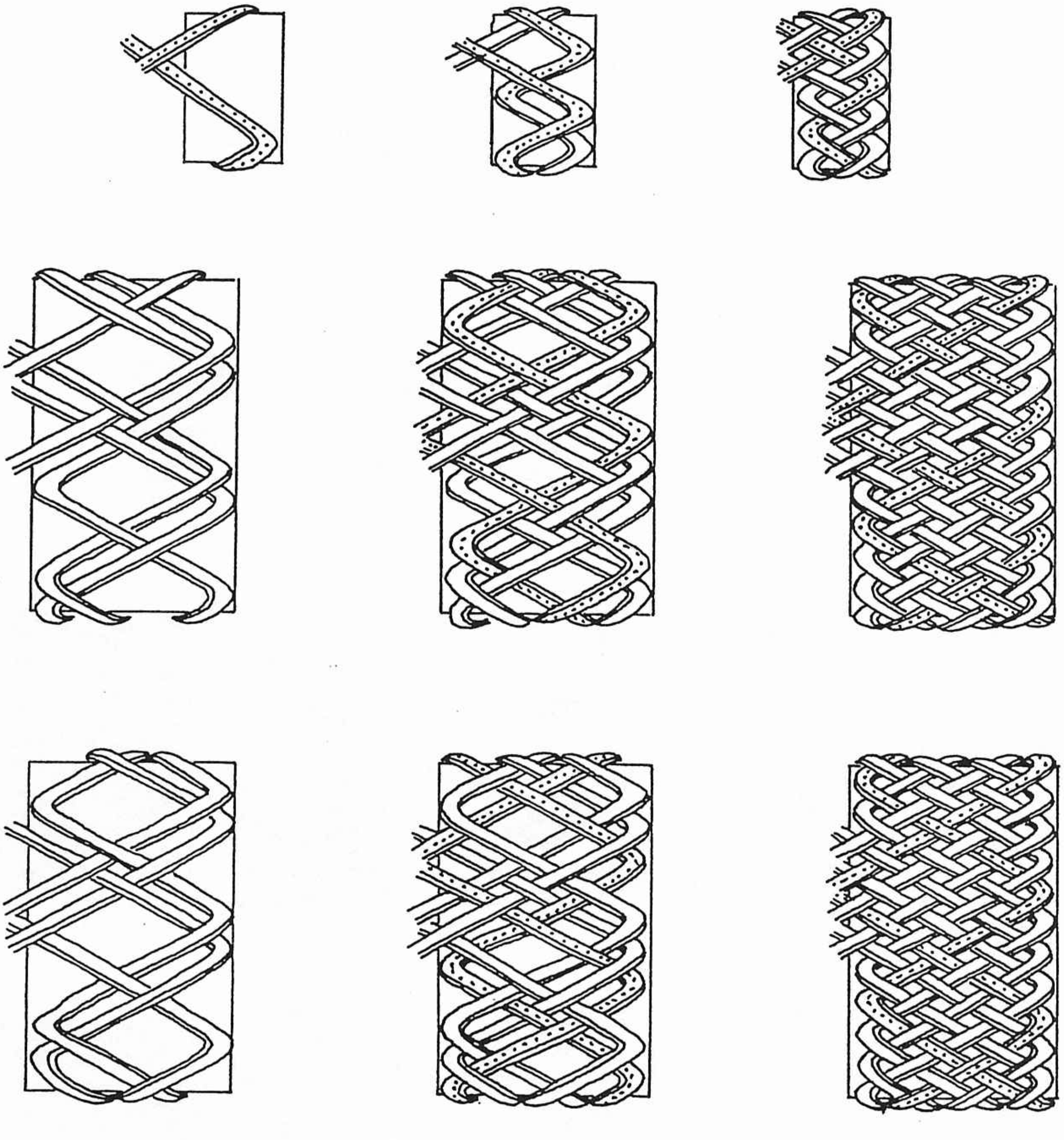


Pictorial drawings associated with pg. 59.

7/11 END-KEEPER	
0 - 7 - 3	1/2
1 - 8 - 4 - 0	2/3
2 - 9 - 5 - 1	2/3
3 - 10 - 6 - 2	2/3

6/9 END-KEEPER	
0 - 6 - 3	1/2
1 - 7 - 4 - 1	2/3
2 - 8 - 5 - 2	2/3
3 - 0	1/1

5/7 END-KEEPER	
0 - 5 - 3	1/2
1 - 6 - 4 - 2	2/3
2 - 0	1/1
3 - 1	1/1

4/6 END-KEEPER	
0 - 4 - 2	1/2
1 - 5 - 3 - 1	2/3
2 - 0	1/1

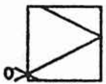
3/5 END-KEEPER	
0 - 3 - 1	1/2
1 - 4 - 2	1/2
2 - 0	1/1

← METHOD I ← METHOD I ← TIE II ← METHOD II

3/4 END-KEEPER	
0 - 3 - 2	1/2
1 - 0	1/1
2 - 1	1/1

3/3 END-KEEPER	
0 - 0	1/1
1 - 1	1/1
2 - 2	1/1

← METHOD I ← METHOD I



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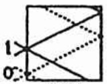

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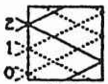

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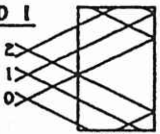

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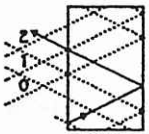

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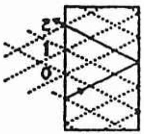

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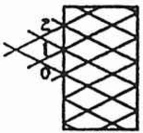

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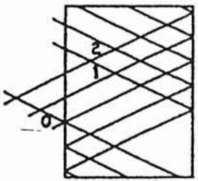

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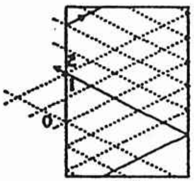

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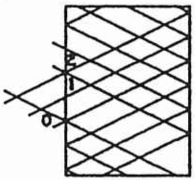

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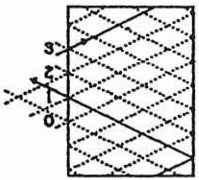

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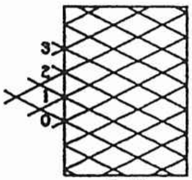

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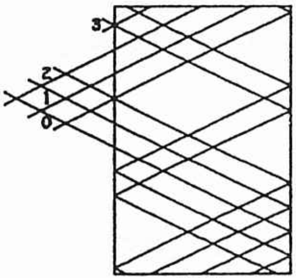

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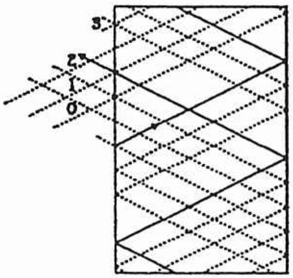
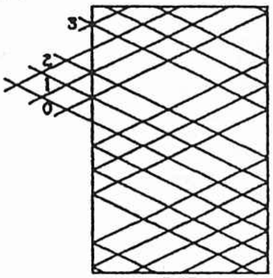
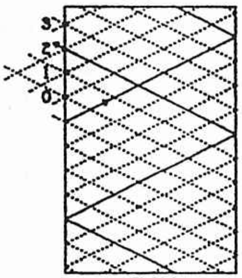


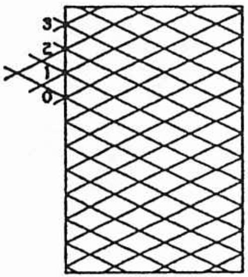
6/9



6/9



7/11



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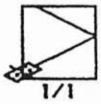
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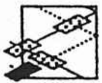
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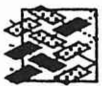
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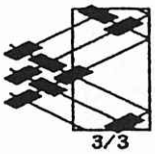
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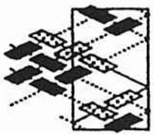
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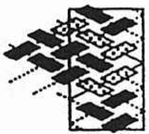
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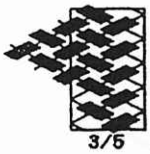
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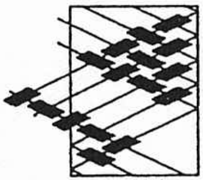
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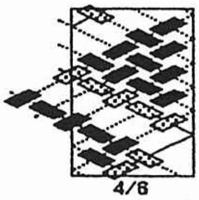
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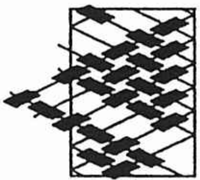
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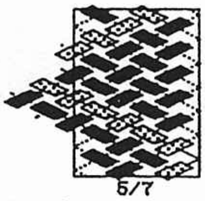
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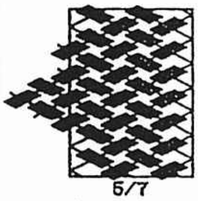
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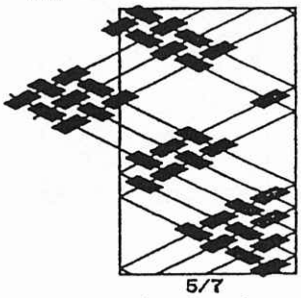
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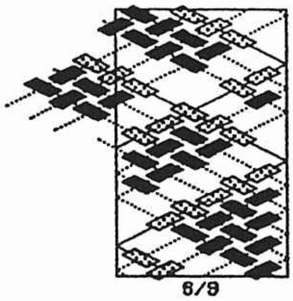
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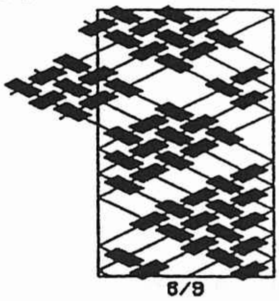
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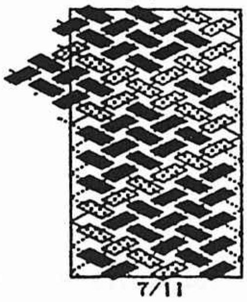
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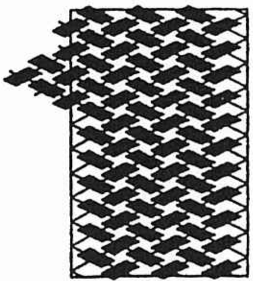
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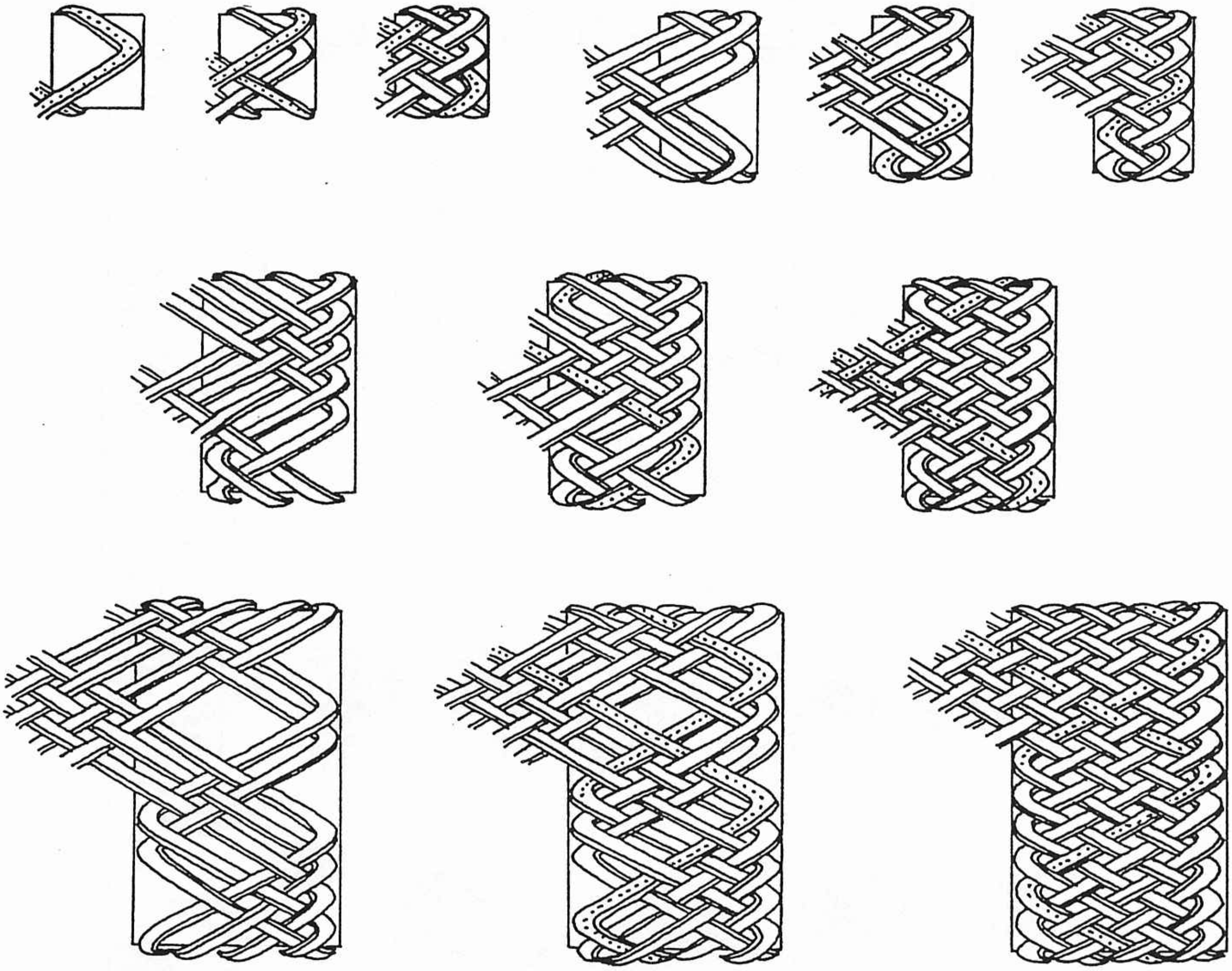


Pictorial drawings associated with pg. 61.

7/11 END-KEEPER	
0 - 7 - 3	1/2
1 - 8 - 4 - 0	2/3
2 - 9 - 5 - 1	2/3
3 - 10 - 6 - 2	2/3

← METHOD I

6/9 END-KEEPER	
0 - 6 - 3	1/2
1 - 7 - 4 - 1	2/3
2 - 8 - 5 - 2	2/3
3 - 0	1/1

← METHOD I

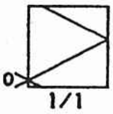
5/7 END-KEEPER	
0 - 5 - 3	1/2
1 - 6 - 4 - 2	2/3
2 - 0	1/1
3 - 1	1/1

← TIE I

4/5 END-KEEPER	
0 - 4 - 3 - 2	2/3
1 - 0	1/1
2 - 1	1/1

3/3 END-KEEPER	
0 - 0	1/1
1 - 1	1/1
2 - 2	1/1

← METHOD I



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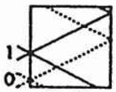

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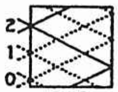

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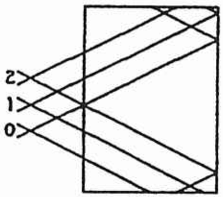

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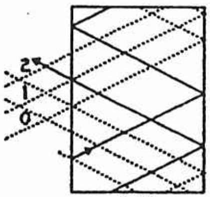

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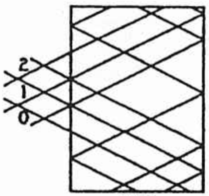

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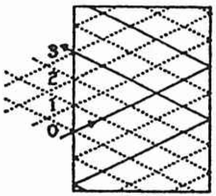

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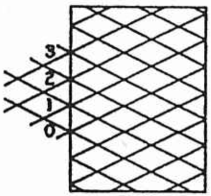

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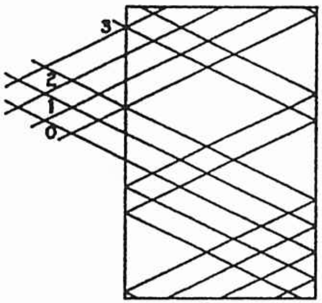

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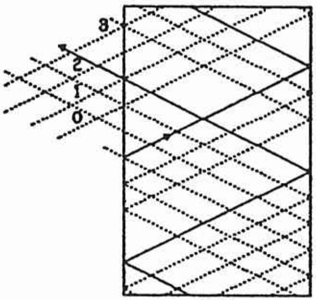

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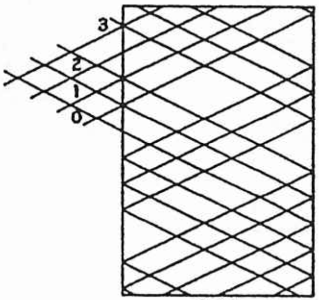

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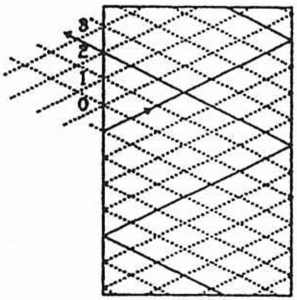
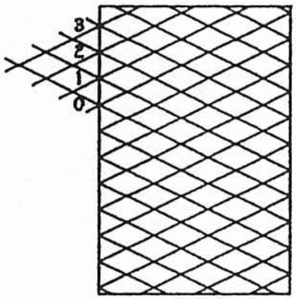


7/11



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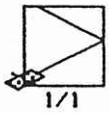
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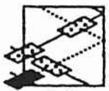
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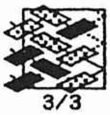
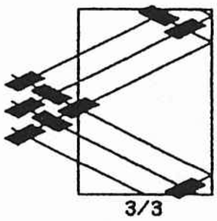

3/3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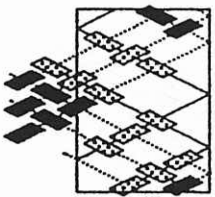
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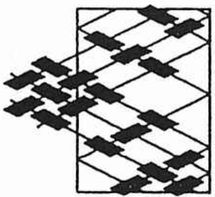
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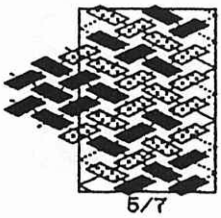
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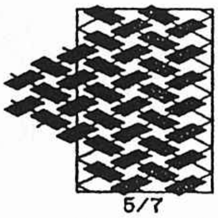
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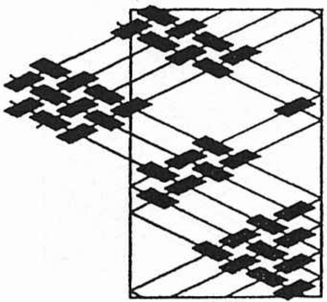
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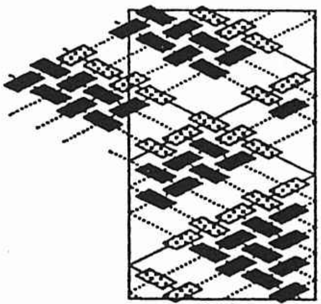
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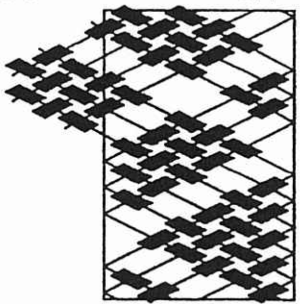
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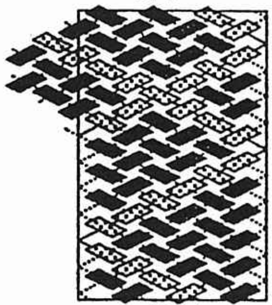
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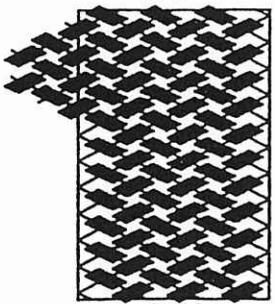
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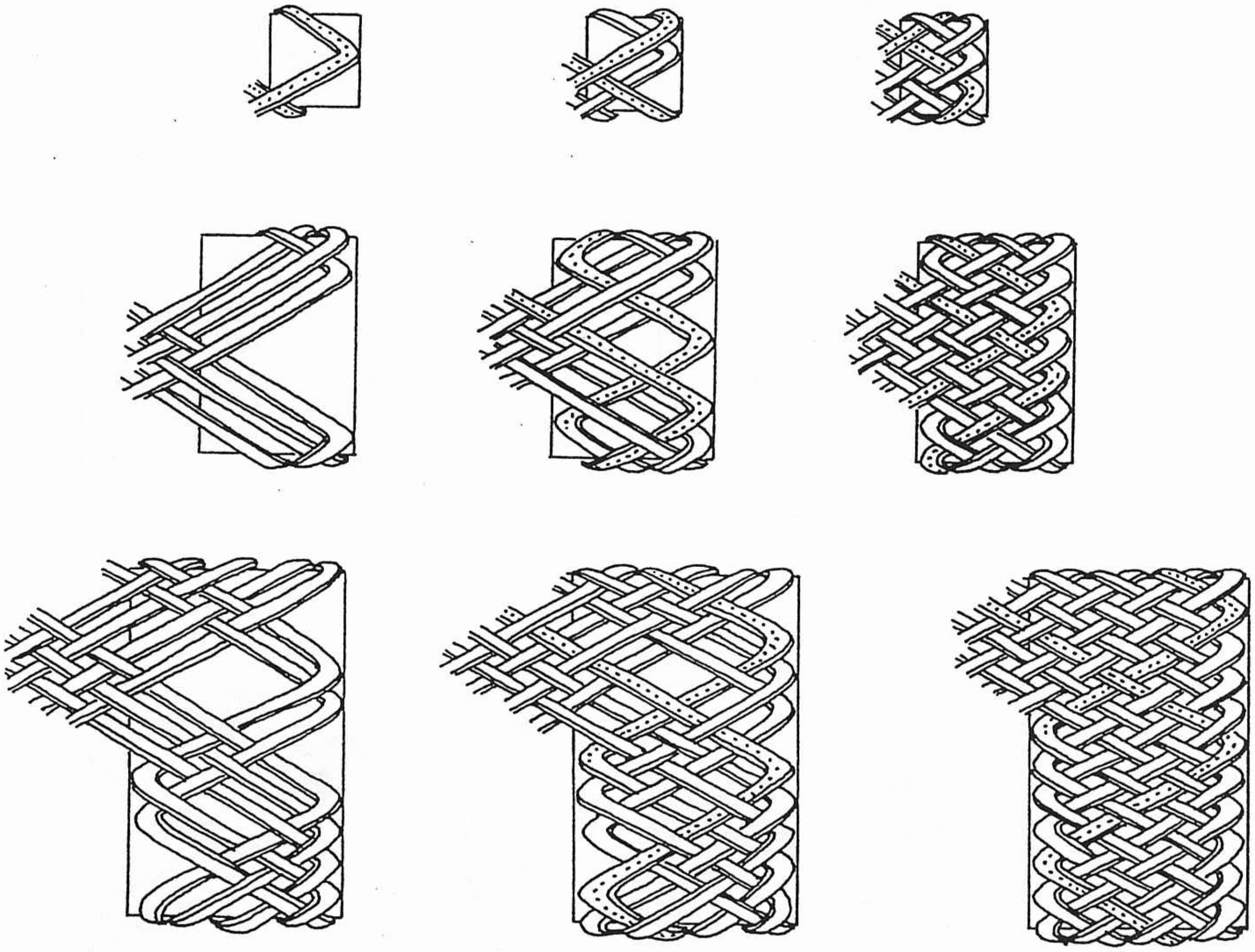


Pictorial drawings associated with pg. 63.

We have seen with the aid of the two examples how different raising procedures can be used in the construction of regular end-keepers. These various raising methods are in general only convenient, and hence useful, when the final regular end-keeper in the construction process has a Casa-coding. Although this coding form will undoubtedly be the most common one, it is by no means always the most pleasing one. In fact, for high grade work, the Casa-coding is rarely used.

The most convenient way to braid a non Casa-coded regular end-keeper is by means of **half-cycle algorithms**, hence to braid such an end-keeper in consecutive half-cycle stages. To enable us to do that, we must know the correct braiding-recipe (algorithm) for each consecutive half-cycle. Thus the first task is the determination of the half-cycle algorithms which belong to the envisaged end-keeper. Of the several methods available for this task, the simplest, no doubt, is the method which uses the **algorithm-diagram** of the envisaged regular end-keeper (see Pamphlet No. 7).

Since a regular end-keeper is a multi-string regular cylindrical braid in which the various strings start and end in adjacent 'bight'-points (note that a string does not necessarily end in its starting-point), the algorithm-diagram of a regular end-keeper is the algorithm-diagram of the corresponding multi-string regular cylindrical braid. In general such a braid has **essential** and **supplementary** starting-points. The number of *essential* starting-points is equal to the g.c.d. (greatest common divisor) of its number of parts and its number of bights. These essential starting-points can always be placed in adjacent 'bight'-points, which is the case in end-keepers. Any further starting-points are *supplementary* ones, and these can be placed of course adjacent to the *essential* ones, which is the case in end-keepers.

As with regular cylindrical braids, regular end-keepers can be divided into three groups with respect to their coding:

(1). column-coded regular end-keepers,
(2). row-coded regular end-keepers,
(3). neither column nor row-coded regular end-keepers.

In Pamphlet No. 7 we have seen how the algorithm-diagram for a column-coded single string regular cylindrical braid (regular knot) is constructed. First we had to obtain *the complementary cyclic bight-number scheme* of the knot.

A regular cylindrical braid with e essential starting-points is an interbraid of e regular knots, all with an identical string-run, and hence an identical complementary cyclic bight-number scheme. Thus the string-run of a regular cylindrical braid with e essential starting-points is a regular 'nesting' of e identical string-runs. Consequently the complementary cyclic bight-number scheme of a regular cylindrical braid with e essential starting-points consists of the complementary cyclic bight-number scheme of one of these e identical string-runs together with a regular **complementary nesting** of the the other $(e-1)$ identical complementary cyclic bight-number scheme **patterns**.

An Example is given on pg. 66, in which $e = 5$. The complementary cyclic bight-number scheme of one of these e (hence 5) interwoven identical string-runs is obtained by means of the calculation process involving $|-p|_b$; these are the numbered dots. The **pattern** of these numbered dots has to be $(e-1)$ times (hence 4 times) complementary nested in this numbered dot arrangement. Hence the dots immediately to the **left** of the numbered dots receive the letter A; those immediately to the **left** of A receive the letter B; those immediately to the **left** of B receive the letter C; and those immediately to the **left** of C receive the letter D. This completes the complementary cyclic bight-number scheme of the 15/25-endkeeper.

EXAMPLE: 15/25 END-KEEPER: 2-PASS HEADHUNTERS CODING.

FOR MOST BRAIDERS THE EASIEST METHOD FOR OBTAINING THE COMPLEMENTARY CYCLIC BIGHT-NUMBER SCHEME IS ITS CONSTRUCTION BY MEANS OF $|-p|_b$.

SET OFF b=25 DOTS ALONG A HORIZONTAL LINE AS SHOWN BELOW:

```
●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●
```

GIVE THE LEFT MOST DOT THE NUMBER 0.

COUNT $|-p|_b$ FURTHER DOTS TO THE RIGHT AND GIVE THE LAST DOT THE NUMBER 1.

COUNT A FURTHER $|-p|_b$ DOTS IN CYCLIC FASHION TO THE RIGHT AND GIVE THE LAST DOT THE NUMBER 2.

AND SO ON TILL WE RETURN AT THE LEFT MOST DOT (THE ONE WITH THE NUMBER 0).

```
0              3              1              4              2
●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●
```

MARK EACH SET OF UNNUMBERED DOTS FROM LEFT TO RIGHT WITH: D, C, B, A.

```
0  D  C  B  A  3  D  C  B  A  1  D  C  B  A  4  D  C  B  A  2  D  C  B  A
●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●
```

THIS GIVES US THE COMPLEMENTARY CYCLIC BIGHT-NUMBER SCHEME OF THE REGULAR CYLINDRICAL BRAID 15/25.

SET OFF ALONG A HORIZONTAL LINE p+1=15+1=16 DOTS AS SHOWN BELOW:

```
●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●
```

THE FIRST AND THE LAST DOT REPRESENT THE LEFT-HAND AND RIGHT-HAND BIGHT-BOUNDARIES OF THE REGULAR CYLINDRICAL BRAID.

SET OFF ABOVE THE DOTS FROM LEFT TO RIGHT, STARTING WITH THE SECOND DOT, THE CODING OF THE REGULAR CYLINDRICAL BRAID FOR A HALF-CYCLE RUNNING FROM LOWER LEFT TO UPPER RIGHT.

ABOVE THIS CODING LINE SET OFF THE COMPLEMENTARY CYCLIC BIGHT-NUMBER SCHEME, REPEAT IF NECESSARY AND/OR AS FAR AS IT CAN GO, BUT LEAVE THE LAST DOT FREE.

STARTING WITH THE SECOND DOT FROM THE RIGHT, SET OFF BELOW THE DOTS THE CODING SEQUENCE OF THE CYLINDRICAL BRAID FOR A HALF-CYCLE RUNNING FROM LOWER RIGHT TO UPPER LEFT. GIVE THE LAST DOT FROM THE RIGHT, HENCE THE LEFT MOST DOT, THE CODING OF THE TRANSITION COLUMN. THE TRANSITION COLUMN IS THE LEFT BIGHT-BOUNDARY OF THE REGULAR CYLINDRICAL BRAID.

BELOW THIS CODING LINE SET OFF THE COMPLEMENTARY CYCLIC BIGHT-NUMBER SCHEME FROM RIGHT TO LEFT, REPEAT IF NECESSARY AND/OR AS FAR AS IT CAN GO, END WITH THE ENTRY UNDER THE LEFT MOST DOT.

```
0  D  C  B  A  3  D  C  B  A  1  D  C  B  A
   u  u  o  o  u  u  o  o  u  u  o  o  u  u
●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●  ●
u  o  o  u  u  o  o  u  u  o  o  u  u  o  o
4  A  B  C  D  1  A  B  C  D  3  A  B  C  D  0
```

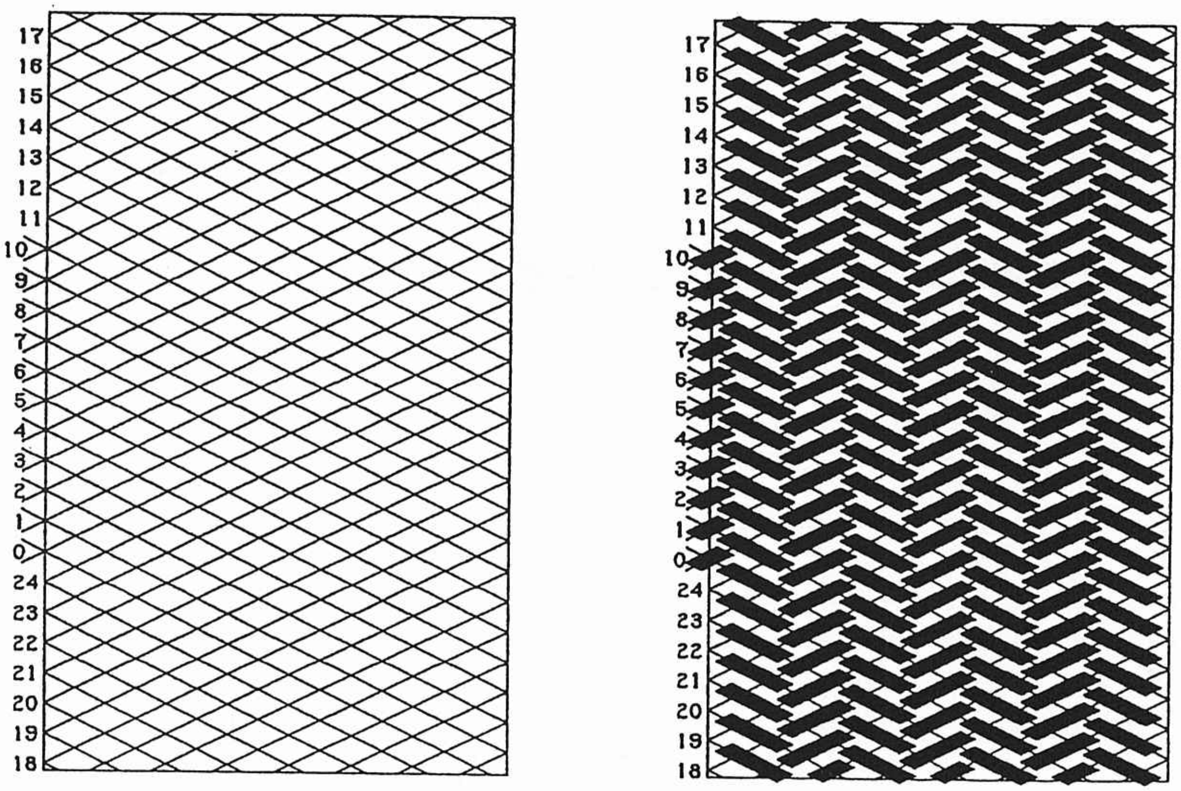
THIS GIVES THE COMPLETE ALGORITHM-DIAGRAM FOR THE REGULAR CYLINDRICAL BRAID, AND HENCE OF THE 15/25 END-KEEPER. THE END-KEEPER CAN NOW BE BRAIDED DIRECTLY FROM THIS ALGORITHM-DIAGRAM, OR ALTERNATIVELY FIRST THE HALF-CYCLE ALGORITHMS CAN BE OBTAINED FROM THIS ALGORITHM-DIAGRAM.

NOTE THAT THE ALGORITHM-DIAGRAM IS INDEPENDENT OF THE NUMBER OF TRANSITION POINTS. THE MINIMUM NUMBER OF TRANSITION POINTS HAS TO BE EQUAL TO THE G.C.D. OF p AND b, HENCE IN THIS EXAMPLE THE MINIMUM NUMBER IS EQUAL TO 5.

It should be noted that the complementary cyclic bight-number scheme of a multi-string regular cylindrical braid is independent of the total number of starting-points; it is only dependent on the number of essential starting-points. The same therefore applies to regular end-keepers.

The algorithm-diagram of a column-coded end-keeper is obtained with the aid of the complementary cyclic bight-number scheme of that end-keeper in exactly the same way as the algorithm-diagram of a column-coded regular knot is obtained with the aid of its complementary cyclic bight-number scheme; this is shown in the Example on pg. 66.

Say this end-keeper has eleven transition-points; see the string-run and grid-diagrams below.



The 2-pass Headhunter's coded 15/25-endkeeper ($x = 11$).

This end-keeper has five essential starting-points and six supplementary starting-points. Each essential starting-point is associated with an essential string-run. The first essential starting-point is transition index-number 0, the second essential starting-point is transition index-number 1, the third essential starting-point is transition index-number 2, the fourth essential starting-point is transition index-number 3, and the fifth essential starting-point is transition index-number 4.

The first essential string-run comprises three components, and hence is made up of three separate strings. The other four essential string-runs each consist of two components, hence two separate strings.

The component-table of this end-keeper is presented on pg. 68.

We start the braiding-process with braiding the first component, hence we start at transition index-number 0 and braid the cycles 0 — 15, and 15 — 5 (hence the first four half-cycles in the essential string-run 0 — 15 — 5 — 20 — 10 — 0):

1). $L \longrightarrow R$: Free Run.
2). $(i = 0)$ $R \longrightarrow L$: Free Run.
3). $(i = 0)$ $L \longrightarrow R$: Free Run.
4). $(i = 1)$ $R \longrightarrow L$: o.

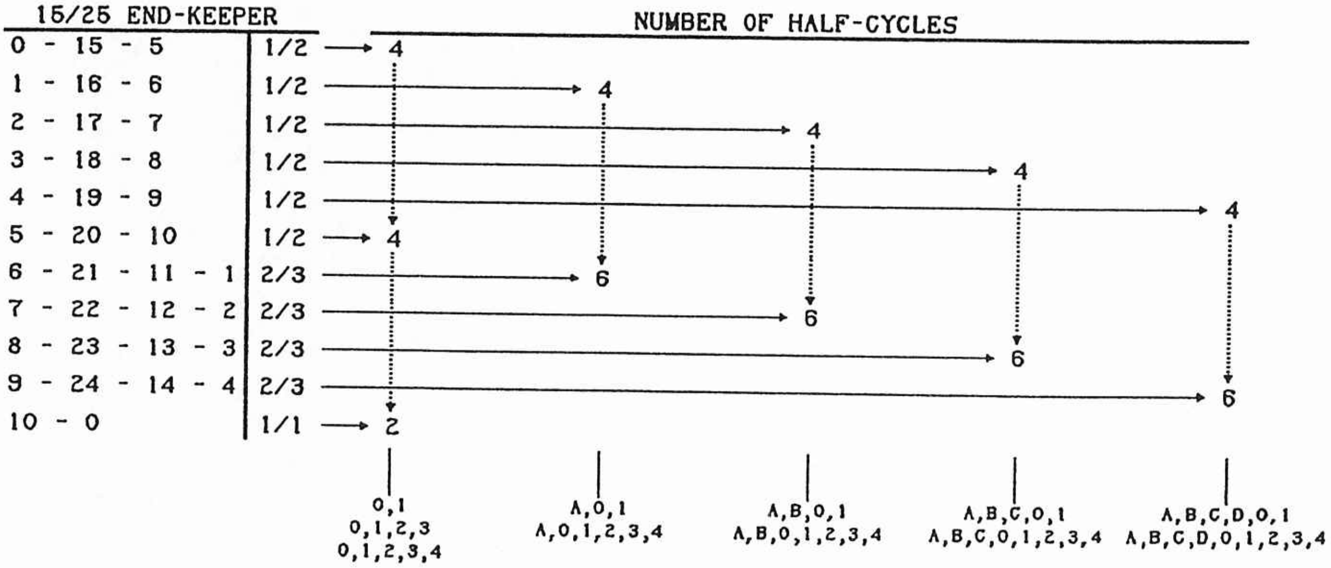
Next we braid with a new string the cycles 5—20, and 20—10 (hence the second four half-cycles in the essential string-run 0—15—5—20—10—0):

5). $(i = 1)$ $L \longrightarrow R$: u.
6). $(i = 2)$ $R \longrightarrow L$: o.
7). $(i = 2)$ $L \longrightarrow R$: u.
8). $(i = 3)$ $R \longrightarrow L$: $2o$.

Then we braid with a further new string the cycle 10—0 (hence the last two half-cycles in the essential string-run 0—15—5—20—10—0):

9). $(i = 3)$ $L \longrightarrow R$: $2u$.
10). $(i = 4)$ $R \longrightarrow L$: $2o - u$.

The first essential string-run, 0—15—5—20—10—0, which consists of three separate strings, hence three components, is now completed.



We continue with braiding the first component (1—16—6) of the second essential string-run (1—16—6—21—11—1):

11). $(i = A)$ $L \longrightarrow R$: $o - 2u$.
12). $(i = A; 0)$ $R \longrightarrow L$: $u - 2o$.
13). $(i = A; 0)$ $L \longrightarrow R$: $o - 2u$.
14). $(i = A; 1)$ $R \longrightarrow L$: $u - 3o$.

Then the second (last) component (6—21—11—1) of the second essential string-run:

15). $(i = A; 1)$ $L \longrightarrow R$: $o - 3u$.
16). $(i = A; 2)$ $R \longrightarrow L$: $u - 3o$.
17). $(i = A; 2)$ $L \longrightarrow R$: $o - 3u$.
18). $(i = A; 3)$ $R \longrightarrow L$: $u - 4o$.
19). $(i = A; 3)$ $L \longrightarrow R$: $o - 4u$.
20). $(i = A; 4)$ $R \longrightarrow L$: $u - 4o - u$.

The second essential string-run, 1—16—6—21—11—1, which consists of two separate strings, hence two components, is now completed.

We continue with braiding the first component (2—17—7) of the third essential string-run (2—17—7—22—12—2):

21). $(i = A, B)$ $L \longrightarrow R:$ $3o - 3u.$
22). $(i = A, B; 0)$ $R \longrightarrow L:$ $3u - 3o.$
23). $(i = A, B; 0)$ $L \longrightarrow R:$ $3o - 3u.$
24). $(i = A, B; 1)$ $R \longrightarrow L:$ $3u - 4o.$

Then the second component (7—22—12—2) of the third essential string-run:

25). $(i = A, B; 1)$ $L \longrightarrow R:$ $3o - 4u.$
26). $(i = A, B; 2)$ $R \longrightarrow L:$ $3u - 4o.$
27). $(i = A, B; 2)$ $L \longrightarrow R:$ $3o - 4u.$
28). $(i = A, B; 3)$ $R \longrightarrow L:$ $2u - o - u - 4o.$
29). $(i = A, B; 3)$ $L \longrightarrow R:$ $2o - u - o - 4u.$
30). $(i = A, B; 4)$ $R \longrightarrow L:$ $2u - o - u - 4o - u.$

The third essential string-run, 2—17—7—22—12—2, which consists of two separate strings, hence two components, is now completed.

Next we braid the first component (3—18—8) of the fourth essential string-run (3—18—8—23—13—3):

31). $(i = A, B, C)$ $L \longrightarrow R:$ $u - 4o - u - o - 2u.$
32). $(i = A, B, C; 0)$ $R \longrightarrow L:$ $o - 4u - o - u - 2o.$
33). $(i = A, B, C; 0)$ $L \longrightarrow R:$ $u - 4o - u - o - 2u.$
34). $(i = A, B, C; 1)$ $R \longrightarrow L:$ $o - 4u - 2o - u - 2o.$

Then the second component (8—23—13—3) of the fourth essential string-run:

35). $(i = A, B, C; 1)$ $L \longrightarrow R:$ $u - 4o - 2u - o - 2u.$
36). $(i = A, B, C; 2)$ $R \longrightarrow L:$ $o - 4u - 2o - u - 2o.$
37). $(i = A, B, C; 2)$ $L \longrightarrow R:$ $u - 4o - 2u - o - 2u.$
38). $(i = A, B, C; 3)$ $R \longrightarrow L:$ $o - 2u - o - 2u - 2o - u - 2o.$
39). $(i = A, B, C; 3)$ $L \longrightarrow R:$ $u - 2o - u - 2o - 2u - o - 2u.$
40). $(i = A, B, C; 4)$ $R \longrightarrow L:$ $o - 2u - o - 2u - 2o - u - 2o - u.$

The fourth essential string-run, 3—18—8—23—13—3, which consists of two separate strings, hence two components, is now completed.

Finally we braid the first component (4—19—9) of the fifth essential string-run (4—19—9—24—14—4):

41). $(i = A, B, C, D)$ $L \longrightarrow R:$ $2u - 2o - u - 2o - u - 2o - 2u.$
42). $(i = A, B, C, D; 0)$ $R \longrightarrow L:$ $2o - 2u - o - 2u - o - 2u - 2o.$
43). $(i = A, B, C, D; 0)$ $L \longrightarrow R:$ $2u - 2o - u - 2o - u - 2o - 2u.$
44). $(i = A, B, C, D; 1)$ $R \longrightarrow L:$ $2o - 2u - o - 2u - 2o - 2u - 2o.$

Then the second (last) component (9—24—14—4) of the fifth essential string-run:

45). $(i = A, B, C, D; 1)$ $L \longrightarrow R:$ $2u - 2o - u - 2o - 2u - 2o - 2u.$
46). $(i = A, B, C, D; 2)$ $R \longrightarrow L:$ $2o - 2u - o - 2u - 2o - 2u - 2o.$
47). $(i = A, B, C, D; 2)$ $L \longrightarrow R:$ $2u - 2o - u - 2o - 2u - 2o - 2u.$
48). $(i = A, B, C, D; 3)$ $R \longrightarrow L:$ $2o - 2u - 2o - 2u - 2o - 2u - 2o.$
49). $(i = A, B, C, D; 3)$ $L \longrightarrow R:$ $2u - 2o - 2u - 2o - 2u - 2o - 2u.$
50). $(i = A, B, C, D; 4)$ $R \longrightarrow L:$ $2o - 2u - 2o - 2u - 2o - 2u - 2o - u.$

This completes the Gaucho-coded 15/25-endkeeper.

On pg. 71 we have shown the algorithm-diagrams of the Casa-coded 6/10 and 7/11-endkeepers ($x = 4$).

The Casa-coded 6/10-endkeeper ($x = 4$) consists of two essential string-runs (g.c.d. of 6 and 10 equals 2), each of which are braided with two separate strings. With the aid of the component table we read from the algorithm-diagram the consecutive half-cycle algorithms:

First component 0—6—2, starting at transition index-number 0:
1). $L \longrightarrow R$: Free Run.
2). $(i = 0)$ $R \longrightarrow L$: Free Run.
3). $(i = 0)$ $L \longrightarrow R$: Free Run.
4). $(i = 1)$ $R \longrightarrow L$: u.

Second component 2—8—4—0, starting at transition index-number 2:
5). $(i = 1)$ $L \longrightarrow R$: o.
6). $(i = 2)$ $R \longrightarrow L$: u.
7). $(i = 2)$ $L \longrightarrow R$: o.
8). $(i = 3)$ $R \longrightarrow L$: $2u$.
9). $(i = 3)$ $L \longrightarrow R$: $2o$.
10). $(i = 4)$ $R \longrightarrow L$: $3u$.

Third component 1—7—3, starting at transition index-number 1:
11). $(i = A)$ $L \longrightarrow R$: $3u$.
12). $(i = A; 0)$ $R \longrightarrow L$: $3o$.
13). $(i = A; 0)$ $L \longrightarrow R$: $3u$.
14). $(i = A; 1)$ $R \longrightarrow L$: $2o - u - o$.

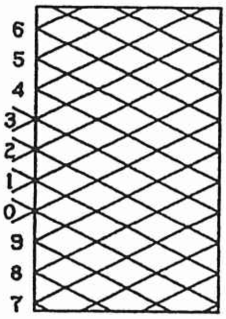
Fourth component 3—9—5—1, starting at transition index-number 3:
15). $(i = A; 1)$ $L \longrightarrow R$: $2u - o - u$.
16). $(i = A; 2)$ $R \longrightarrow L$: $2o - u - o$.
17). $(i = A; 2)$ $L \longrightarrow R$: $2u - o - u$.
18). $(i = A; 3)$ $R \longrightarrow L$: $o - u - o - u - o$.
19). $(i = A; 3)$ $L \longrightarrow R$: $u - o - u - o - u$.
20). $(i = A; 4)$ $R \longrightarrow L$: $o - u - o - u - o - u$.

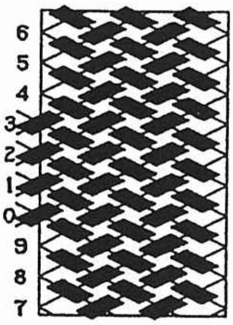
This completes the braiding-process of the Casa-coded 6/10-endkeeper.

The Casa-coded 7/11-endkeeper ($x = 4$) consists of one essential string-run (g.c.d. of 7 and 11 equals 1). With the aid of the component table we read from the algorithm-diagram the consecutive half-cycle algorithms:

First component 0—7—3, starting at transition index-number 0:
1). $L \longrightarrow R$: Free Run.
2). $(i = 0)$ $R \longrightarrow L$: Free Run.
3). $(i = 0)$ $L \longrightarrow R$: Free Run.
4). $(i = 1)$ $R \longrightarrow L$: o.

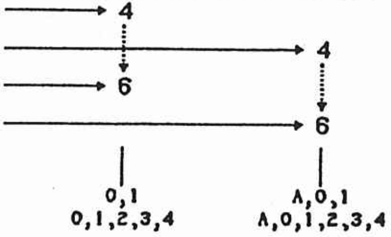
Second component 3—10—6—2, starting at transition index-number 3:
5). $(i = 1)$ $L \longrightarrow R$: o.
6). $(i = 2)$ $R \longrightarrow L$: o.
7). $(i = 2)$ $L \longrightarrow R$: o.
8). $(i = 3)$ $R \longrightarrow L$: $u - o$.
9). $(i = 3)$ $L \longrightarrow R$: $u - o$.
10). $(i = 4)$ $R \longrightarrow L$: $u - o - u$.





6/10 END-KEEPER	
0 - 6 - 2	1/2
1 - 7 - 3	1/2
2 - 8 - 4 - 0	2/3
3 - 9 - 5 - 1	2/3

NUMBER OF HALF-CYCLES



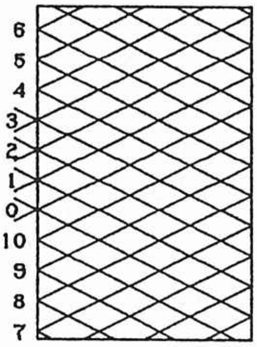
$$|{}^{-}p|_b = |{}^{-}6|_{10} = 4$$

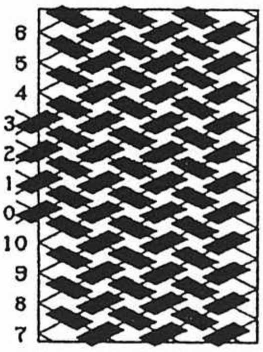
```
0  Λ  3  Λ  1  Λ  4  Λ  2  Λ
•  •  •  •  •  •  •  •  •  •
```

THE COMPLEMENTARY CYCLIC BIGHT-NUMBER SCHEME

```
——→ LEFT TO RIGHT HALF-CYCLES
0  Λ  3  Λ  1  Λ
   u  o  u  o  u
•  •  •  •  •  •  •
u  o  u  o  u  o
4  Λ  1  Λ  3  Λ  0
                ←——
RIGHT TO LEFT HALF-CYCLES
```

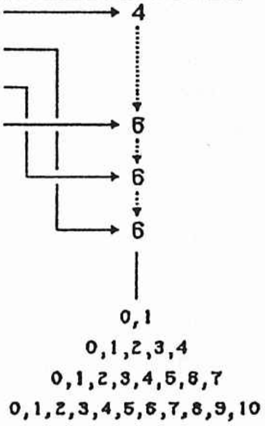
THE ALGORITHM-DIAGRAM





7/11 END-KEEPER	
0 - 7 - 3	1/2
1 - 8 - 4 - 0	2/3
2 - 9 - 5 - 1	2/3
3 - 10 - 6 - 2	2/3

NUMBER OF HALF-CYCLES



$$|{}^{-}p|_b = |{}^{-}7|_{11} = 4$$

```
0  3  6  9  1  4  7  10  2  5  8
•  •  •  •  •  •  •  •   •  •  •
```

THE COMPLEMENTARY CYCLIC BIGHT-NUMBER SCHEME

```
——→ LEFT TO RIGHT HALF-CYCLES
0   3  6  9  1  4  7
    u  o  u  o  u  o
•   •  •  •  •  •  •  •
u   o  u  o  u  o  u
10  7  4  1  9  6  3  0
                   ←——
RIGHT TO LEFT HALF-CYCLES
```

THE ALGORITHM-DIAGRAM

Third component 2—9—5—1, starting at transition index-number 2:

11). $(i = 4)$ $L \longrightarrow R$: $u - o - u$.
12). $(i = 5)$ $R \longrightarrow L$: $u - o - u$.
13). $(i = 5)$ $L \longrightarrow R$: $u - o - u$.
14). $(i = 6)$ $R \longrightarrow L$: $u - 2o - u$.
15). $(i = 6)$ $L \longrightarrow R$: $u - 2o - u$.
16). $(i = 7)$ $R \longrightarrow L$: $u - 2o - u - o$.

Fourth component 1—8—4—0, starting at transition index-number 1:

17). $(i = 7)$ $L \longrightarrow R$: $u - 2o - u - o$.
18). $(i = 8)$ $R \longrightarrow L$: $u - 2o - u - o$.
19). $(i = 8)$ $L \longrightarrow R$: $u - 2o - u - o$.
20). $(i = 9)$ $R \longrightarrow L$: $u - o - u - o - u - o$.
21). $(i = 9)$ $L \longrightarrow R$: $u - o - u - o - u - o$.
22). $(i = 10)$ $R \longrightarrow L$: $u - o - u - o - u - o - u$.

This completes the braiding-process of the Casa-coded 7/11-endkeeper.

The construction and use of the algorithm-diagrams for row-coded regular knots (row-coded single-string regular cylindrical braids) has been discussed in Pamphlet No. 9. In the same way as the algorithm-diagrams for column-coded regular end-keepers (column-coded multi-string regular cylindrical braids) are an extension of the algorithm-diagrams for column-coded regular knots, so are the algorithm-diagrams for row-coded regular end-keepers (row-coded multi-string regular cylindrical braids) an extension of the algorithm-diagrams for row-coded regular knots. We shall assume that the reader is fully conversant with the contents of the Pamphlets No. 7 and No. 9.

With the aid of two Examples we shall outline the construction of algorithm-diagrams for row-coded regular end-keepers. For most braiders the simplest way to construct these algorithm-diagrams will be by making use of the quantities $|-p|_b$ and $|p|_r$.

In the Example on pg. 73 we have: $p = 8$; $b = 12$; $x = 5$; $r = 3$. Since the g.c.d.$(8, 12) = 4$, it follows that this end-keeper has four essential transition-points; hence there are four essential starting-points (the transition index-numbers $0, 1, 2, 3$) and one supplementary starting-point (the transition index-number 4). We shall assume that the braiding directions of the half-cycles are from lower-left to upper-right and from lower-right to upper-left.

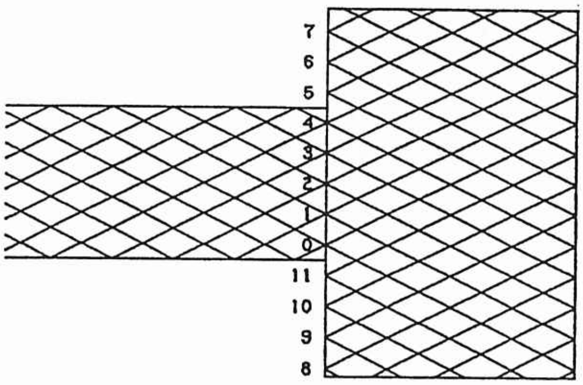
With $|-p|_b = |-8|_{12} = 4$ we construct the complementary cyclic bight-number scheme (see pg. 73)

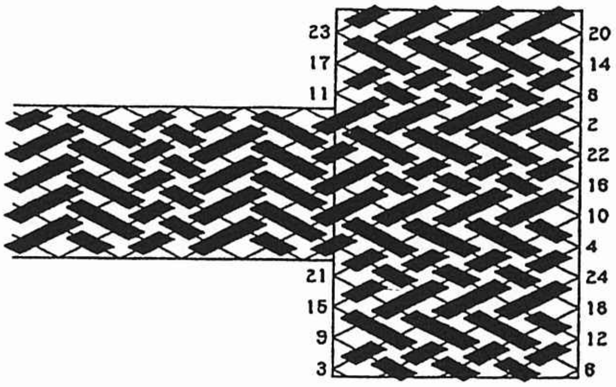
The algorithm-diagram is constructed in the same way as the algorithm-diagrams for row-coded regular knots (see Pamphlet No. 9). Since $r = 3$, there are three 'j' positions $(j = 1, j = 2, j = 3)$ for the *Left to Right* half-cycles, and three 'j' positions $(j = 1, j = 2, j = 3)$ for the *Right to Left* half-cycles.

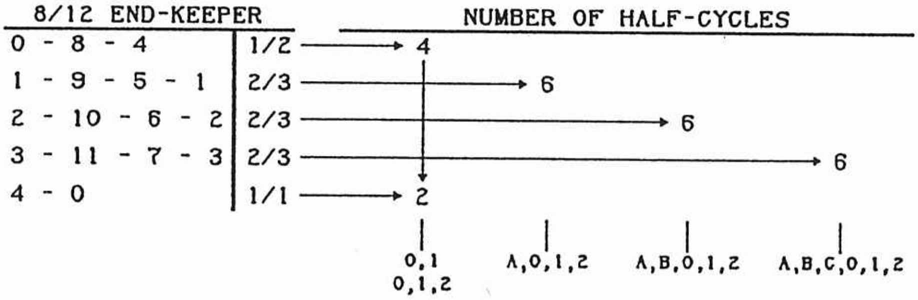
The first essential string-run consists of two components: 0—8—4, and 4—0. These components are associated with the left to right half-cycles $1, 3; 5$, and the right to left half-cycles $2, 4; 6$.

For the left to right direction we obtain: Half-cycle 1 comes under $j = 1$. Count $|p|_r = 2$ positions to the right (in cyclic fashion), hence half-cycle 3 comes under $j = 3$. Count a further 2 positions to the right in cyclic fashion, hence half-cycle 5 comes under $j = 2$.

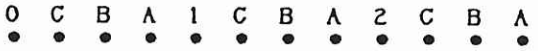
For the right to left direction we obtain: Half-cycle 2 comes under $j = 1$. Count $|p|_r = 2$ positions to the left in cyclic fashion, hence half-cycle 4 comes under $j = 3$.



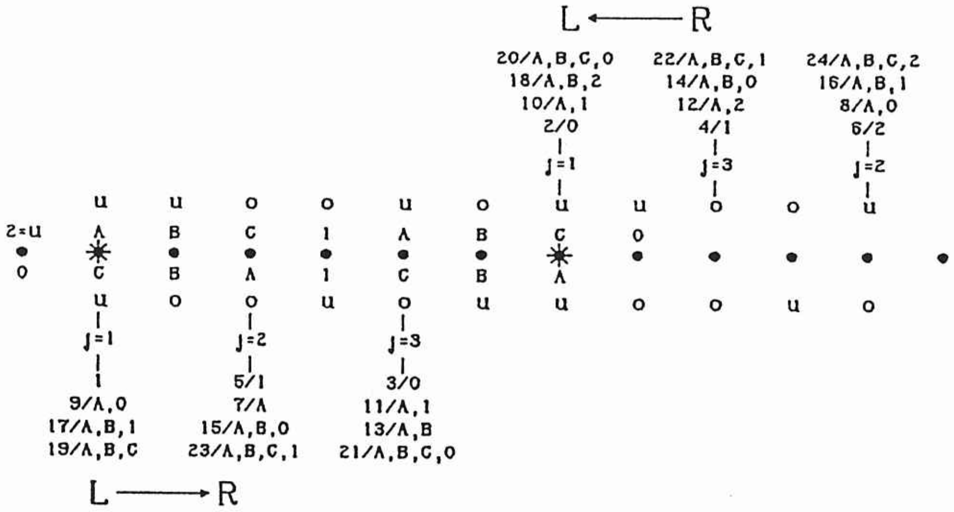


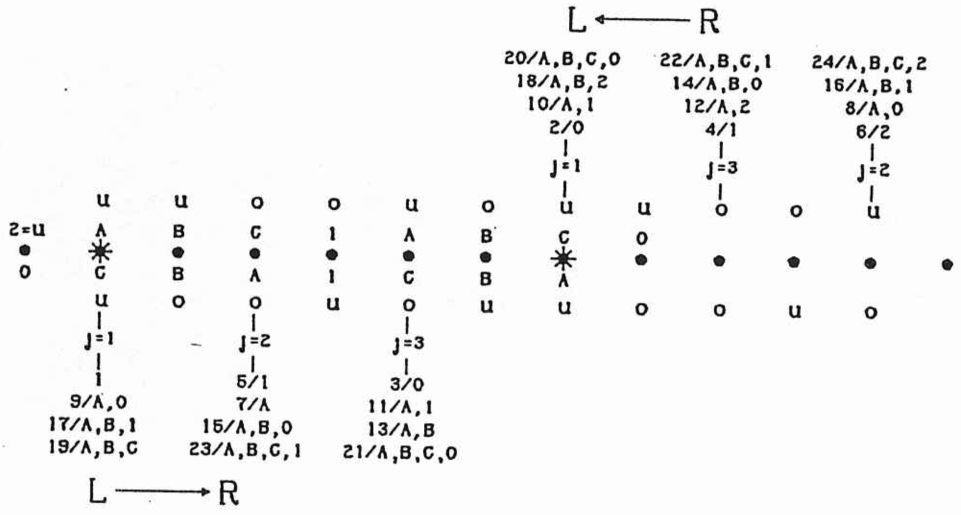


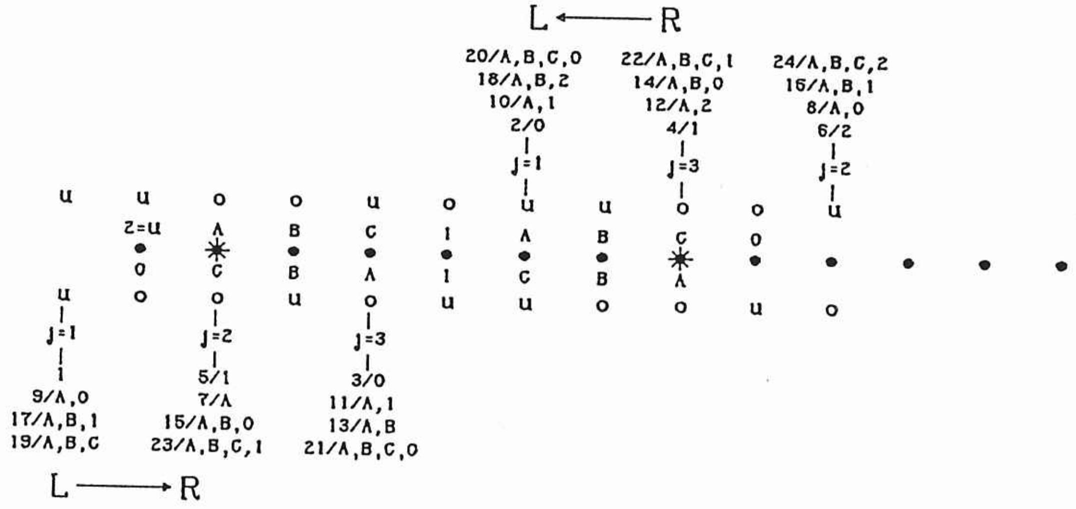
$|-P|_b = |-8|_{12} = 4$ $|P|_r = |8|_3 = 2$

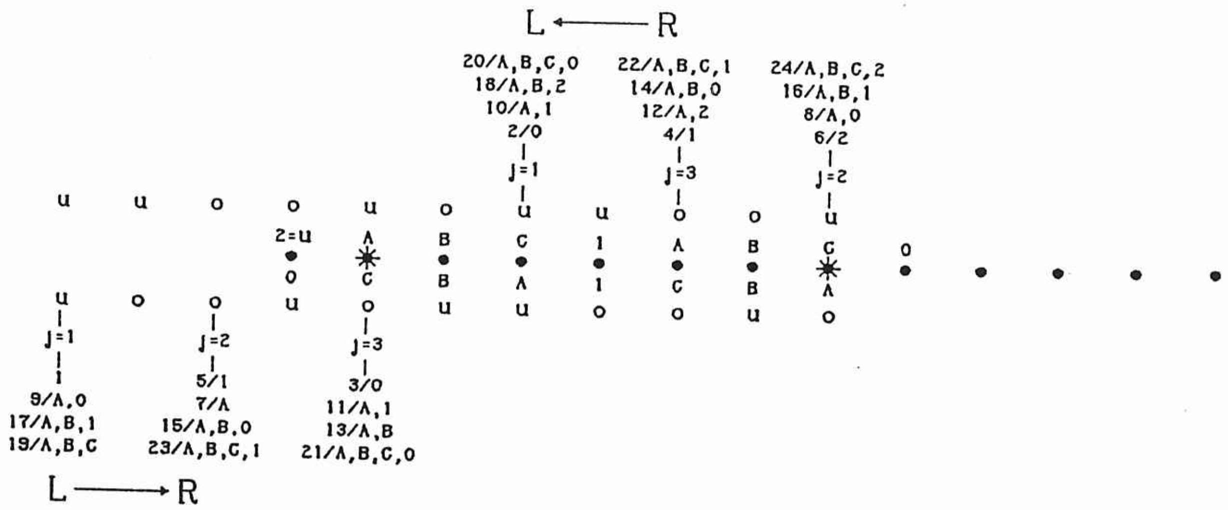


THE COMPLEMENTARY CYCLIC BIGHT-NUMBER SCHEME




L ← R
20/A,B,C,0
18/A,B,2
10/A,1
2/0
j=1
22/A,B,C,1
14/A,B,0
12/A,2
4/1
j=3
24/A,B,C,2
16/A,B,1
8/A,0
6/2
j=2
2=u
u
j=1
1
9/A,0
17/A,B,1
19/A,B,C
j=2
5/1
7/A
15/A,B,0
23/A,B,C,1
j=3
3/0
11/A,1
13/A,B
21/A,B,C,0
L → R


L ← R
20/A,B,C,0
18/A,B,2
10/A,1
2/0
j=1
22/A,B,C,1
14/A,B,0
12/A,2
4/1
j=3
24/A,B,C,2
16/A,B,1
8/A,0
6/2
j=2
2=u
j=1
1
9/A,0
17/A,B,1
19/A,B,C
j=2
5/1
7/A
15/A,B,0
23/A,B,C,1
j=3
3/0
11/A,1
13/A,B
21/A,B,C,0
L → R


L ← R
20/A,B,C,0
18/A,B,2
10/A,1
2/0
j=1
22/A,B,C,1
14/A,B,0
12/A,2
4/1
j=3
24/A,B,C,2
16/A,B,1
8/A,0
6/2
j=2
2=u
j=1
1
9/A,0
17/A,B,1
19/A,B,C
j=2
5/1
7/A
15/A,B,0
23/A,B,C,1
j=3
3/0
11/A,1
13/A,B
21/A,B,C,0
L → R

Count a further 2 positions to the left in cyclic fashion, hence half-cycle 6 comes under $j = 2$.

The second essential string-run consists of one component: 1—9—5—1. This component is associated with the left to right half-cycles 7, 9, 11, and the right to left half-cycles 8, 10, 12.

For the left to right direction we obtain: Half-cycle 7 comes under $j = 2$ ($j = |1+1|_r = |2|_3 = 2$). Count $|p|_r = 2$ positions to the right in cyclic fashion, hence half-cycle 9 comes under $j = 1$. Count a further 2 positions to the right in cyclic fashion, hence half-cycle 11 comes under $j = 3$.

For the right to left direction we obtain: Half-cycle 8 comes under $j = 2$. Count $|p|_r = 2$ positions to the left in cyclic fashion, hence half-cycle 10 comes under $j = 1$. Count a further 2 positions to the left in cyclic fashion, hence half-cycle 12 comes under $j = 3$.

The third essential string-run consists of one component: 2—10—6—2. This component is associated with the left to right half-cycles 13, 15, 17, and the right to left half-cycles 14, 16, 18.

For the left to right direction we obtain: Half-cycle 13 comes under $j = 3$ ($j = |2+1|_r = |3|_3 = 3$). Count $|p|_r = 2$ positions to the right in cyclic fashion, hence half-cycle 15 comes under $j = 2$. Count a further 2 positions to the right in cyclic fashion, hence half-cycle 17 comes under $j = 1$.

For the right to left direction we obtain: Half-cycle 14 comes under $j = 3$. Count $|p|_r = 2$ positions to the left in cyclic fashion, hence half-cycle 16 comes under $j = 2$. Count a further 2 positions to the left in cyclic fashion, hence half-cycle 18 comes under $j = 1$.

The fourth essential string-run consists of one component: 3—11—7—3. This component is associated with the left to right half-cycles 19, 21, 23, and the right to left half-cycles 20, 22, 24.

For the left to right direction we obtain: Half-cycle 19 comes under $j = 1$ ($j = |3+1|_r = |4|_3 = 1$). Count $|p|_r = 2$ positions to the right in cyclic fashion, hence half-cycle 21 comes under $j = 3$. Count a further 2 positions to the right in cyclic fashion, hence half-cycle 23 comes under $j = 2$.

For the right to left direction we obtain: Half-cycle 20 comes under $j = 1$. Count $|p|_r = 2$ positions to the left in cyclic fashion, hence half-cycle 22 comes under $j = 3$. Count a further 2 positions to the left in cyclic fashion, hence half-cycle 24 comes under $j = 2$.

As discussed in Pamphlet No. 9, the central strip in the algorithm-diagram can slide in order for the "star" to line up with the appropriate half-cycle positions (see pg. 74). The left-most dot of the central strip represents the left boundary of the end-keeper, and since all the transition points have the same coding, this dot is in this case always associated with an *under-coding* for the right to left half-cycles which go through the transition-points.

The algorithm-diagram gives us the following half-cycle algorithms (see pg. 74):

First component 0—8—4, starting at transition index-number 0:

1). $L \longrightarrow R$: Free Run.
2). ($i = 0$) $R \longrightarrow L$: Free Run.
3). ($i = 0$) $L \longrightarrow R$: Free Run.
4). ($i = 1$) $R \longrightarrow L$: *o*.

Second component 4—0, starting at transition index-number 4:

5). $(i = 1)$ $L \longrightarrow R:$ $o - u.$
6). $(i = 2)$ $R \longrightarrow L:$ $2u.$

Note that the 'o' in 5) is in transition point '4', and that the last 'u' in 6) is in transition-point '0'.

Third component 1—9—5—1, starting at transition index-number 1:

7). $(i = A)$ $L \longrightarrow R:$ $2o.$
8). $(i = A;0)$ $R \longrightarrow L:$ $o - u.$
9). $(i = A;0)$ $L \longrightarrow R:$ $o - u.$
10). $(i = A;1)$ $R \longrightarrow L:$ $u - o - u.$
11). $(i = A;1)$ $L \longrightarrow R:$ $u - 2o.$
12). $(i = A;2)$ $R \longrightarrow L:$ $u - 2o - u.$

Note that the last 'u' in 12) is in transition-point '1'.

Fourth component 2—10—6—2, starting at transition index-number 2:

13). $(i = A,B)$ $L \longrightarrow R:$ $3u - o.$
14). $(i = A,B;0)$ $R \longrightarrow L:$ $2u - 2o.$
15). $(i = A,B;0)$ $L \longrightarrow R:$ $u - 3o.$
16). $(i = A,B;1)$ $R \longrightarrow L:$ $2o - u - o - u.$
17). $(i = A,B;1)$ $L \longrightarrow R:$ $2o - 3u.$
18). $(i = A,B;2)$ $R \longrightarrow L:$ $o - u - o - 3u.$

Note that the last 'u' in 18) is in transition-point '2'.

Fifth component 3—11—7—3, starting at transition index-number 3:

19). $(i = A,B,C)$ $L \longrightarrow R:$ $u - 3o - 2u.$
20). $(i = A,B,C;0)$ $R \longrightarrow L:$ $u - o - u - o - 2u.$
21). $(i = A,B,C;0)$ $L \longrightarrow R:$ $o - 2u - o - u - o.$
22). $(i = A,B,C;1)$ $R \longrightarrow L:$ $o - 2u - o - u - 2o.$
23). $(i = A,B,C;1)$ $L \longrightarrow R:$ $o - u - o - 2u - 2o.$
24). $(i = A,B,C;2)$ $R \longrightarrow L:$ $u - 2o - 2u - o - 2u.$

Note that the last 'u' in 24) is in transition-point '3'.

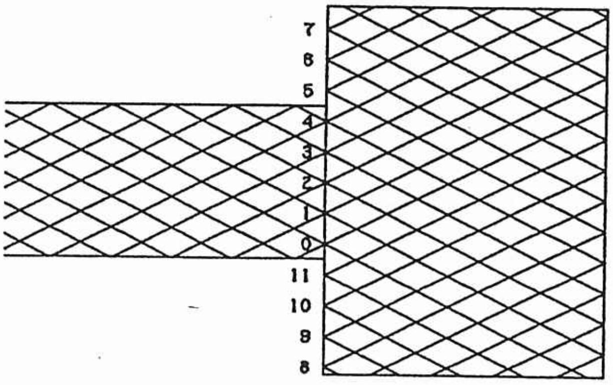
In the Example on pg. 77 we have: $p = 9$; $b = 12$; $x = 5$; $r = 3$. Since the g.c.d.$(9,12) = 3$, it follows that this end-keeper has three essential transition-points; hence there are three essential starting-points (the transition index-numbers 0, 1, 2) and two supplementary starting-points (the transition index-numbers 3, 4). We shall assume that the braiding directions of the half-cycles are again from lower-left to upper-right and from lower-right to upper-left.

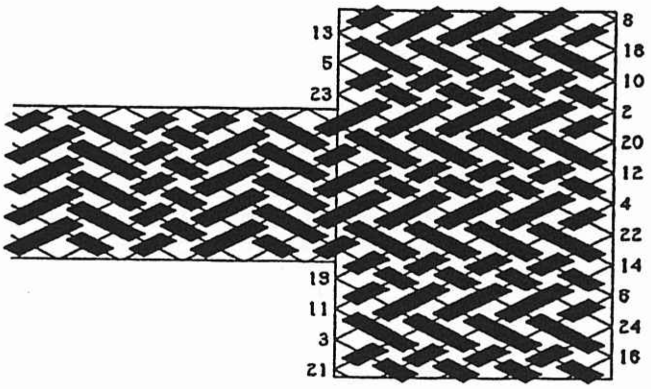
With $|-p|_b = |-9|_{12} = 3$ we construct the complementary cyclic bight-number scheme (see pg. 77)

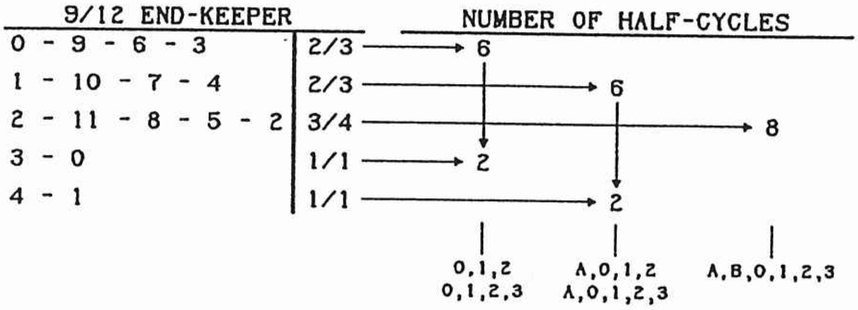
The algorithm-diagram is constructed in the same way as the algorithm-diagrams for row-coded regular knots (see Pamphlet No. 9). Since $r = 3$, there are three 'j' positions ($j = 1$, $j = 2$, $j = 3$) for the *Left to Right* half-cycles, and three 'j' positions ($j = 1$, $j = 2$, $j = 3$) for the *Right to Left* half-cycles.

The first essential string-run consists of two components: 0—9—6—3, and 3—0. These components are associated with the left to right half-cycles 1, 3, 5; 7, and the right to left half-cycles 2, 4, 6; 8.

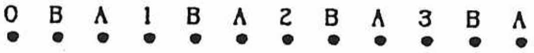
For the left to right direction we obtain: Half-cycle 1 comes under $j = 1$. Count $|p|_r = 0$ positions to the right (in cyclic fashion), hence half-cycle 3 comes under $j = 1$.



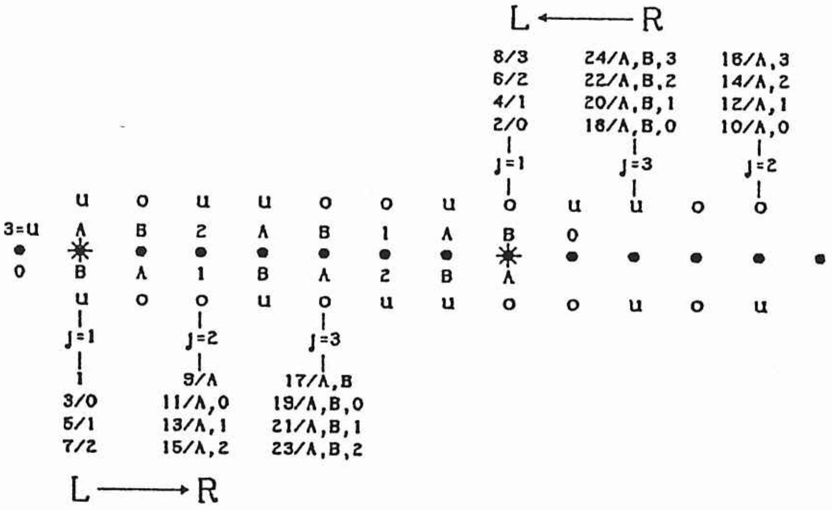


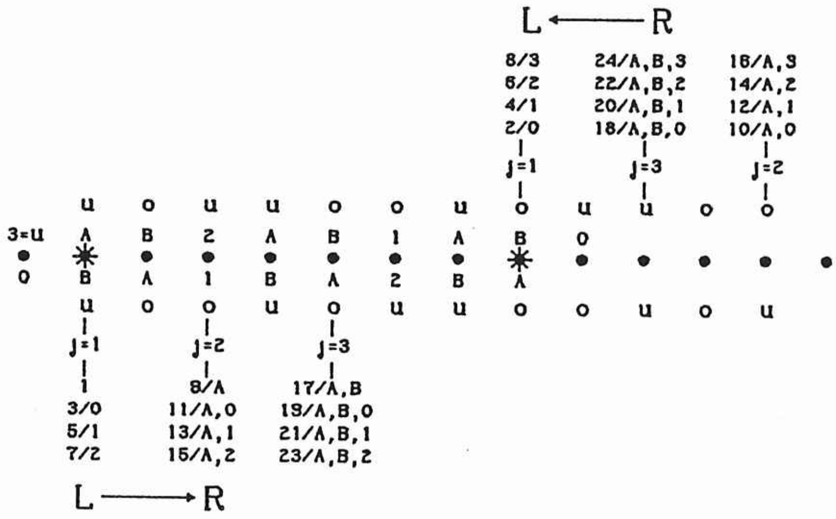


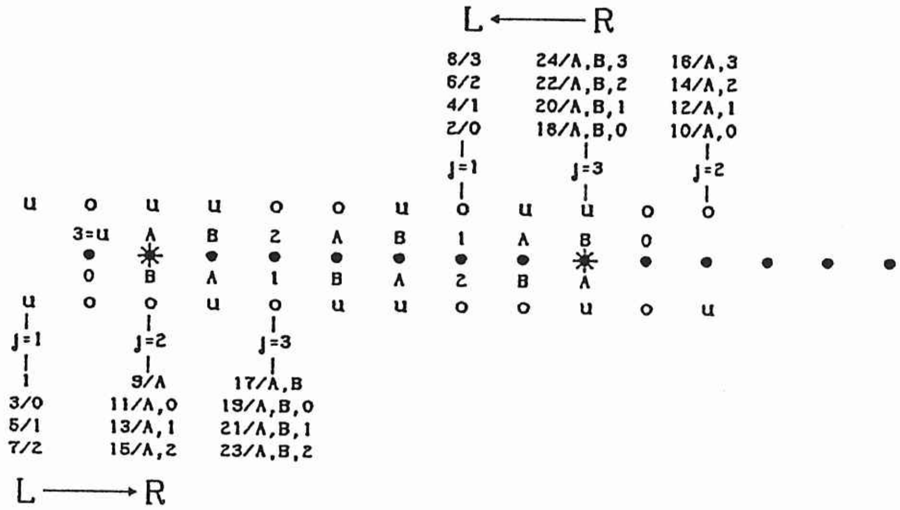
$|-p|_b = |-9|_{12} = 3$ $\qquad$ $|p|_r = |9|_3 = 0$

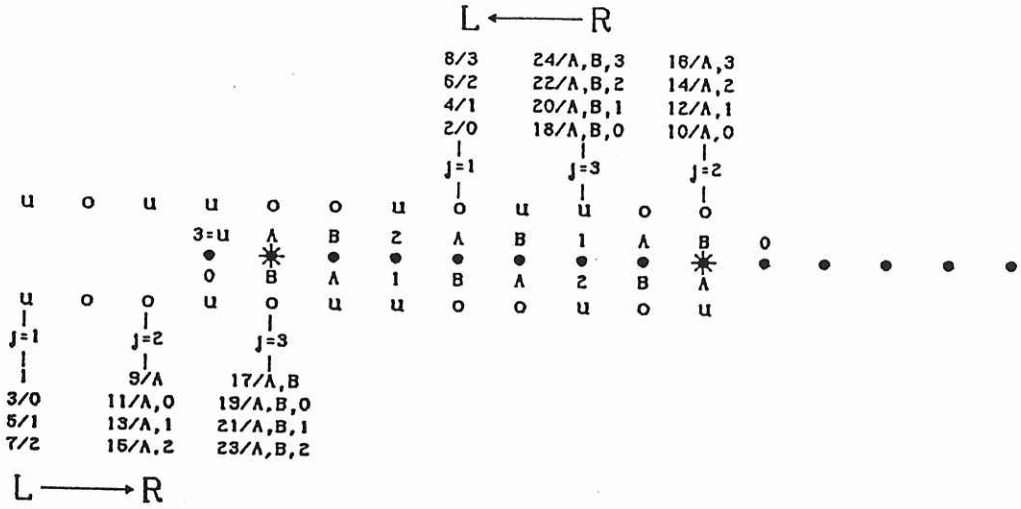


THE COMPLEMENTARY CYCLIC BIGHT-NUMBER SCHEME




L ← R
8/3
6/2
4/1
2/0
j=1
24/A,B,3
22/A,B,2
20/A,B,1
18/A,B,0
j=3
16/A,3
14/A,2
12/A,1
10/A,0
j=2
u o u u o o u o u u o o
3=u A B 2 A B 1 A B 0
0 B A 1 B A 2 B A
u o o u o u u o o u o u
j=1
1
3/0
5/1
7/2
j=2
9/A
11/A,0
13/A,1
15/A,2
j=3
17/A,B
19/A,B,0
21/A,B,1
23/A,B,2
L → R


L ← R
8/3
6/2
4/1
2/0
j=1
24/A,B,3
22/A,B,2
20/A,B,1
18/A,B,0
j=3
16/A,3
14/A,2
12/A,1
10/A,0
j=2
u o u u o o u o u u o o
3=u A B 2 A B 1 A B 0
0 B A 1 B A 2 B A
u o o u o u u o o u o u
j=1
1
3/0
5/1
7/2
j=2
9/A
11/A,0
13/A,1
15/A,2
j=3
17/A,B
19/A,B,0
21/A,B,1
23/A,B,2
L → R


L ← R
8/3
6/2
4/1
2/0
j=1
24/A,B,3
22/A,B,2
20/A,B,1
18/A,B,0
j=3
16/A,3
14/A,2
12/A,1
10/A,0
j=2
u o u u o o u o u u o o
3=u A B 2 A B 1 A B 0
0 B A 1 B A 2 B A
u o o u o u u o o u o u
j=1
1
3/0
5/1
7/2
j=2
9/A
11/A,0
13/A,1
15/A,2
j=3
17/A,B
19/A,B,0
21/A,B,1
23/A,B,2
L → R

Count a further 0 positions to the right in cyclic fashion, hence half-cycle 5 comes under $j = 1$. Count another 0 positions to the right in cyclic fashion, hence half-cycle 7 comes under $j = 1$.

For the right to left direction we obtain: Half-cycle 2 comes under $j = 1$. Count $|p|_r = 0$ positions to the left in cyclic fashion, hence half-cycle 4 comes under $j = 1$. Count a further $|p|_r = 0$ positions to the left in cyclic fashion, hence half-cycle 6 comes under $j = 1$. Count another $|p|_r = 0$ positions to the left in cyclic fashion, hence half-cycle 8 comes under $j = 1$.

The second essential string-run consists of two components: $1 — 10 — 7 — 4$, and $4 — 1$. These components are associated with the left to right half-cycles 9, 11, 13; 15, and the right to left half-cycles 10, 12, 14; 16.

For the left to right direction we obtain: Half-cycle 9 comes under $j = 2$ ($j = |1+1|_r = |2|_3 = 2$). Count $|p|_r = 0$ positions to the right (in cyclic fashion), hence half-cycle 11 comes under $j = 2$. Count a further 0 positions to the right in cyclic fashion, hence half-cycle 13 comes under $j = 2$. Count another 0 positions to the right in cyclic fashion, hence half-cycle 15 comes under $j = 2$.

For the right to left direction we obtain: Half-cycle 10 comes under $j = 2$. Count $|p|_r = 0$ positions to the left in cyclic fashion, hence half-cycle 12 comes under $j = 2$. Count a further $|p|_r = 0$ positions to the left in cyclic fashion, hence half-cycle 14 comes under $j = 2$. Count another $|p|_r = 0$ positions to the left in cyclic fashion, hence half-cycle 16 comes under $j = 2$.

The third essential string-run consists of one component: $2 — 11 — 8 — 5 — 2$. This component is associated with the left to right half-cycles 17, 19, 21, 23, and the right to left half-cycles 18, 20, 22, 24.

For the left to right direction we obtain: Half-cycle 17 comes under $j = 3$ ($j = |2+1|_r = |3|_3 = 3$). Count $|p|_r = 0$ positions to the right in cyclic fashion, hence half-cycle 19 comes under $j = 3$. Count a further 0 positions to the right in cyclic fashion, hence half-cycle 21 comes under $j = 3$. Count another 0 positions to the right in cyclic fashion, hence half-cycle 23 comes under $j = 3$.

For the right to left direction we obtain: Half-cycle 18 comes under $j = 3$. Count $|p|_r = 0$ positions to the left in cyclic fashion, hence half-cycle 20 comes under $j = 3$. Count a further 0 positions to the left in cyclic fashion, hence half-cycle 22 comes under $j = 3$. Count another 0 positions to the left in cyclic fashion, hence half-cycle 24 comes under $j = 3$.

The left-most dot of the central strip represents the left boundary of the end-keeper, and since all the transition points have the same coding, this dot is in this case always associated with an *under-coding* for the right to left half-cycles which go through the transition-points.

The algorithm-diagram gives us the following half-cycle algorithms (see pg. 78):

First component $0 — 9 — 6 — 3$, starting at transition index-number 0:

1). $L \longrightarrow R$: Free Run.
2). $(i = 0)$ $R \longrightarrow L$: Free Run.
3). $(i = 0)$ $L \longrightarrow R$: Free Run.
4). $(i = 1)$ $R \longrightarrow L$: o.
5). $(i = 1)$ $L \longrightarrow R$: o.
6). $(i = 2)$ $R \longrightarrow L$: $o - u$.

Second component 3 — 0, starting at transition index-number 3:
7). $(i = 2)$ $L \longrightarrow R:$ $2o - u.$
8). $(i = 3)$ $R \longrightarrow L:$ $o - 2u.$

Note that the first 'o' in 7) is in transition point '3', and that the last 'u' in 8) is in transition-point '0'.

Third component 1 — 10 — 7 — 4, starting at transition index-number 1:
9). $(i = A)$ $L \longrightarrow R:$ $3u.$
10). $(i = A; 0)$ $R \longrightarrow L:$ $3o.$
11). $(i = A; 0)$ $L \longrightarrow R:$ $3u.$
12). $(i = A; 1)$ $R \longrightarrow L:$ $o - u - 2o.$
13). $(i = A; 1)$ $L \longrightarrow R:$ $u - o - 2u.$
14). $(i = A; 2)$ $R \longrightarrow L:$ $o - u - o - u - o.$

Fourth component 4 — 1, starting at transition index-number 4:
15). $(i = A; 2)$ $L \longrightarrow R:$ $o - u - o - u - o - u.$
16). $(i = A; 3)$ $R \longrightarrow L:$ $o - u - o - u - o - u.$

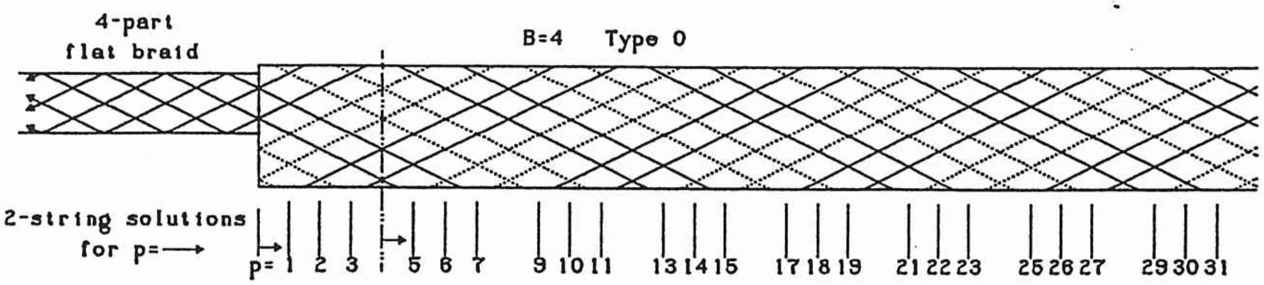
Note that the first 'o' in 15) is in transition point '4', and that the last 'u' in 16) is in transition-point '1'.

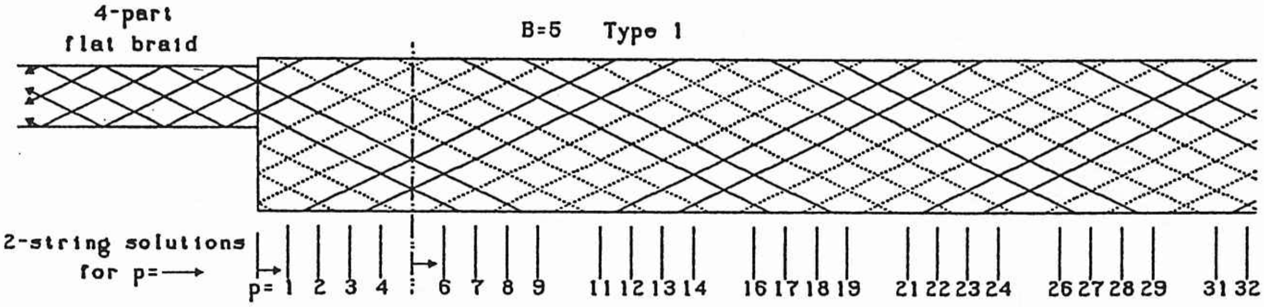
Fifth component 2 — 11 — 8 — 5 — 2, starting at transition index-number 2:
17). $(i = A, B)$ $L \longrightarrow R:$ $o - u - 3o - u.$
18). $(i = A, B; 0)$ $R \longrightarrow L:$ $3u - o - 2u.$
19). $(i = A, B; 0)$ $L \longrightarrow R:$ $o - u - 3o - u.$
20). $(i = A, B; 1)$ $R \longrightarrow L:$ $2u - o - u - o - 2u.$
21). $(i = A, B; 1)$ $L \longrightarrow R:$ $o - 2u - 3o - u.$
22). $(i = A, B; 2)$ $R \longrightarrow L:$ $2u - o - u - 2o - 2u.$
23). $(i = A, B; 2)$ $L \longrightarrow R:$ $o - 2u - 2o - u - o - u.$
24). $(i = A, B; 3)$ $R \longrightarrow L:$ $2u - o - u - 2o - 3u.$

Note that the last 'u' in 24) is in transition-point '2'.

Although the half-cycle algorithms of regular end-keepers which are neither column-coded nor row-coded can be obtained by calculation, however, since the procedures are in general a little more cumbersome it is therefore more convenient to obtain them via a suitable computer program. We shall treat this in a future Pamphlet.

On the following eleven pages (81 — 91) are depicted the general string-run diagrams for some common end-keepers. The valid p-values are indicated in each of these general string-run diagrams.


4-part
flat braid
B=4 Type 0
2-string solutions
for p=
p= 1 2 3 5 6 7 9 10 11 13 14 15 17 18 19 21 22 23 25 26 27 29 30 31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flat braid
B=5 Type 1
2-string solutions
for p=
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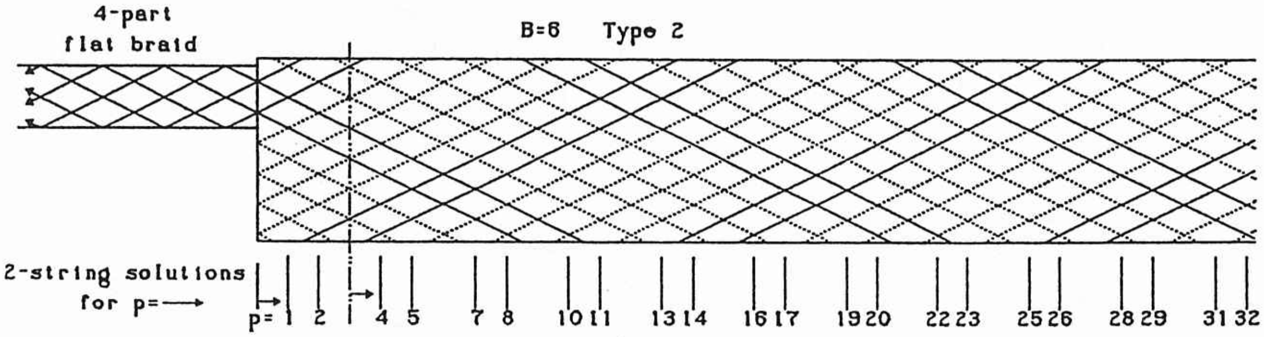
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B=6 Type 2
2-string solutions
for p=
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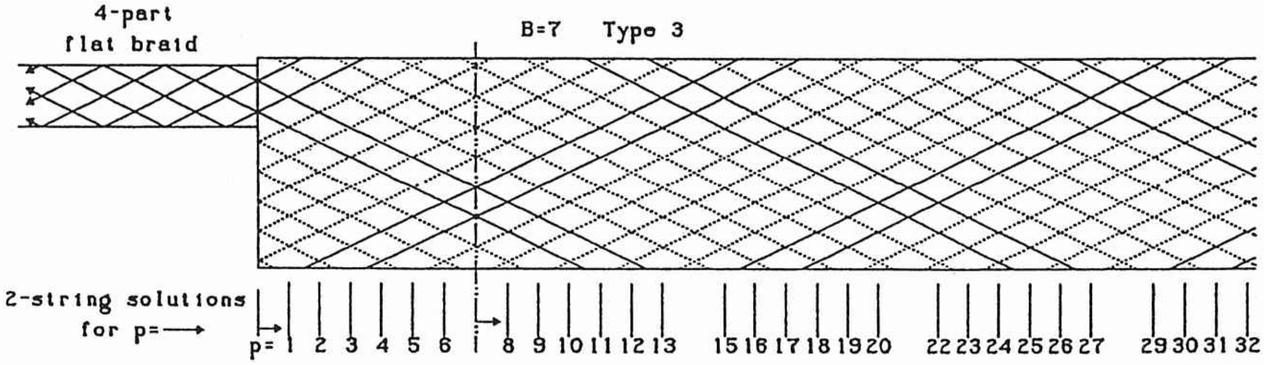
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B=7 Type 3
2-string solutions
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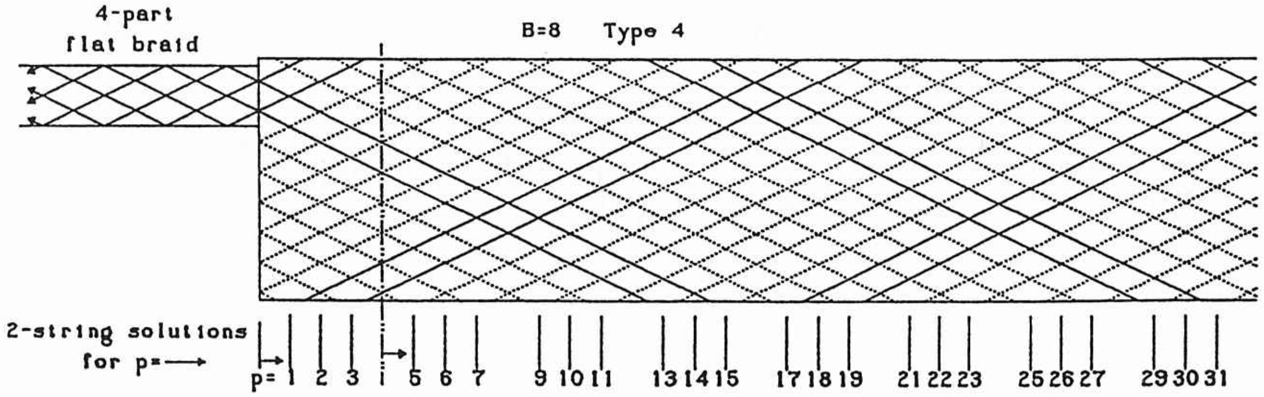
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flat braid
B=8 Type 4
2-string solutions
for p=
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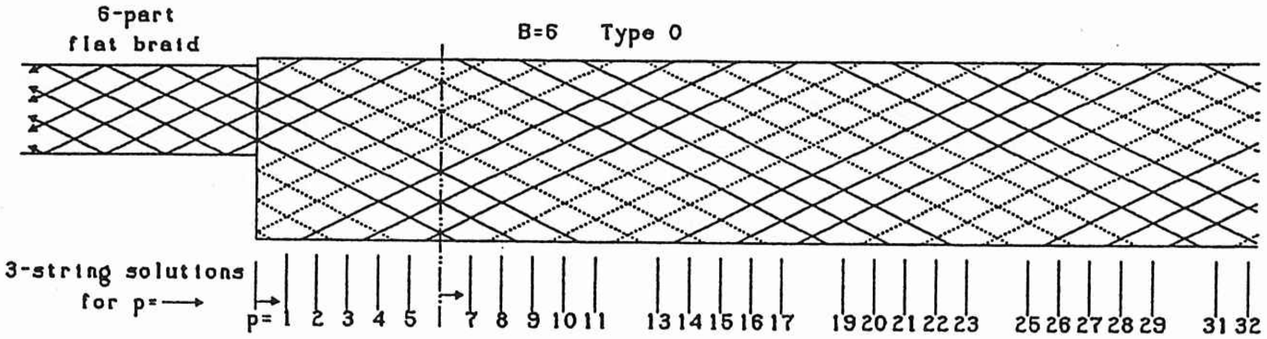
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flat braid
B=6 Type 0
3-string solutions
for p=
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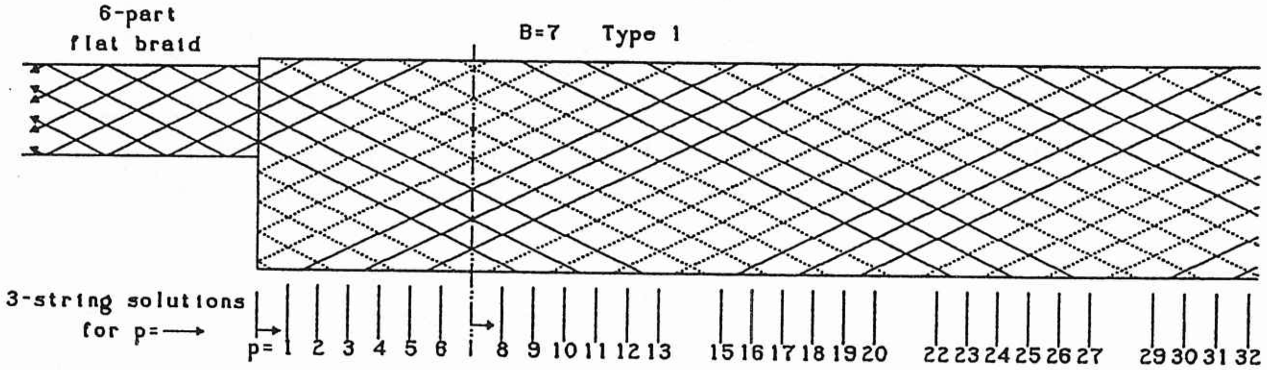
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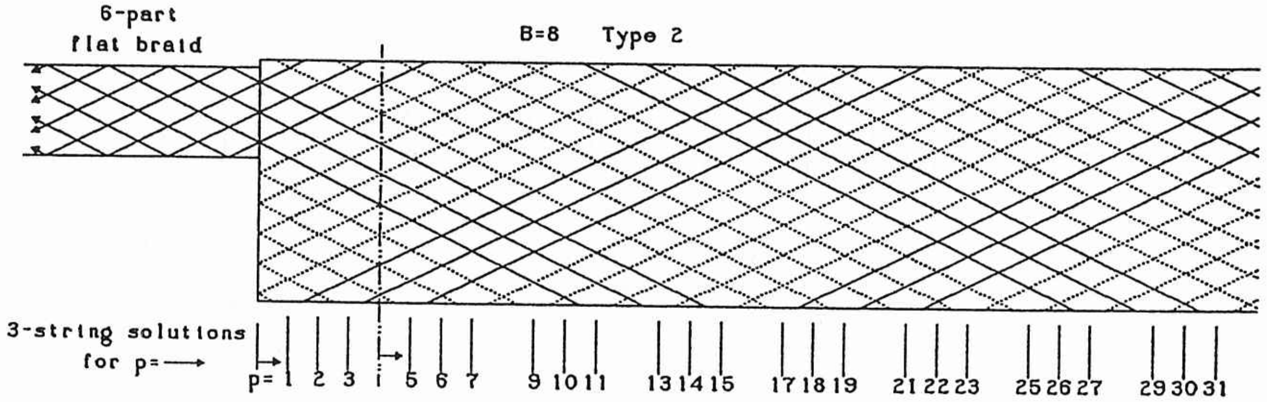
B=7 Type 1
3-string solutions
for p=
p= 1 2 3 4 5 6 8 9 10 11 12 13 15 16 17 18 19 20 22 23 24 25 26 27 29 30 31 32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3-string solutions
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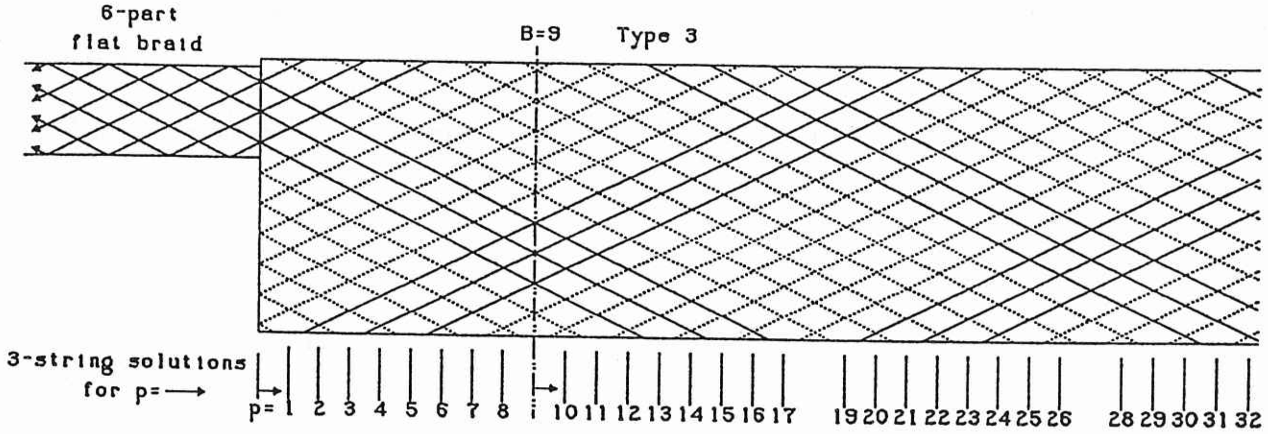
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flat braid
B=9 Type 3
3-string solutions
for p=
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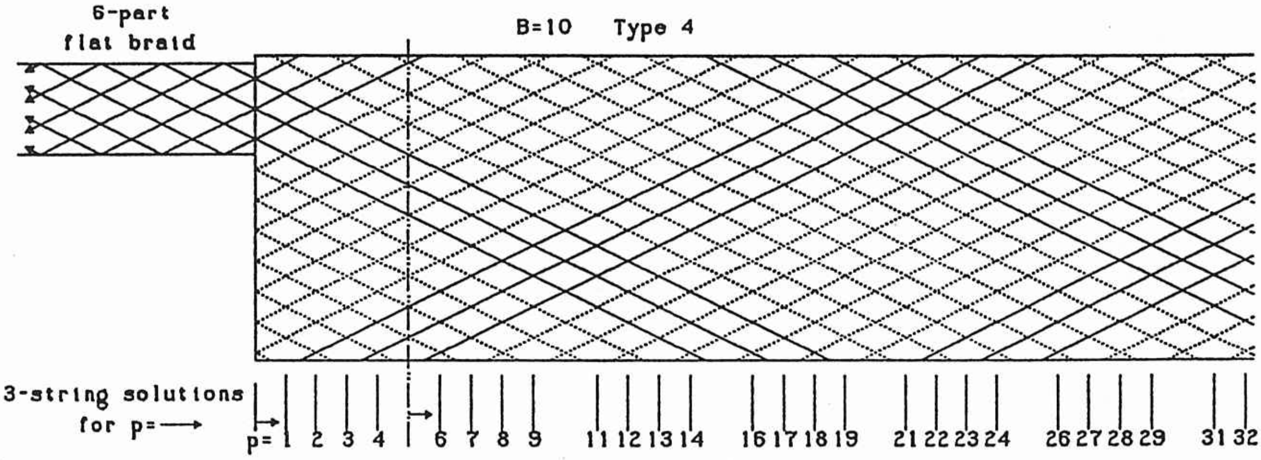
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B=10
Type 4
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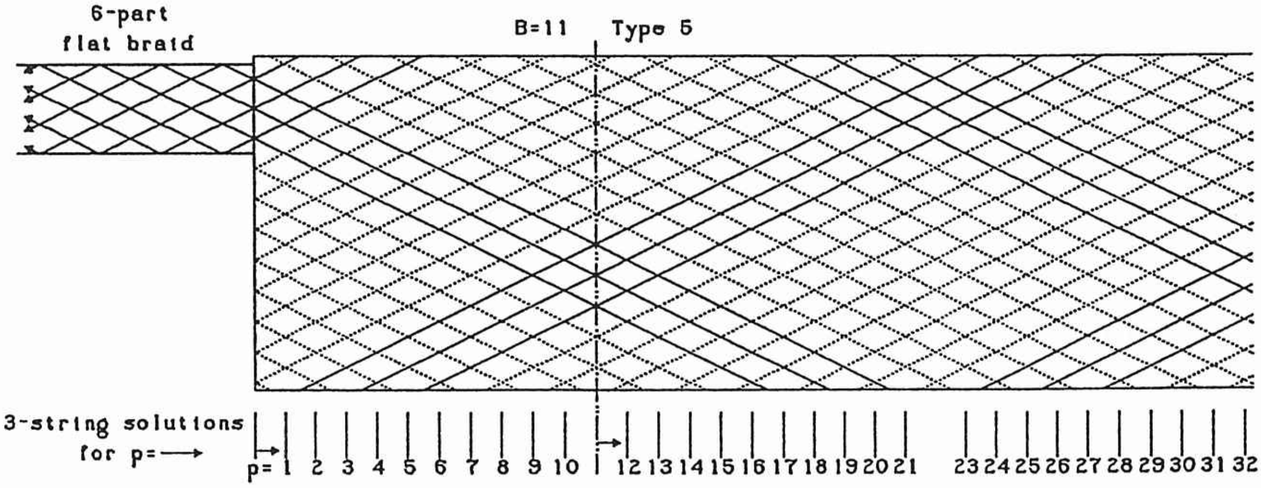
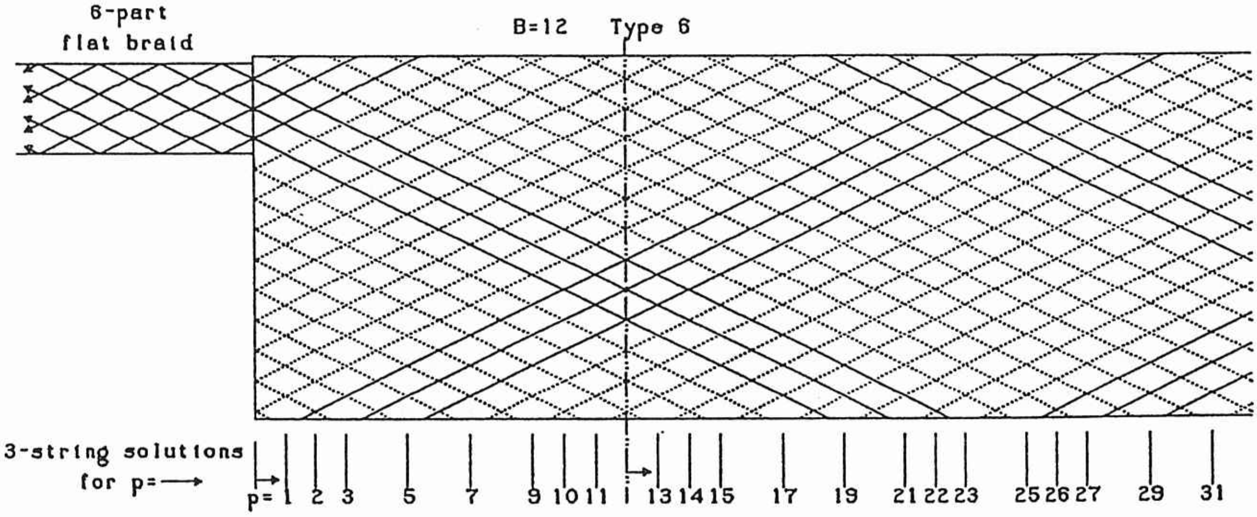

6-part
flat braid
B=11
Type 5
3-string solutions
for p=
p= 1 2 3 4 5 6 7 8 9 10 12 13 14 15 16 17 18 19 20 21 23 24 25 26 27 28 29 30 31 32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flat braid
B=12
Type 6
3-string solutions
for p=
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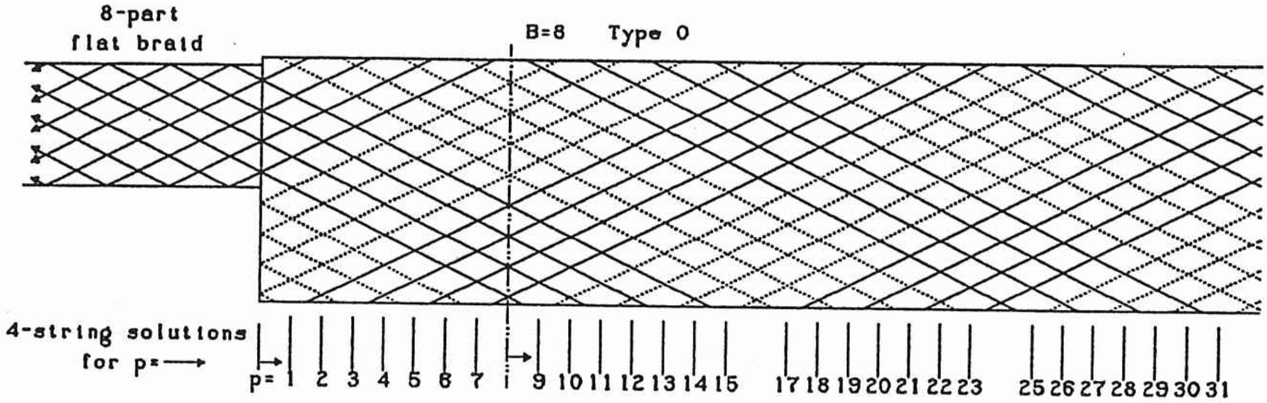
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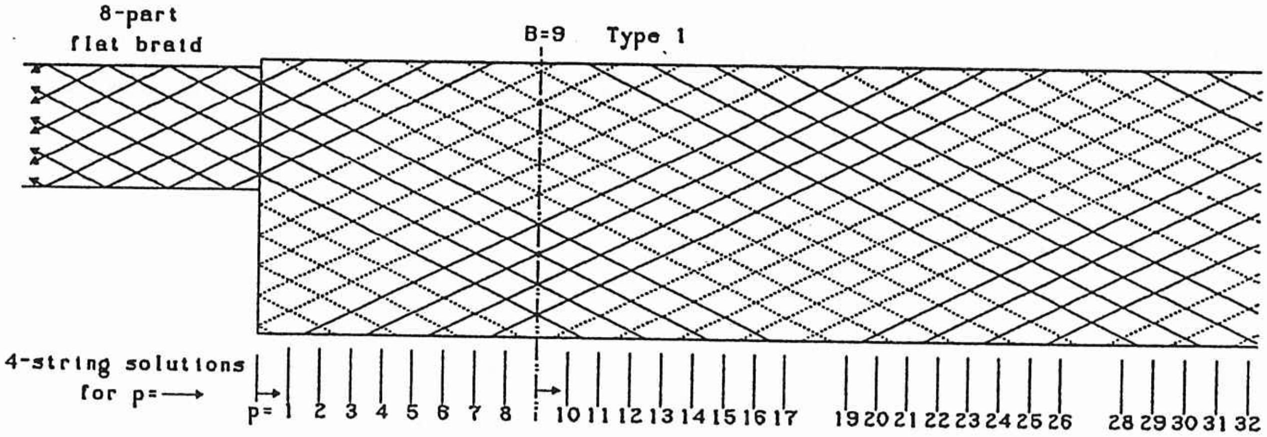
B=8 Type 0
4-string solutions for p=⟶
p= 1 2 3 4 5 6 7 9 10 11 12 13 14 15 17 18 19 20 21 22 23 25 26 27 28 29 30 31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B=9 Type 1
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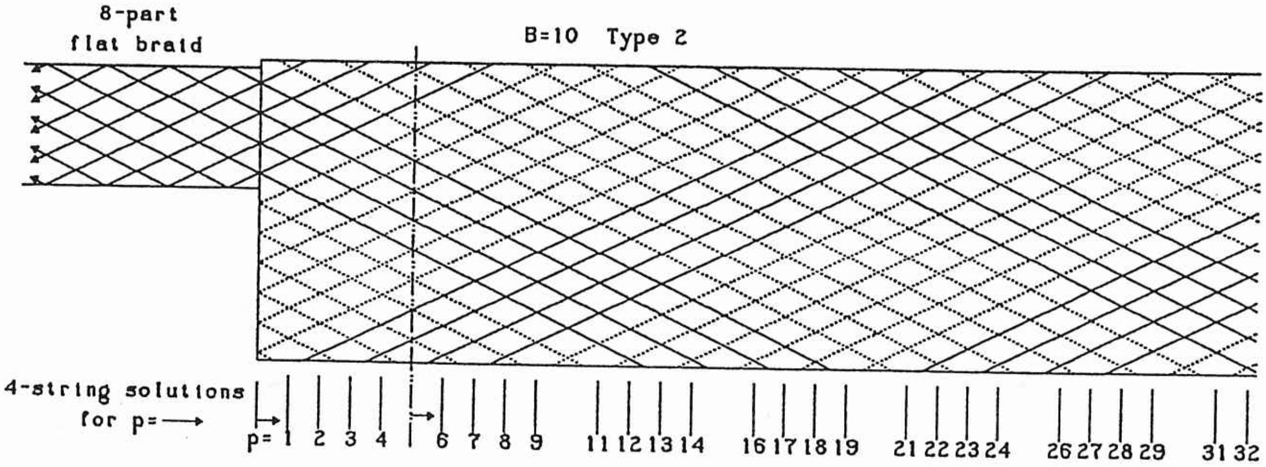
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B=10 Type 2
4-string solutions for p=⟶
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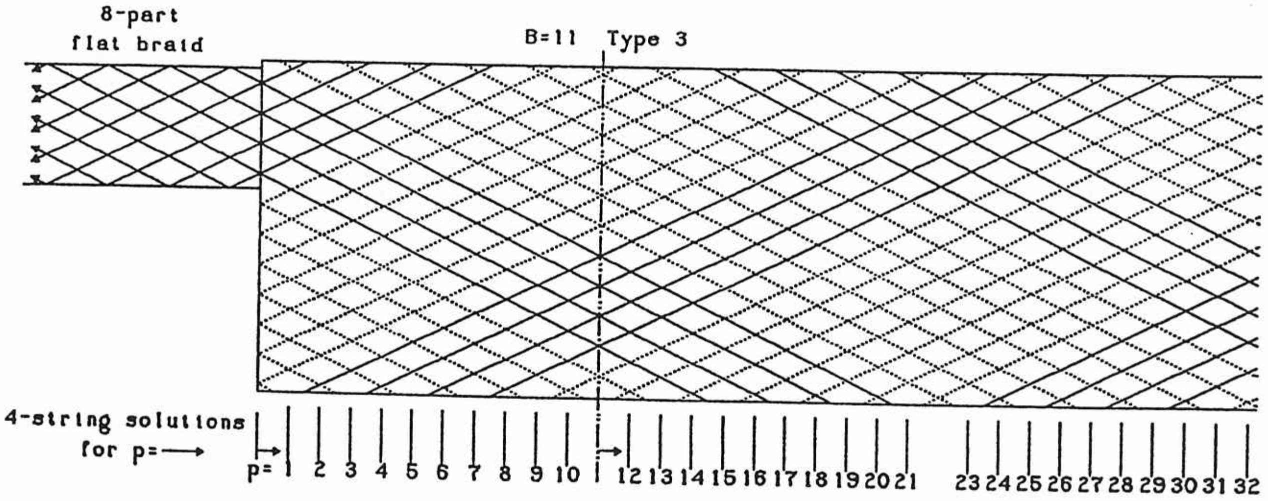
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B=11 Type 3
4-string solutions for p=⟶
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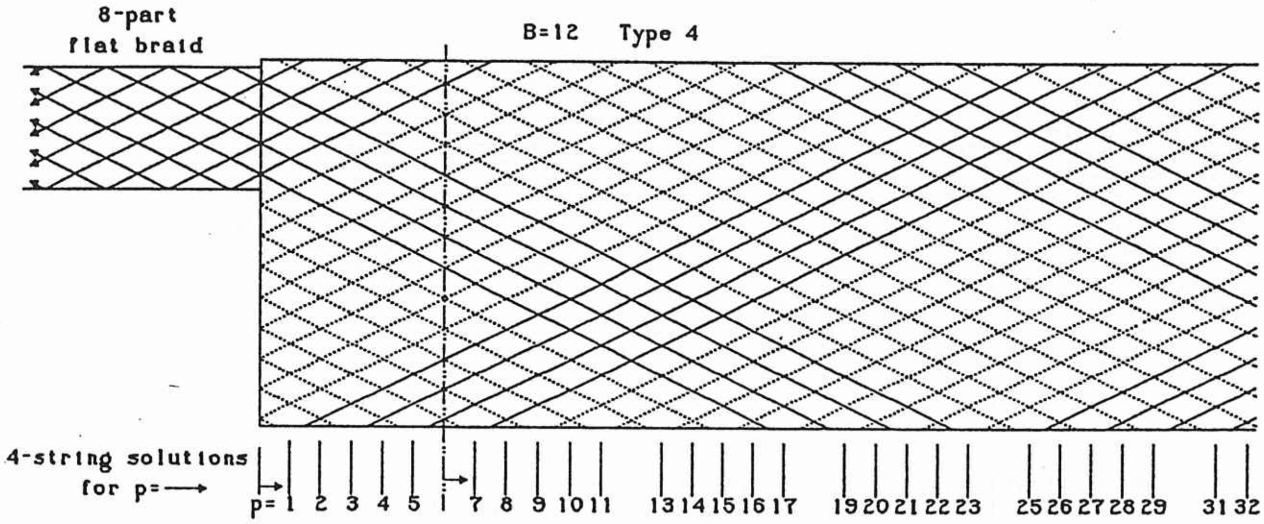
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B=12 Type 4
4-string solutions
for p=
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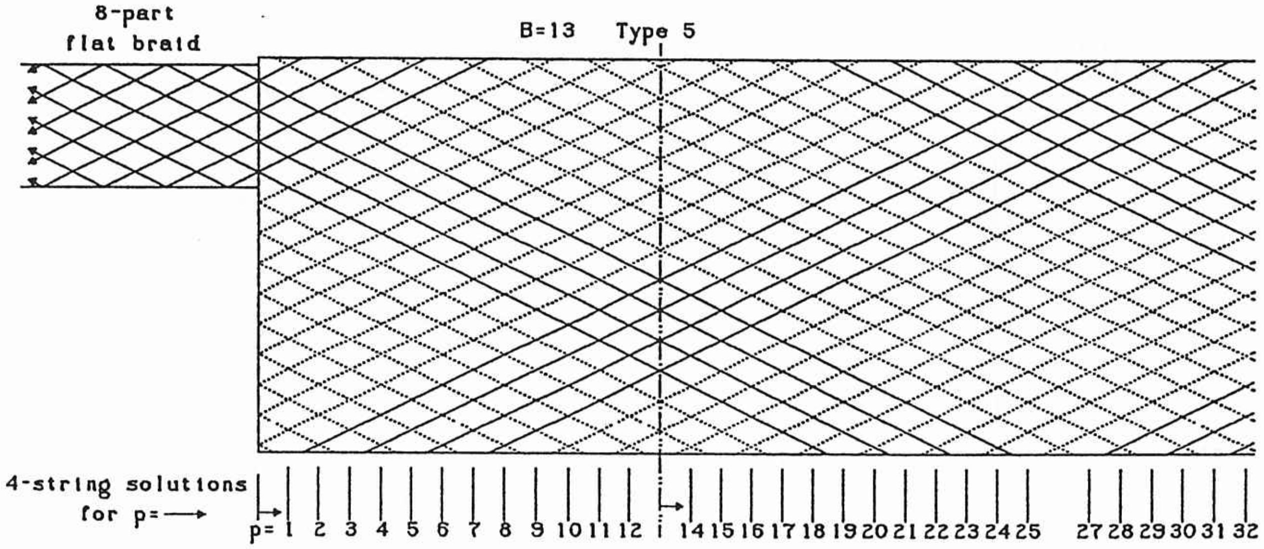
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B=13 Type 5
4-string solutions
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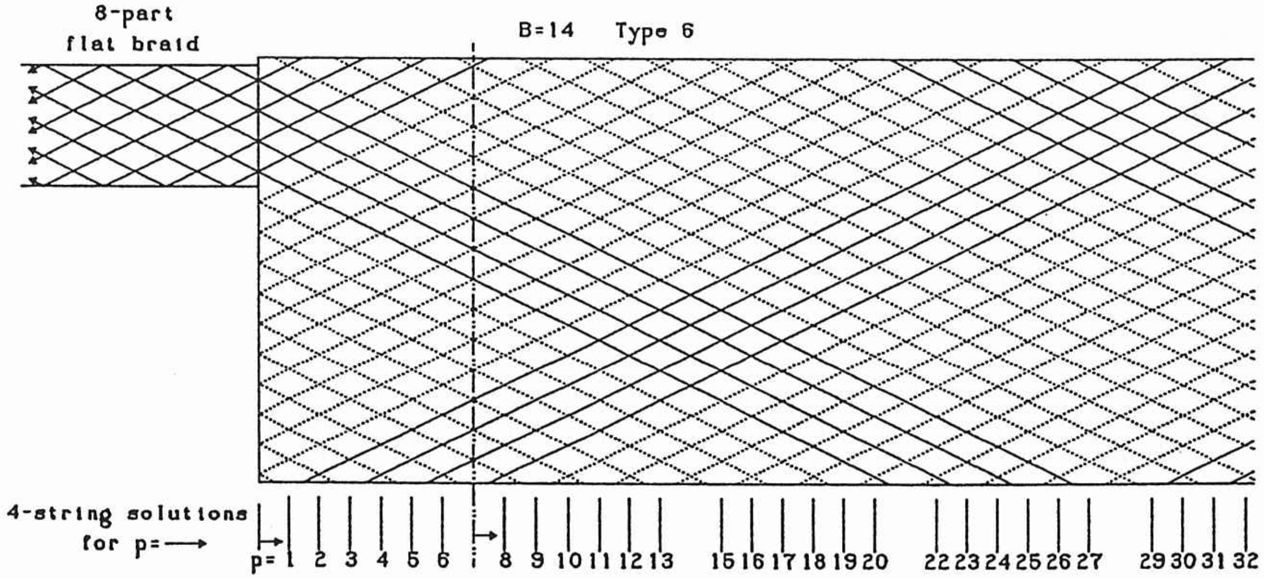
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B=14 Type 6
4-string solutions
for p=
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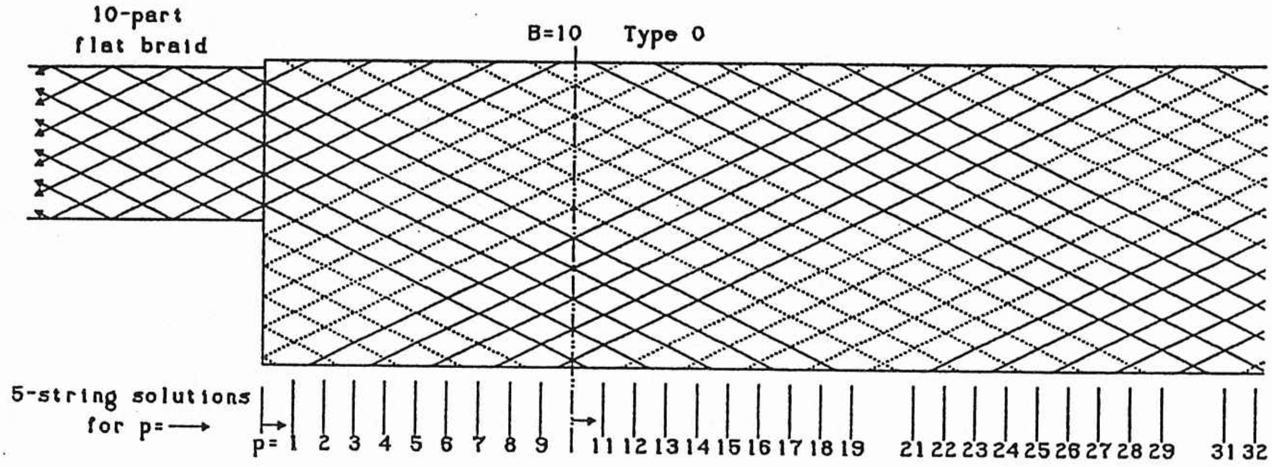
10-part
flat braid
B=10
Type 0
5-string solutions
for p=⟶
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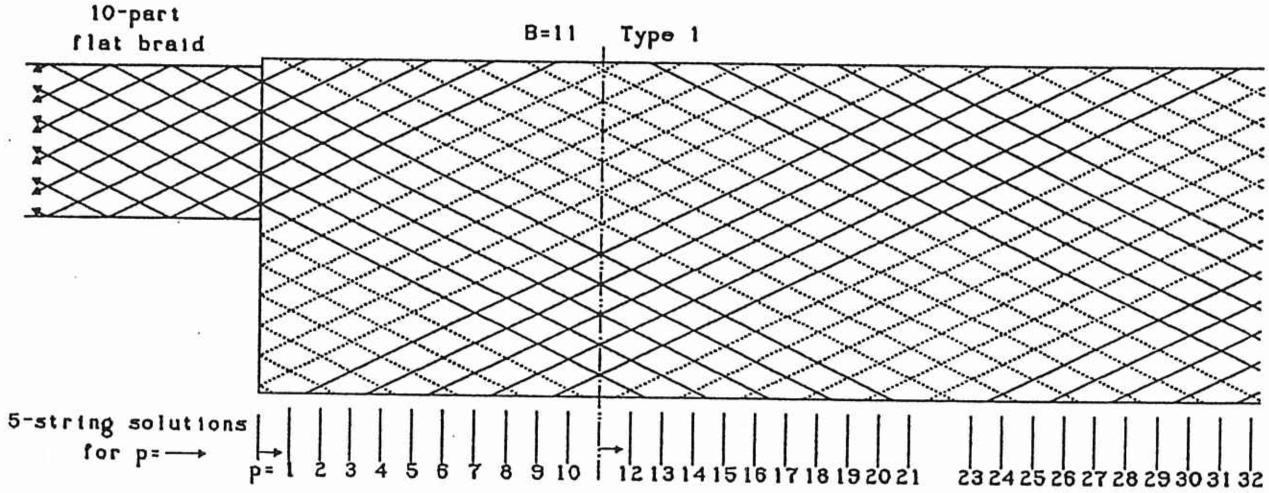
11 12 13 14 15 16 17 18 19
21 22 23 24 25 26 27 28 29
31 32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flat braid
B=11
Type 1
5-string solutions
for p=⟶
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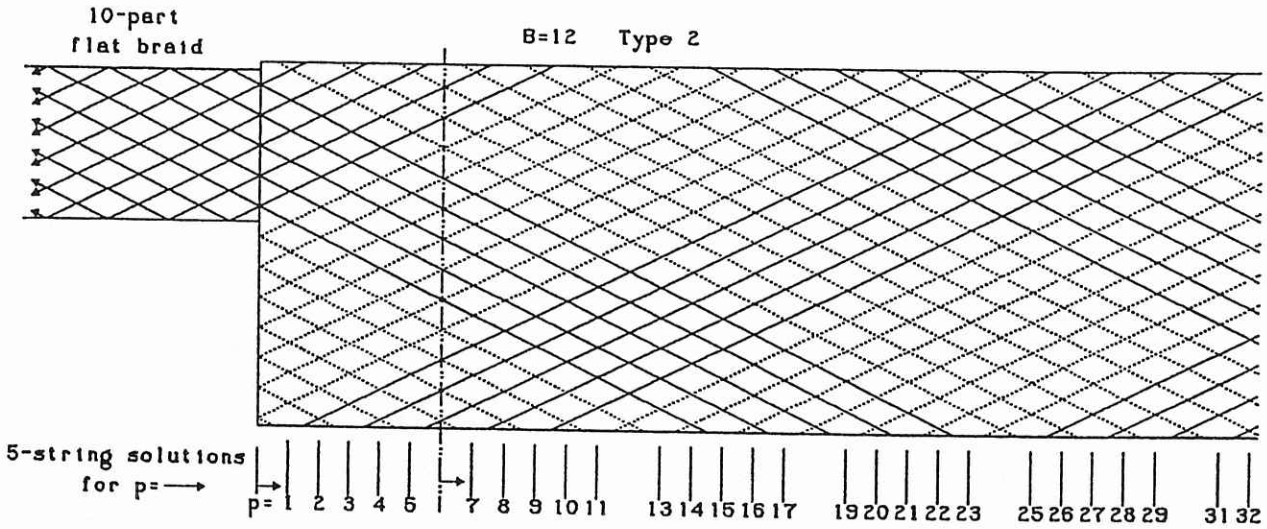
12 13 14 15 16 17 18 19 20 21
23 24 25 26 27 28 29 30 31 32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flat braid
B=12
Type 2
5-string solutions
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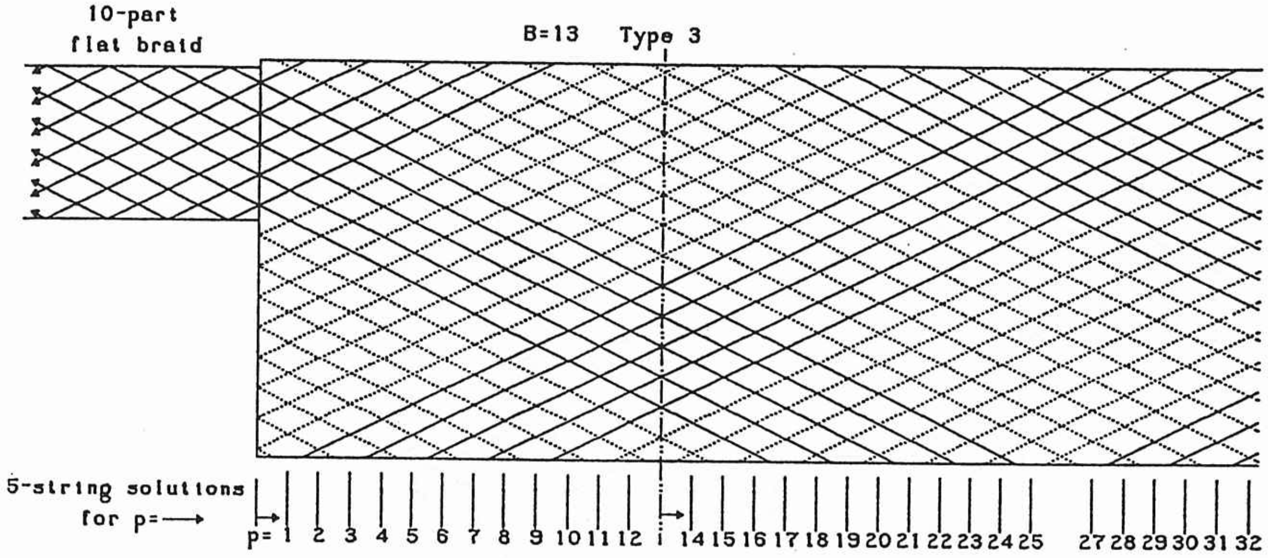
13 14 15 16 17
19 20 21 22 23
25 26 27 28 29
31 32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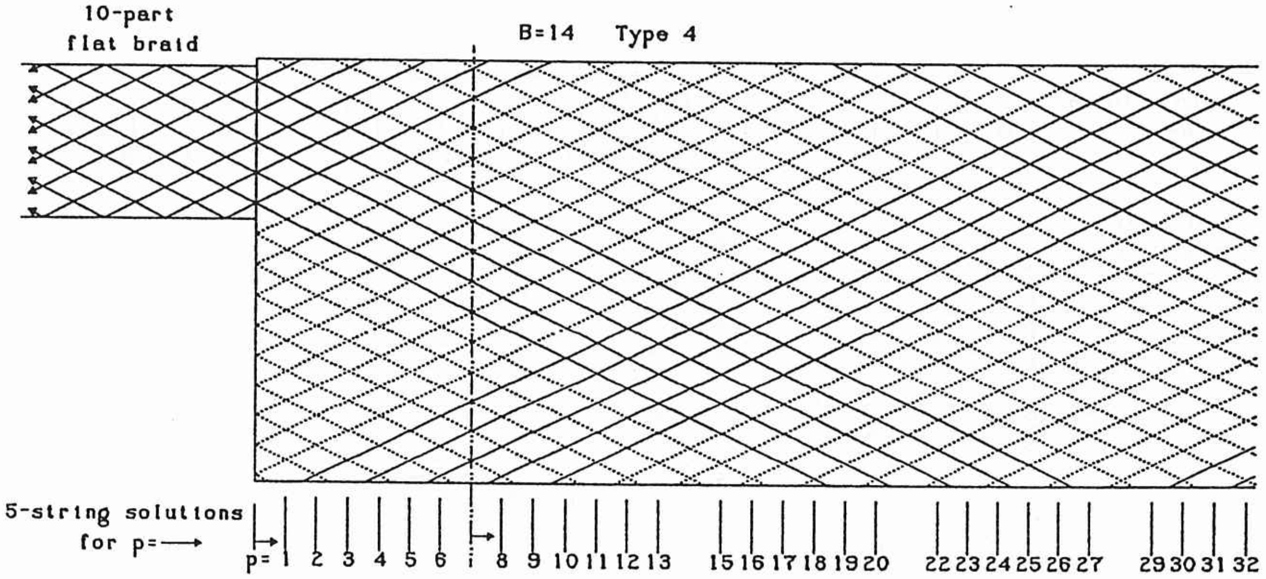
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for p=
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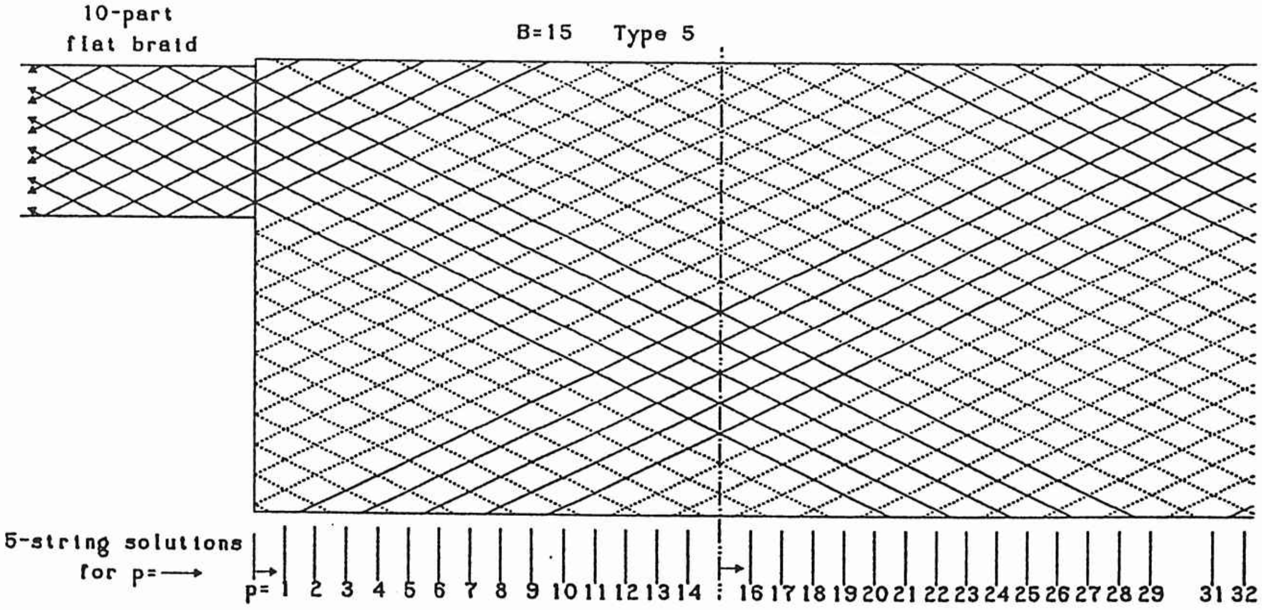
10-part
flat braid
B=15 Type 5
5-string solutions
for p=
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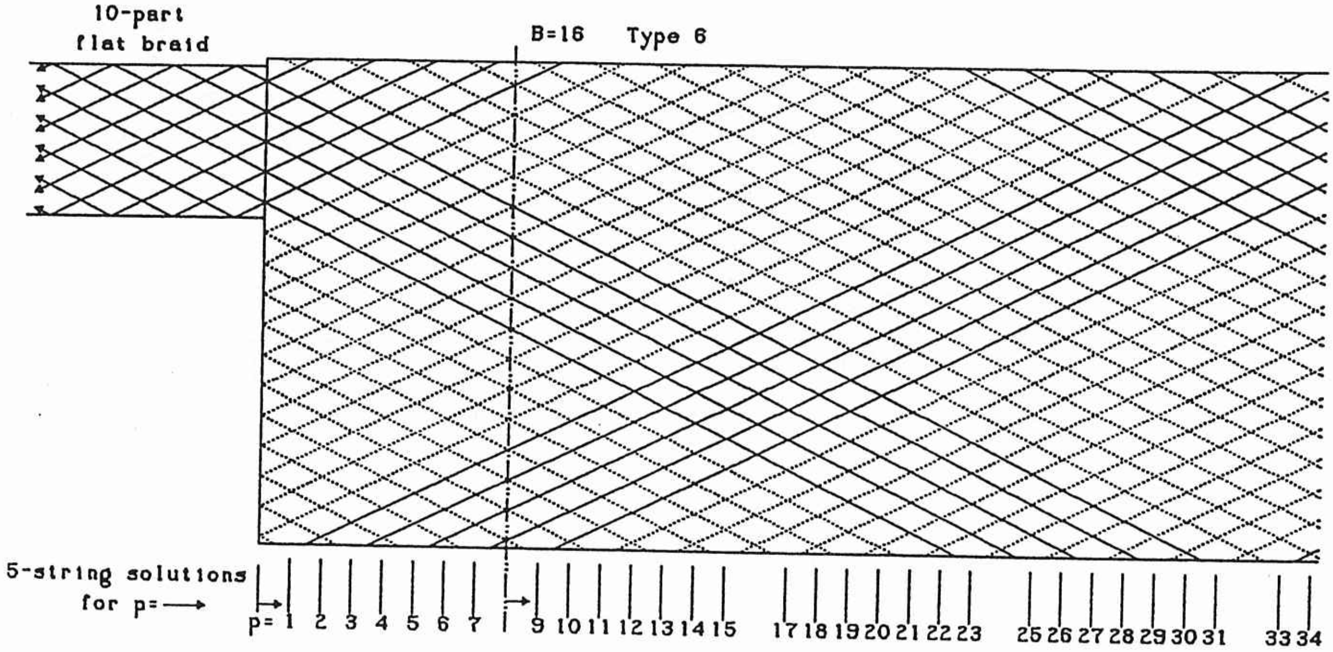
10-part
flat braid
B=16
Type 6
5-string solutions
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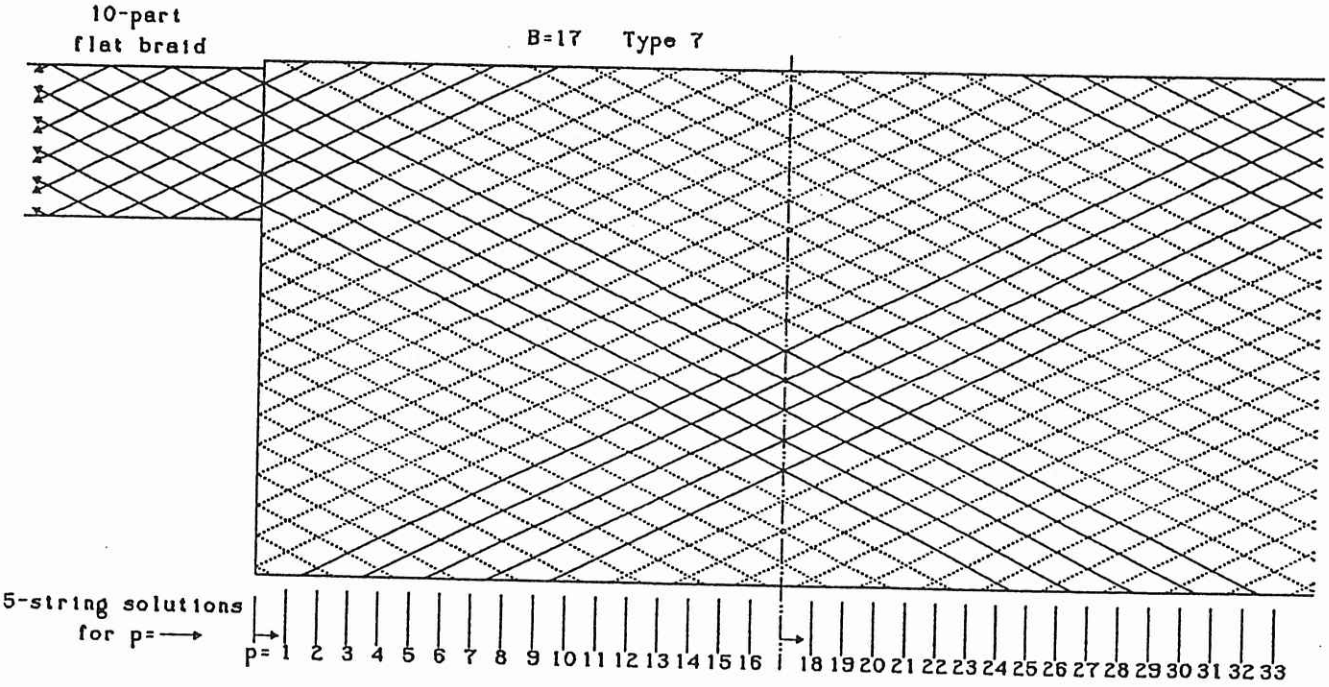
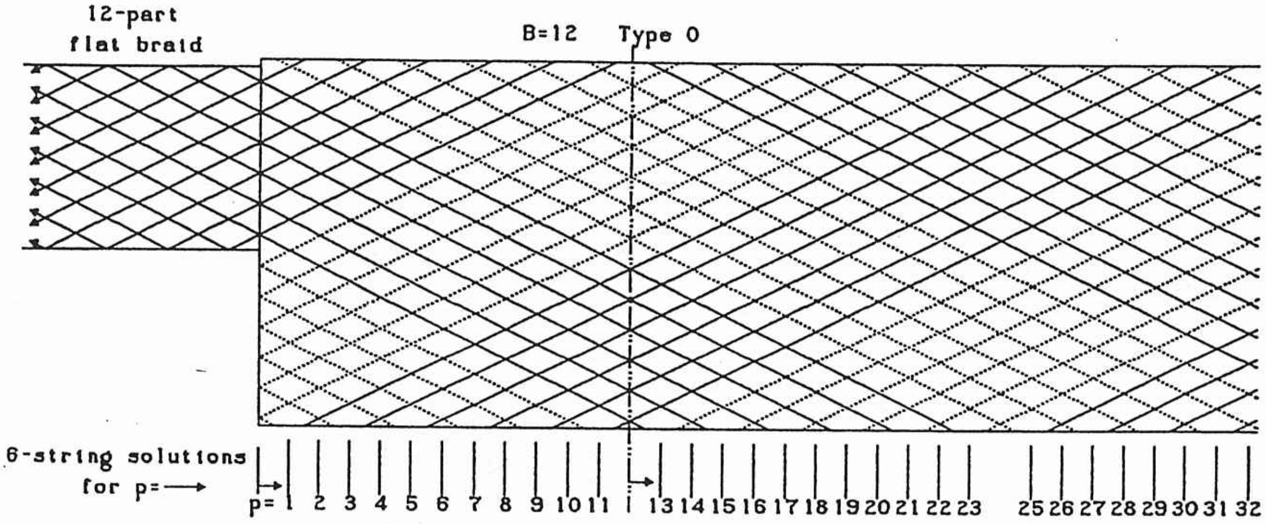
9 10 11 12 13 14 15
17 18 19 20 21 22 23
25 26 27 28 29 30 31
33 34


10-part
flat braid
B=17
Type 7
5-string solutions
for p=
p= 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33


12-part
flat braid
B=12
Type 0
6-string solutions
for p=
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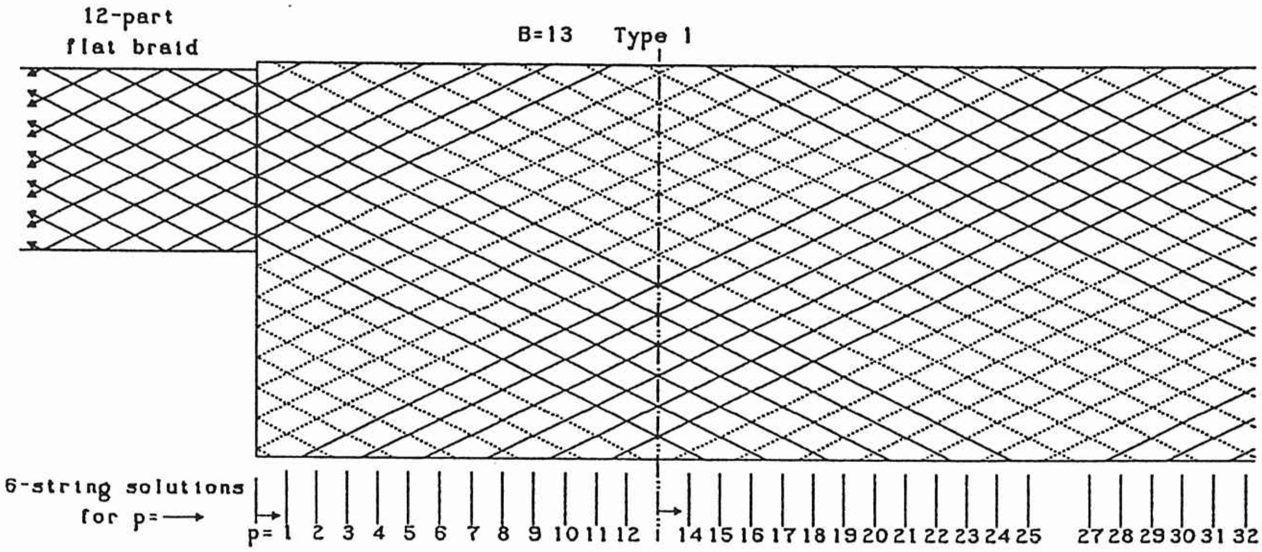
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flat braid
B=13
Type 1
6-string solutions
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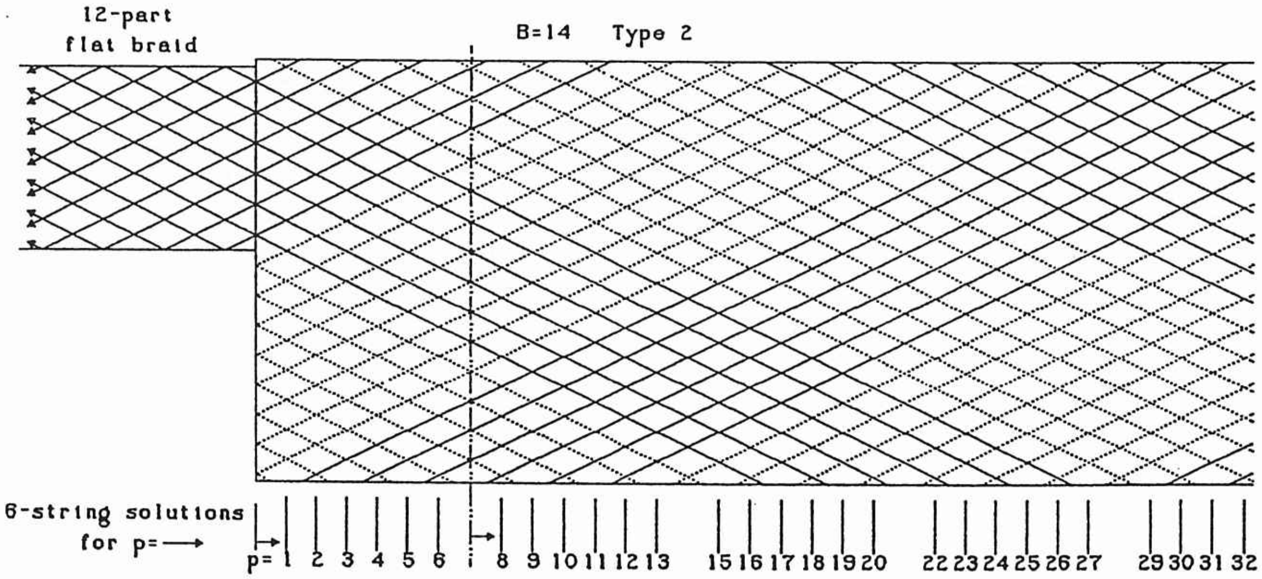
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B=14
Type 2
6-string solutions
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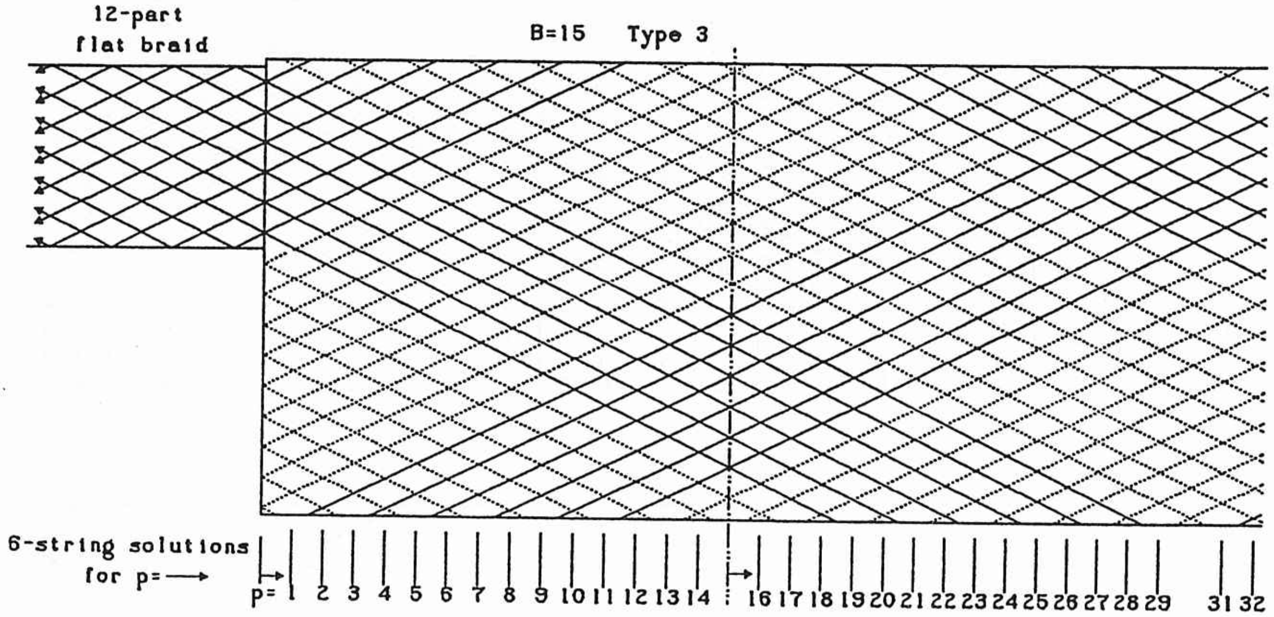
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6-string solutions
for p=⟶
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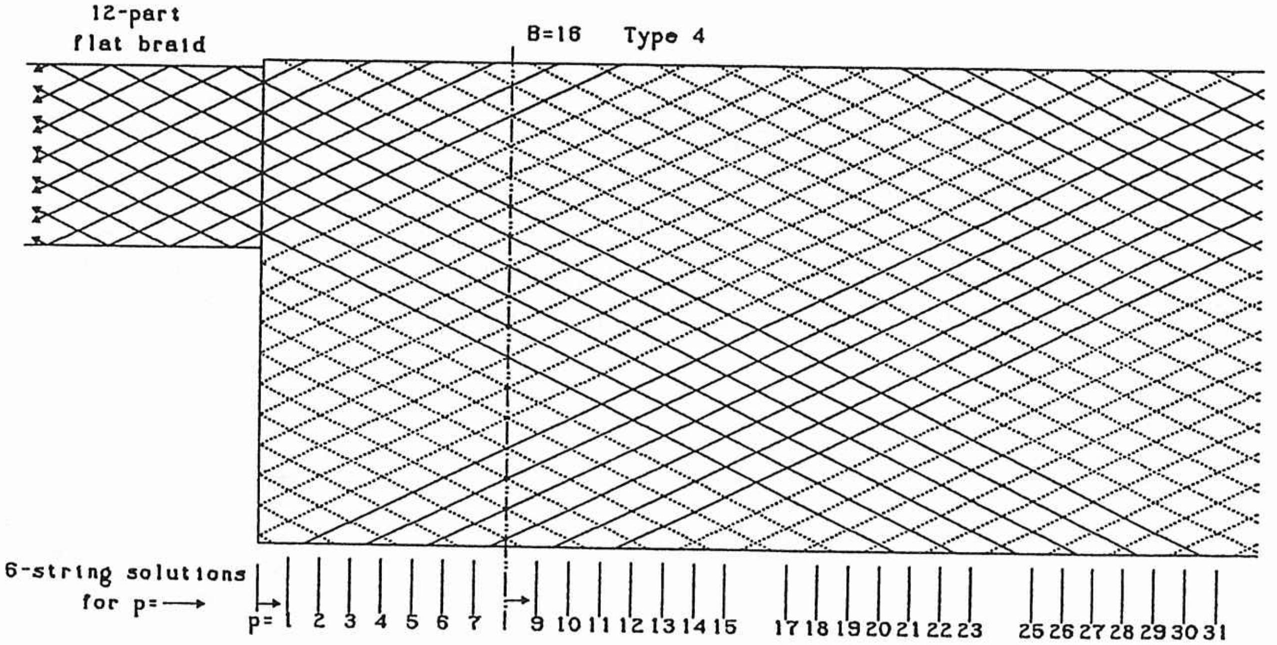
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for p=⟶
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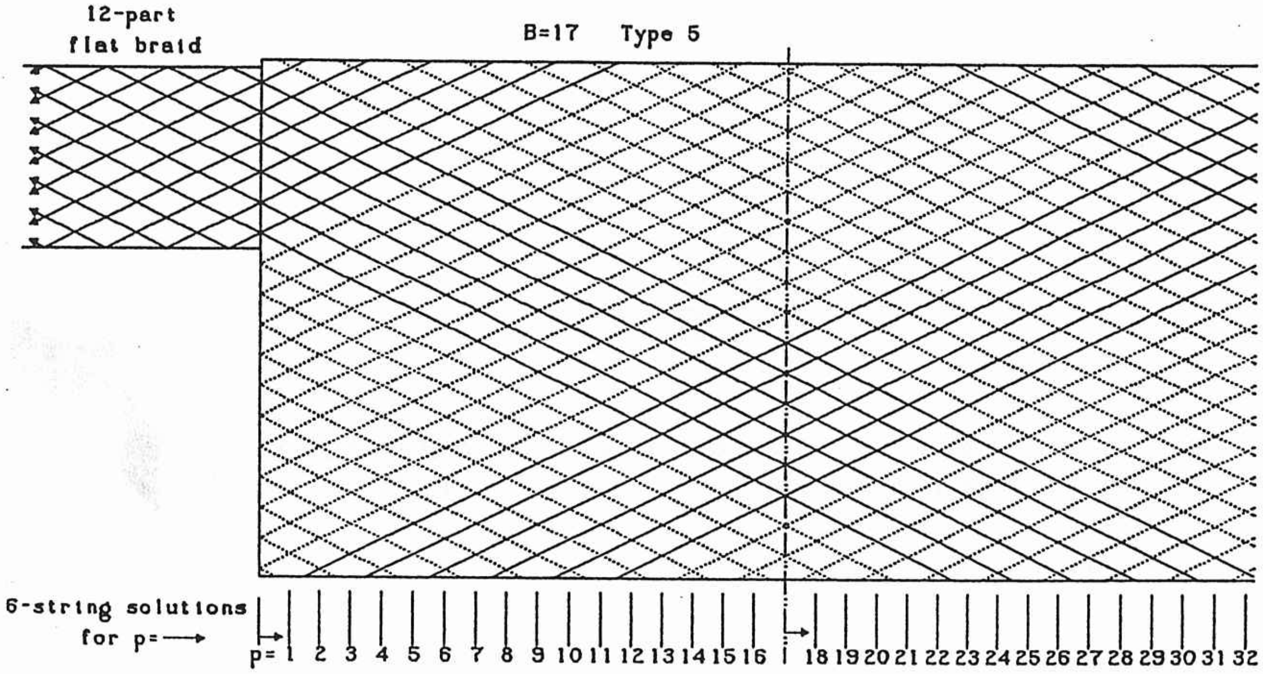
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for p=
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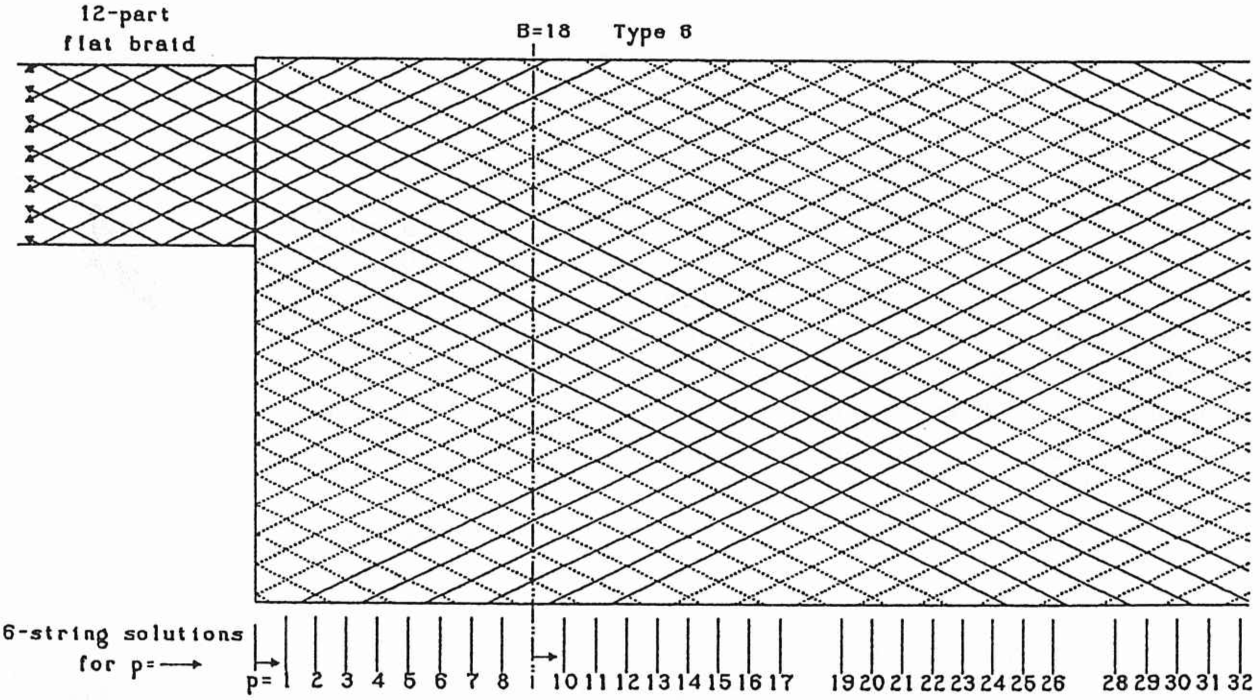
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B=18 Type 6
6-string solutions
for p=
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PROOFS

The component-table can contain at most three different parts/bights specifications:

start transition index-number: end transition index-number:

$$\begin{array}{lcl}
0 & \longrightarrow & y \\
1 & \longrightarrow & y+1 \\
2 & \longrightarrow & y+2 \\
 & \vdots & \\
x-1-y & \longrightarrow & x-1 = y+(x-1-y) \\
x-y & \longrightarrow & z \\
x-y+1 & \longrightarrow & z+1 \\
 & \vdots & \\
x-y+(y-1-z) = x-1-z & \longrightarrow & y-1 = z+(y-1-z) \\
x-1-z+1 = x-z & \longrightarrow & 0
\end{array}$$

Since $x-1-z+1 = x-z \longrightarrow y$ and since $0 \longrightarrow y$, hence the component string-run starting at transition index-number $x-z$ goes through transition index-number 0, and hence ends at transition index-number 0 instead of continuing to transition index-number y. There are still $x-1-(x-z) = z-1$ transition index-points left where the string-runs of further components will start.

$$\begin{array}{lcl}
x-z+1 & \longrightarrow & 1 \\
x-z+2 & \longrightarrow & 2 \\
 & \vdots & \\
x-z+(z-1) = x-1 & \longrightarrow & z-1
\end{array}$$

All transition index-points possess now a start of a component string-run, and hence also an end of a component string-run.

When $z=0$ the final tabulated component starts with the transition index-number $x-y+(y-1-z) = x-1-z = x-1$, and ends with the transition index-number $y-1 = z+(y-1-z)$ for its string-run. Consequently the component-table contains two different parts/bights specifications.

When $y=0$ the final tabulated component starts with the transition index-number $x-1-y = x-1$, and ends with the transition index-number $x-1 = y+(x-1-y)$. Consequently the component-table contains only one parts/bights specification.

Component-tables with three different parts/bights specifications.

From the component entries

$$\begin{array}{lcl} x-1-y & \longrightarrow & x-1=y+(x-1-y) \\ x-y & \longrightarrow & z \end{array}$$

with the parts/bights specifications p_1/b_1 and p_2/b_2 respectively it follows that the last component entry $x-y \longrightarrow z$ can be written as $x-y \longrightarrow x \longrightarrow z$ (note that $z>0$). The part $x-y \longrightarrow x$ has the parts/bights specification p_1/b_1 and the part $x \longrightarrow z$ has the parts/bights specification p_3/b_3 since it has the same parts/bights specification as $x-z+(z-1)=x-1 \longrightarrow z-1$. Hence $p_2=p_1+p_3$ and $b_2=b_1+b_3$. Obviously a Method II enlargement is associated with the component entry $x-1-y \longrightarrow x-1$, and enlarges this component fom p_1/b_1 to p_2/b_2. The parts/bights specification of the overall end-keeper goes from $\{(x-y)p_1+(y-z)p_2+zp_3\}/\{(x-y)b_1+(y-z)b_2+zb_3\}=$ $\{(x-z)p_1+yp_3\}/\{(x-z)b_1+yb_3\}$ to
$\{(x-y-1)p_1+(y-z+1)p_2+zp_3\}/\{(x-y-1)b_1+(y-z+1)b_2+zb_3\}=$
$\{(x-z)p_1+(y+1)p_3\}/\{(x-z)b_1+(y+1)b_3\}$.

From the component entries

$$\begin{array}{lcl} x-y+(y-1-z)=x-1-z & \longrightarrow & y-1=z+(y-1-z) \\ x-1-z+1=x-z & \longrightarrow & 0 \end{array}$$

with the parts/bights specifications p_2/b_2 and p_3/b_3 respectively it follows that a Method I enlargement is obviously associated with the component entry $x-1-z+1=x-z \longrightarrow 0$, and enlarges this component fom p_3/b_3 to p_2/b_2. We have already seen above that $p_2=p_3+p_1$ and $b_2=b_3+b_1$. The parts/bights specification of the overall end-keeper goes from $\{(x-y)p_1+(y-z)p_2+zp_3\}/\{(x-y)b_1+(y-z)b_2+zb_3\}=$ $\{(x-z)p_1+yp_3\}/\{(x-z)b_1+yb_3\}$ to
$\{(x-y)p_1+(y-z+1)p_2+(z-1)p_3\}/\{(x-y)b_1+(y-z+1)b_2+(z-1)b_3\}=$
$\{(x-z+1)p_1+yp_3\}/\{(x-z+1)b_1+yb_3\}$.

Since an enlargement of component $x-1-z+1=x-z \longrightarrow 0$ by following component $0 \longrightarrow y$ is identical to subjecting component $x-1-z+1=x-z \longrightarrow 0$ to enlargement Method I, and since an enlargement of component $x-1-y \longrightarrow x-1=y+(x-1-y)$ by following component $x-z+(z-1)=x-1 \longrightarrow z-1$ is identical to subjecting component $x-1-y \longrightarrow x-1=y+(x-1-y)$ to enlargement Method II, it follows that an end-keeper with a component-table having three different parts/bights specifications can be enlarged by two enlargement processes only.

Component-tables with two different parts/bights specifications.

A component-table with only two parts/bights specifications will have the following make up:

$$\begin{array}{lcl} \text{start transition index-number:} & & \text{end transition index-number:} \\ 0 & \longrightarrow & y \\ 1 & \longrightarrow & y+1 \\ 2 & \longrightarrow & y+2 \\ & \vdots & \\ x-1-y & \longrightarrow & x-1=y+(x-1-y) \\ x-y & \longrightarrow & z=0 \\ x-y+1 & \longrightarrow & 1 \\ & \vdots & \\ x-y+(y-1)=x-1 & \longrightarrow & y-1 \end{array}$$

Let the set of entries starting with the transition index-numbers 0 to $x-1-y$ have the parts/bights specification p_1/b_1, and let the set of entries starting with the transition index-numbers $x-y$ to $x-1$ have the parts/bights specification p_2/b_2.

When p_2/b_2 is at a higher level in the RKT (see Pamphlet No. 4) than p_1/b_1, then the entry $x-y \longrightarrow z=0$ can be written as $x-y \longrightarrow x \longrightarrow z=0$. The part $x-y \longrightarrow x$ has the parts/bights specification p_1/b_1, and the part $x \longrightarrow z=0$ represents the Method II enlargement-part of a Method II enlargement performed on the component $x-1-y \longrightarrow x-1=y+(x-1-y)$. Hence when this end-keeper undergoes an enlargement Method II (thus an enlargement Method II performed on component $x-1-y \longrightarrow x-1$), its overall parts/bights specification
$\{(x-y)p_1+yp_2\}/\{(x-y)b_1+yb_2\}$ goes to
$\{(x-y-1)p_1+(y+1)p_2\}/\{(x-y-1)b_1+(y+1)b_2\}$. Note that the parts/bights specification of the component $x-1-y \longrightarrow x-1$ goes from p_1/b_1 to p_2/b_2.

When this end-keeper undergoes an enlargement Method I, thus an enlargement Method I performed on component $x-y \longrightarrow z=0$, then the enlargement of this component is accomplished by letting its end follow the string-run of component $0 \longrightarrow y$. Consequently the parts/bights specification of component $x-y \longrightarrow z=0$ goes from p_2/b_2 to $(p_2+p_1)/(b_2+b_1)$, and the overall parts/bights specification of the end-keeper goes from $\{(x-y)p_1+yp_2\}/\{(x-y)b_1+yb_2\}$ to $\{(x-y+1)p_1+yp_2\}/\{(x-y+1)b_1+yb_2\}$.

There is still a further enlargement process available for this end-keeper, namely the enlargement process whereby the end of component $x-1-y \longrightarrow x-1$ follows the string-run of component $x-1 \longrightarrow y-1$ (this is a further Method II enlargement (see Pamphlet No. 4)). With this enlargement the parts/bights specification of component $x-1-y \longrightarrow x-1$ goes from p_1/b_1 to $(p_1+p_2)/b_1+b_2)$, and the overall parts/bights specification of the end-keeper goes from $\{(x-y)p_1+yp_2\}/\{(x-y)b_1+yb_2\}$ to $\{(x-y)p_1+(y+1)p_2\}/\{(x-y)b_1+(y+1)b_2\}$.

When p_1/b_1 is at a higher level in the RKT (see Pamphlet No. 4) than p_2/b_2, then the entry $x-1-y \longrightarrow x-1=y+(x-1-y)$ can be written as $x-1-y \longrightarrow z-1=-1=b-1 \longrightarrow x-1=y+(x-1-y)$.

The part $x-1-y \longrightarrow z-1=-1$ has the parts/bights specification p_2/b_2, and the part $-1 \longrightarrow x-1$ represents the Method I enlargement-part of a Method I enlargement performed on the component $x-y \longrightarrow z=0$. Hence when this end-keeper undergoes an enlargement Method I (thus an enlargement Method I performed on component $x-y \longrightarrow z=0$), its overall parts/bights specification $\{(x-y)p_1+yp_2\}/\{(x-y)b_1+yb_2\}$ goes to $\{(x-y+1)p_1+(y-1)p_2\}/\{(x-y+1)b_1+(y-1)b_2\}$. Note that the parts/bights specification of the component $x-y \longrightarrow z=0$ goes from p_2/b_2 to p_1/b_1.

When this end-keeper undergoes an enlargement Method II, thus an enlargement Method II performed on component $x-1-y \longrightarrow x-1$, then the enlargement of this component is accomplished by letting its end follow the string-run of component $x-1 \longrightarrow y-1$. Consequently the parts/bights specification of component $x-1-y \longrightarrow x-1$ goes from p_1/b_1 to $(p_1+p_2)/(b_1+b_2)$, and the overall parts/bights specification of the end-keeper goes from $\{(x-y)p_1+yp_2\}/\{(x-y)b_1+yb_2\}$ to $\{(x-y)p_1+(y+1)p_2\}/\{(x-y)b_1+(y+1)b_2\}$.

There is still a further enlargement process available for this end-keeper, namely the enlargement process whereby the end of component $x-y \longrightarrow z=0$ follows the string-run of component $0 \longrightarrow y$ (this is a further Method I enlargement (see Pamphlet No. 4)). With this enlargement the parts/bights specification of component $x-y \longrightarrow z=0$ goes from p_2/b_2 to $(p_2+p_1)/b_2+b_1)$, and the overall parts/bights specification of the end-keeper goes from $\{(x-y)p_1 + yp_2\}/\{(x-y)b_1 + yb_2\}$ to $\{(x-y+1)p_1 + yp_2\}/\{(x-y+1)b_1 + yb_2\}$.

Hence an end-keeper which has a component-table with two different parts/bights specifications can be enlarged by three different enlargements. When p_2/b_2 is at a higher level in the RKT, then the end-keeper can be enlarged by two different Method II enlargements and one Method I enlargement. When p_1/b_1 is at a higher level in the RKT, then the end-keeper can be enlarged by two different Method I enlargements and one Method II enlargement.

Component-tables with only one parts/bights specification.

A component-table with only one parts/bights specification will have the following make up:

$$\begin{array}{lcl}
\text{start transition index-number:} & & \text{end transition index-number:} \\
0 & \longrightarrow & y=0 \\
1 & \longrightarrow & 1 \\
2 & \longrightarrow & 2 \\
 & \vdots & \\
x-1-y=x-1 & \longrightarrow & x-1=y+(x-1-y)
\end{array}$$

Let the parts/bights specification of the components be equal to p_1/b_1. An enlargement Method I process is associated with the component $0 \longrightarrow y=0$. The new parts/bights specification of this enlarged component is the fundamental Method I enlargement in the RKT of its original parts/bight specification.

An enlargement Method II process is associated with the component $x-1 \longrightarrow x-1$. The new parts/bights specification of this enlarged component is the fundamental Method II enlargement in the RKT of its original parts/bight specification.

This are the only two enlargement processes which can be applied to this end-keeper since an enlargement by following the appropriate component does not exist. Such an enlargement would change the original overall parts/bights specification $p/b = xp_1/xb_1$ to $(x+1)p_1/(x+1)b_1$. However, this new overall parts/bights specification needs at least $(x+1)$ transition-points, which is one more than we got, hence such an enlargement is not possible.

DETERMINING THE ENLARGEMENT PATHS

In order to know how a given p/b-endkeeper may be obtained by means of enlargement processes, we have to know its various enlargement-paths from the trunk-node x/x to its node p/b. When suitable Enlargement Graphs are available, these enlargement-paths can be read from them after locating the node p/b in question. However, suitable Enlargement Graphs are often not available, or a suitable Enlargement Graph may not be extensive enough to contain the required node p/b. If no suitable Enlargement Graph is avaible, we either have to draw the appropriate part of one for the envisaged end-keeper, or calculate its enlargement-paths.

It is in general much less cumbersome to read the various enlargement-paths for an end-keeper from an Enlargement Graph than to obtain them by means of manual calculation. However, in order to draw the appropriate part of an Enlargement Graph, we have to be thoroughly conversant with its general layout and hence construction process. Neither the manual calculation process nor the Enlargement Graph method give a proper overview of the various enlargement-paths available. In order to obtain such an overview, we have to assemble the various enlargement-paths in a suitable **Enlargement-path table**. The compiling of such a table from an Enlargement Graph is however rather cumbersome. The best overall solution is to obtain all the relevant details by means of a computer program, for which the calculation method of the various enlargement-paths is perfectly suited. At the same time such a computer program can assemble the *Enlargement-path table.*

We shall first discuss, for an envisaged end-keeper, in general terms the construction of the appropriate part of an Enlagement Graph. Then we shall discuss the calculation process for obtaining the various enlargement-paths, and finally we shall present a suitable Enlargement-path table.

The construction of the appropriate part of an Enlargement Graph:

We have already seen that a p/b-endkeeper with x transition-points is presented by the node p/b in the Enlargement Graph with root-node x/x. Since this end-keeper consists of x components, we can say that the node p/b consists of x components. We have also seen that among these x components there is/are:

(1). Only one parts/bights specification (only one type), or
(2). Only two different parts/bights specifications (only two types), or
(3). Only three different parts/bights specifications (only three types).

Each of these different component types is represented by a node in the RKT, and these nodes all lie on the evolution-path from the trunk-node 1/1 to the largest node in the set. Thus an end-keeper is associated with only one of three types of evolution-paths in the RKT:

(1*). A path in which the end-node only is associated with the components, or
(2*). A path in which the end-node and one other node are associated with the components, or
(3*). A path in which the end-node and two other nodes are associated with the components.

In the case (2*): the other node is either adjacent to the end-node (case **(2*a)**), or it is the penultimate node on the penultimate branch (case **(2*b)**).

In the case (3*): one of the other two nodes is adjacent to the end-node, and one is the penultimate node on the penultimate branch.

The evolution-path in the RKT, associated with the end-keeper, is represented in its Enlargement Graph by the corresponding **principal-path** with its **principal-nodes**. The values of the principal-nodes are $x\times$ the values of the corresponding nodes on the evolution-path in the RKT.

On pg. 98 is presented an end part of an Enlargement Graph for an end-keeper (x transition-points) in which the largest component specification has for example an evolution-vector (=path-formula) in the RKT of $[\cdot;\cdot,\cdot,\cdot,\cdot,3,2,1]$, and corresponds with the principal-node **A**.

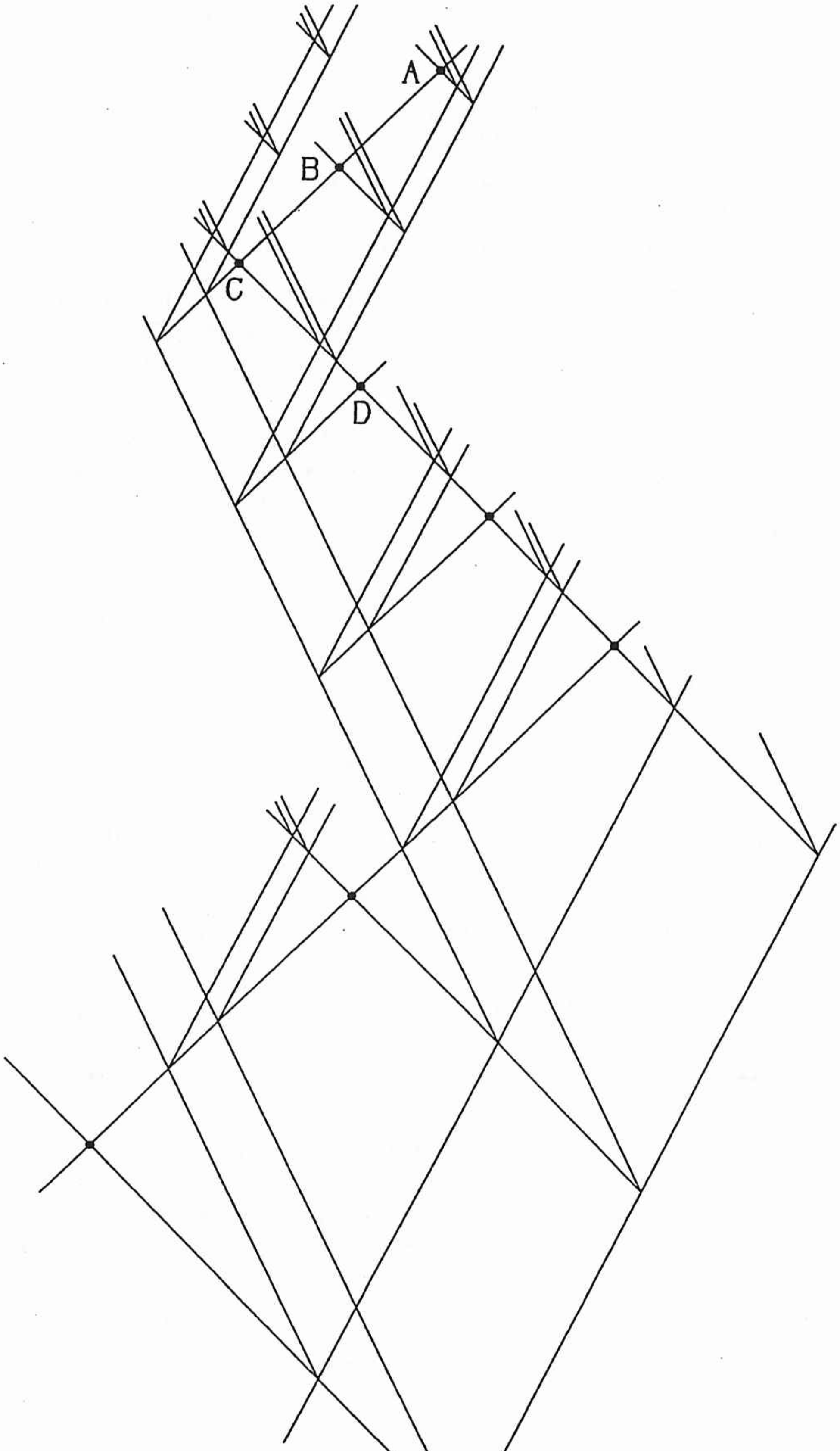
The enlargement-paths for case (1), and hence (1*), are presented on pg. 99 by the thick lines along the principal-lines and the band-lines (there are $(x-1)$ lines in a band).

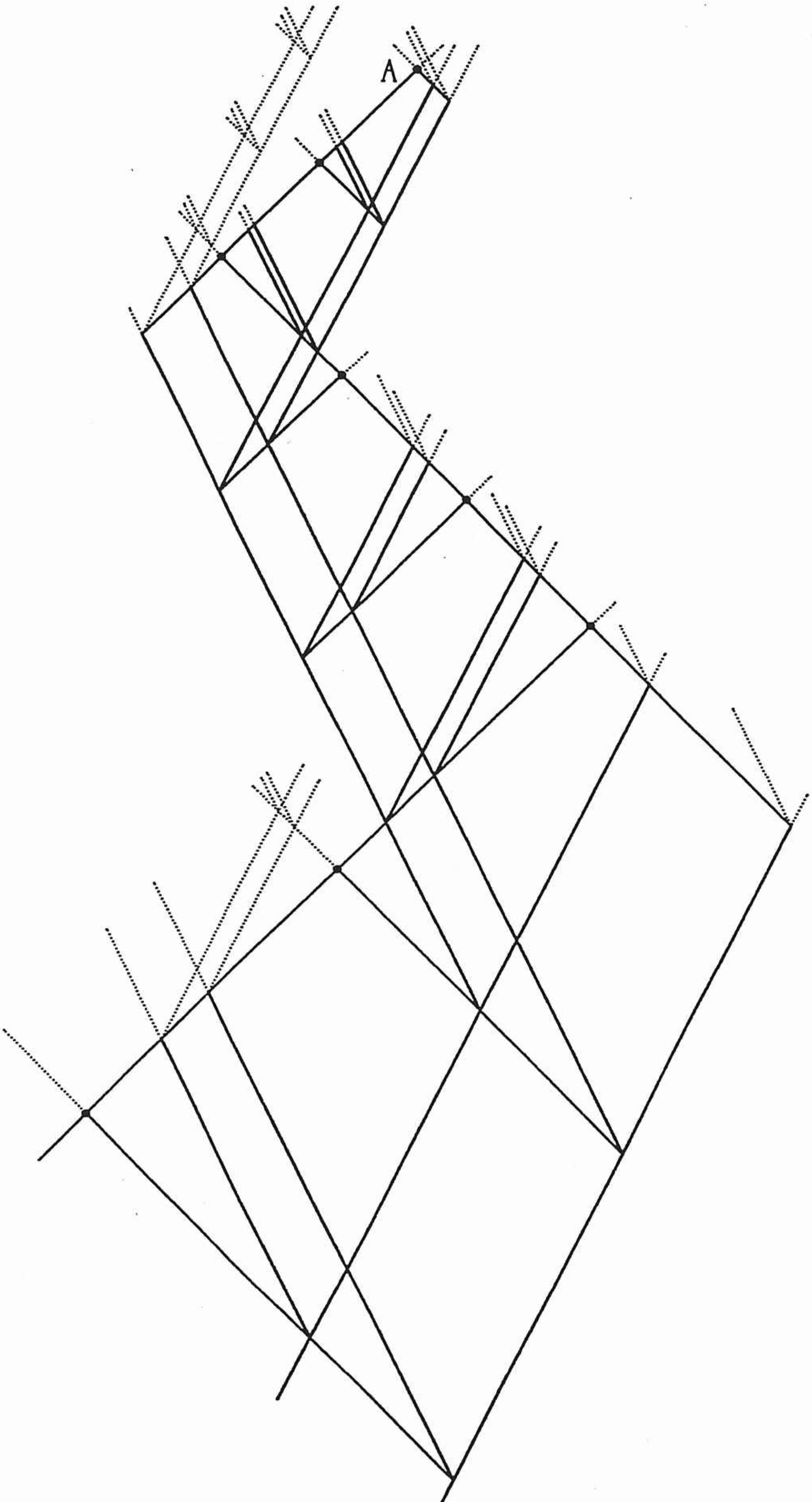
The enlargement-paths for case (2*a) are presented on pg. 100 by the thick lines along the principal-lines, the band-lines, and the appropriate thin solid lines along the principal-line and band-lines. The node representing the end-keeper lies on the thin solid principal line-segment between the principal-nodes **A** and **B**.

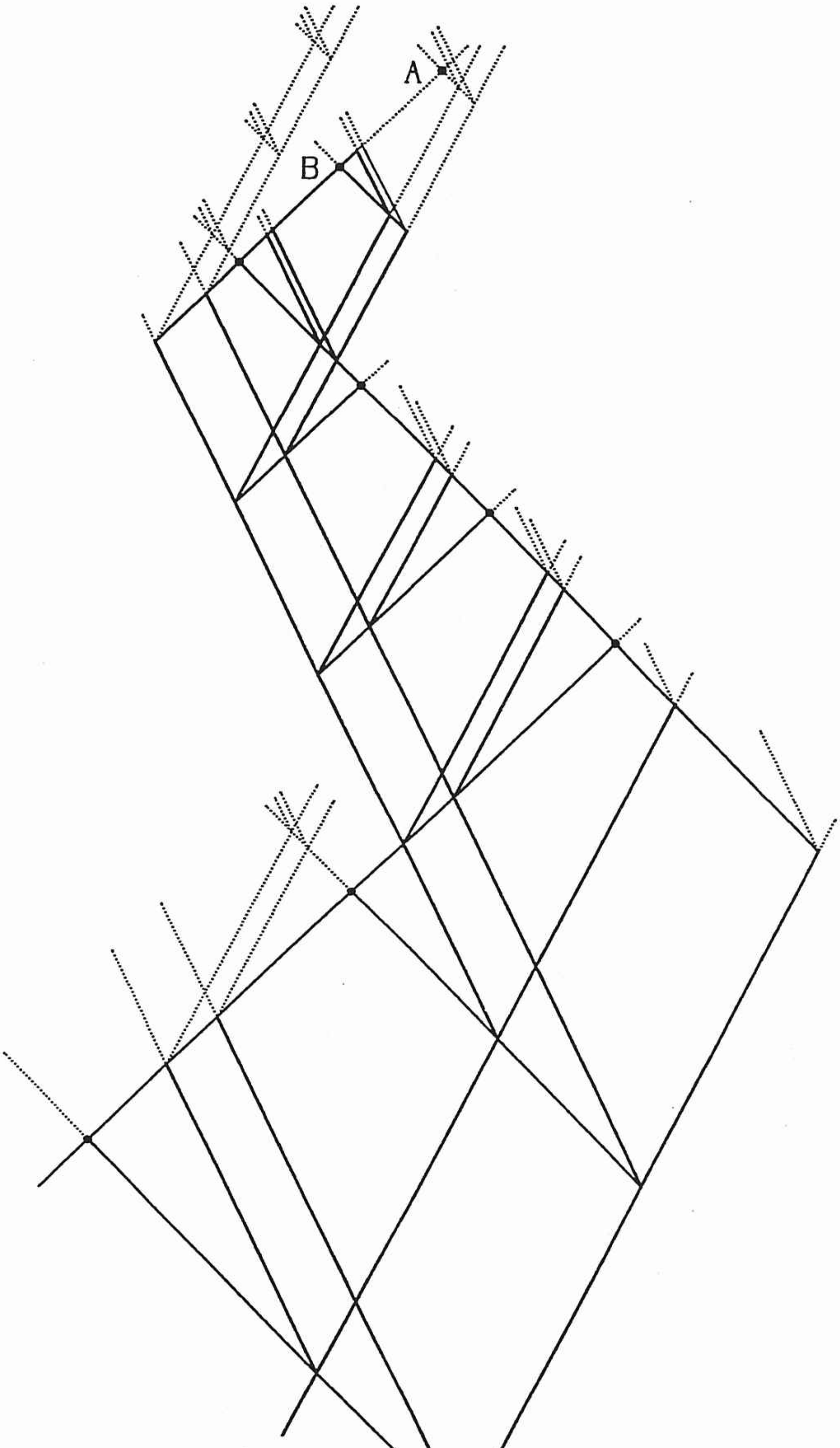
The enlargement-paths for case (2*b) are presented on pg. 101 by the thick lines along the principal-lines, the band-lines, and the appropriate thin solid lines along the principal-lines and the band-lines. The node representing the end-keeper lies on the thin solid principal line-segment between the principal-node **A** and the thick band-line to the extreme right of **A**.

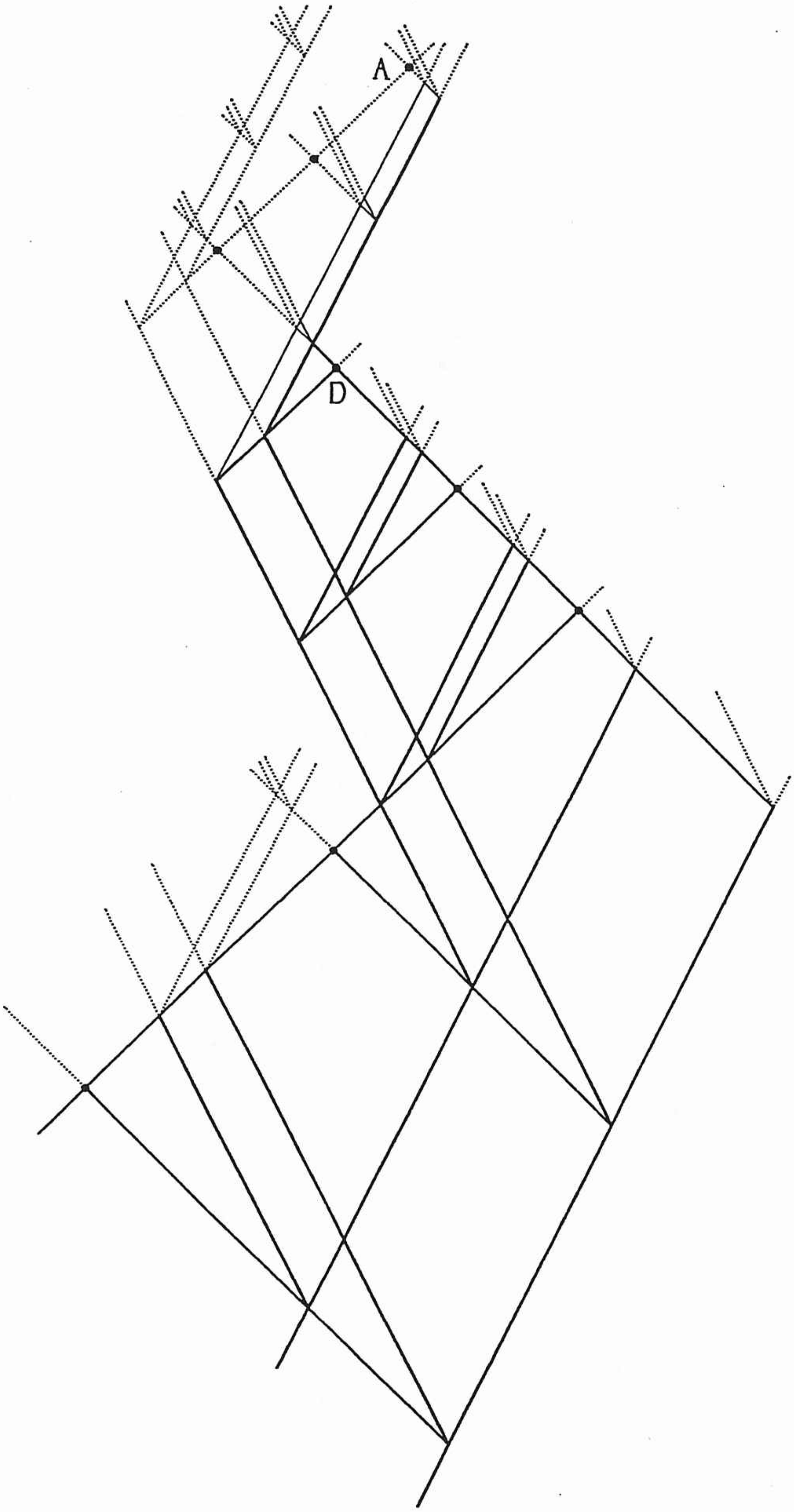
The enlargement-paths for case (3), and hence (3*), are presented on pg. 102 by the thick lines along the principal-lines, the band-lines, and the appropriate thin solid lines along the principal-lines and the band-lines. The node representing the end-keeper lies on one of the thin solid line-segments of the left-slanting band to the right of **B**. Note that it does not lie on the thin solid line-segment, along a principal-line, to the right of **B**.

A similar set of part Enlargement Graphs for an end-keeper (x transition-points) in which the largest component specification has for example an evolution-vector (=path-formula) in the RKT of $[\cdot;\cdot,\cdot,\cdot,3,2,1]$ are presented on pgs. 103 — 107.


A
B
C
D


A


A
B


A
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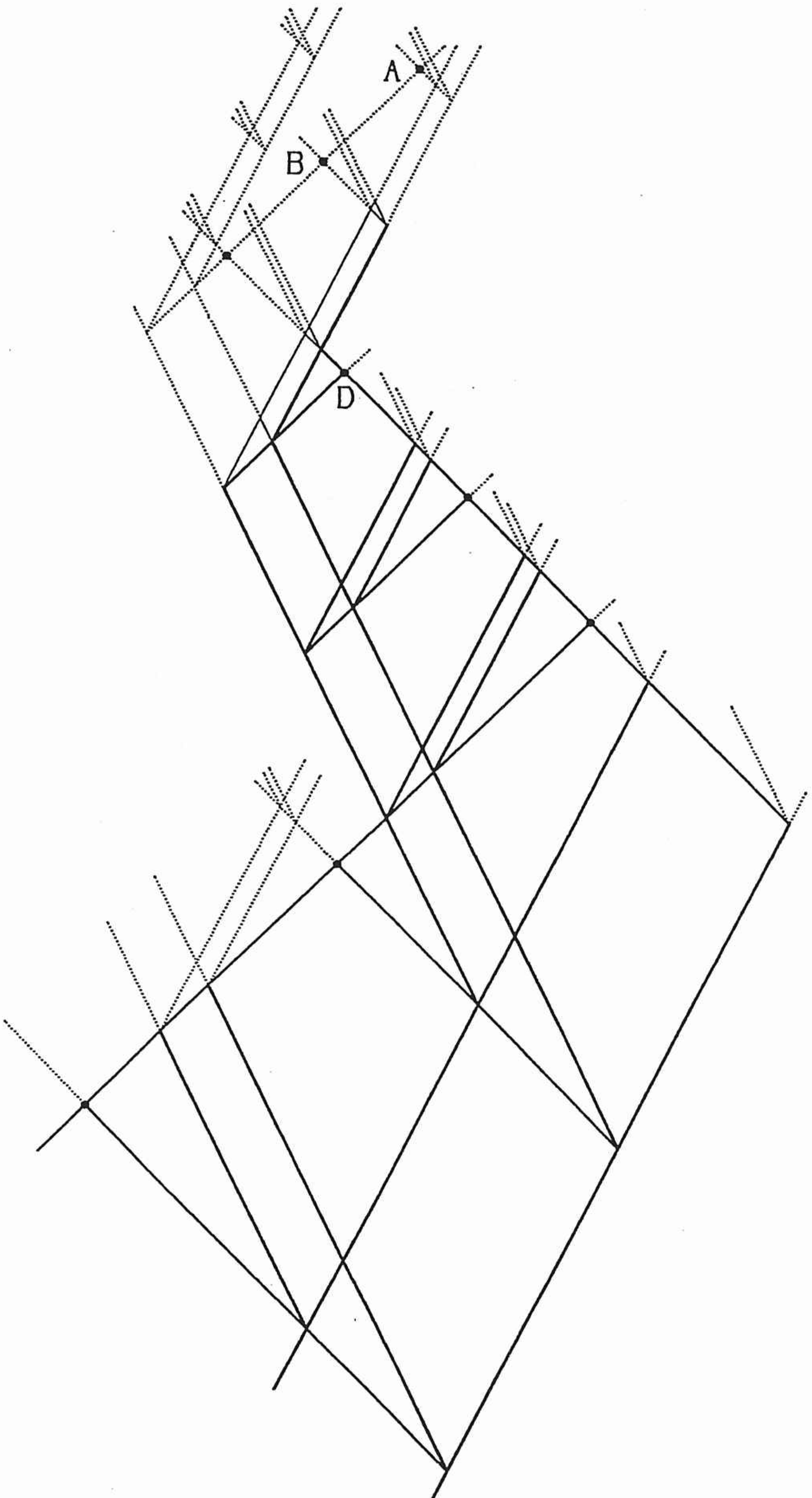
A
B
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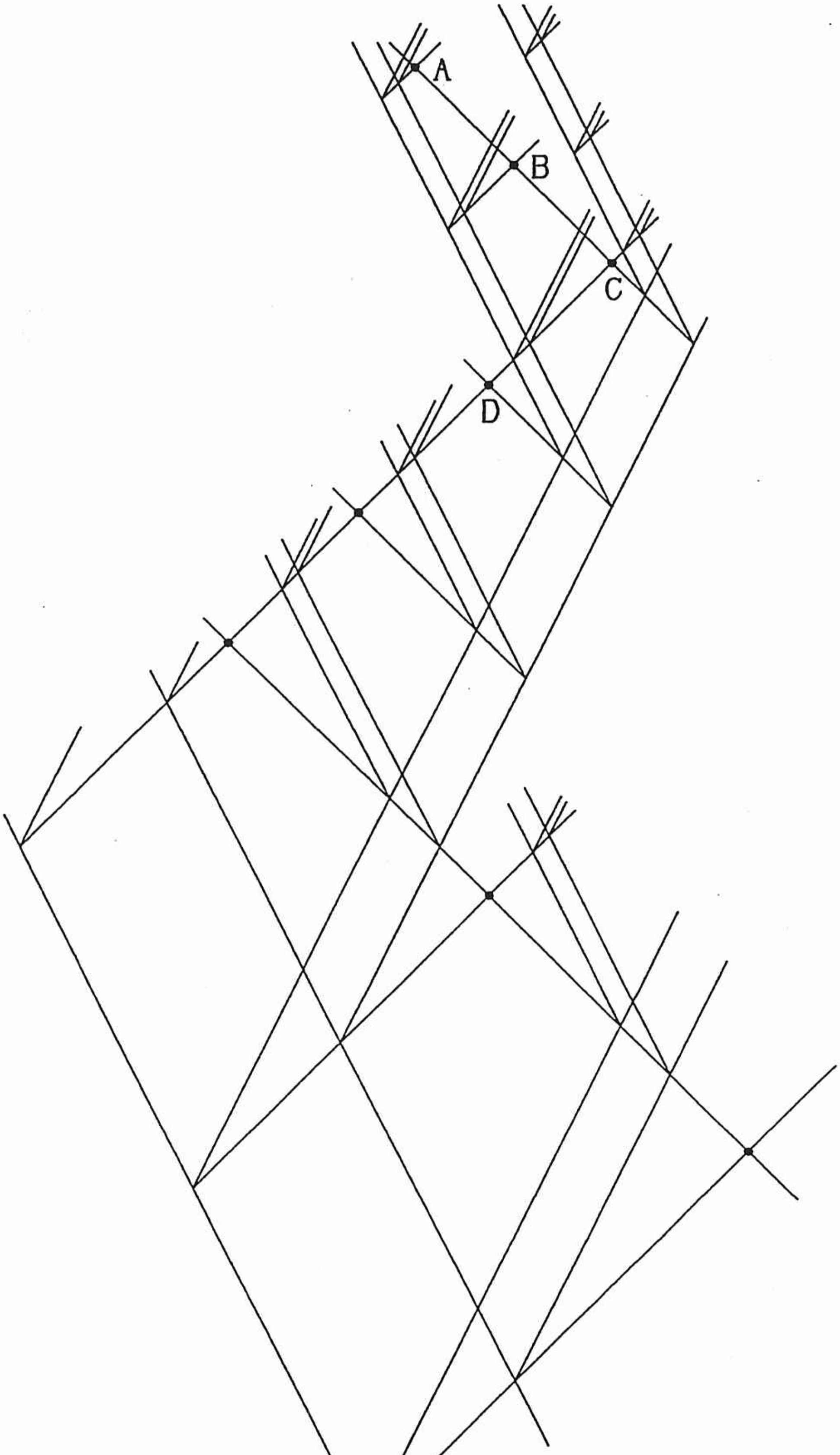
A
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C
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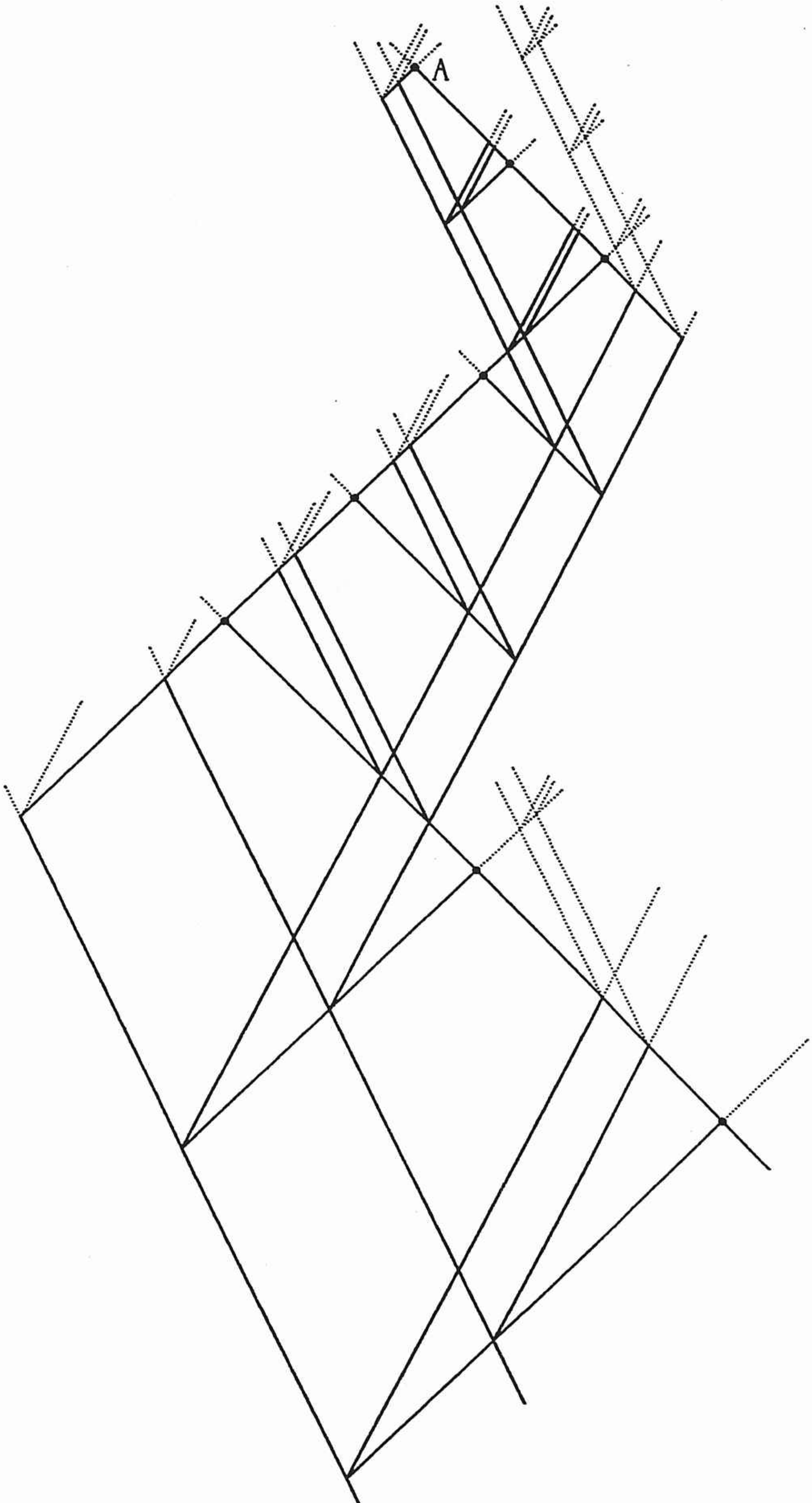
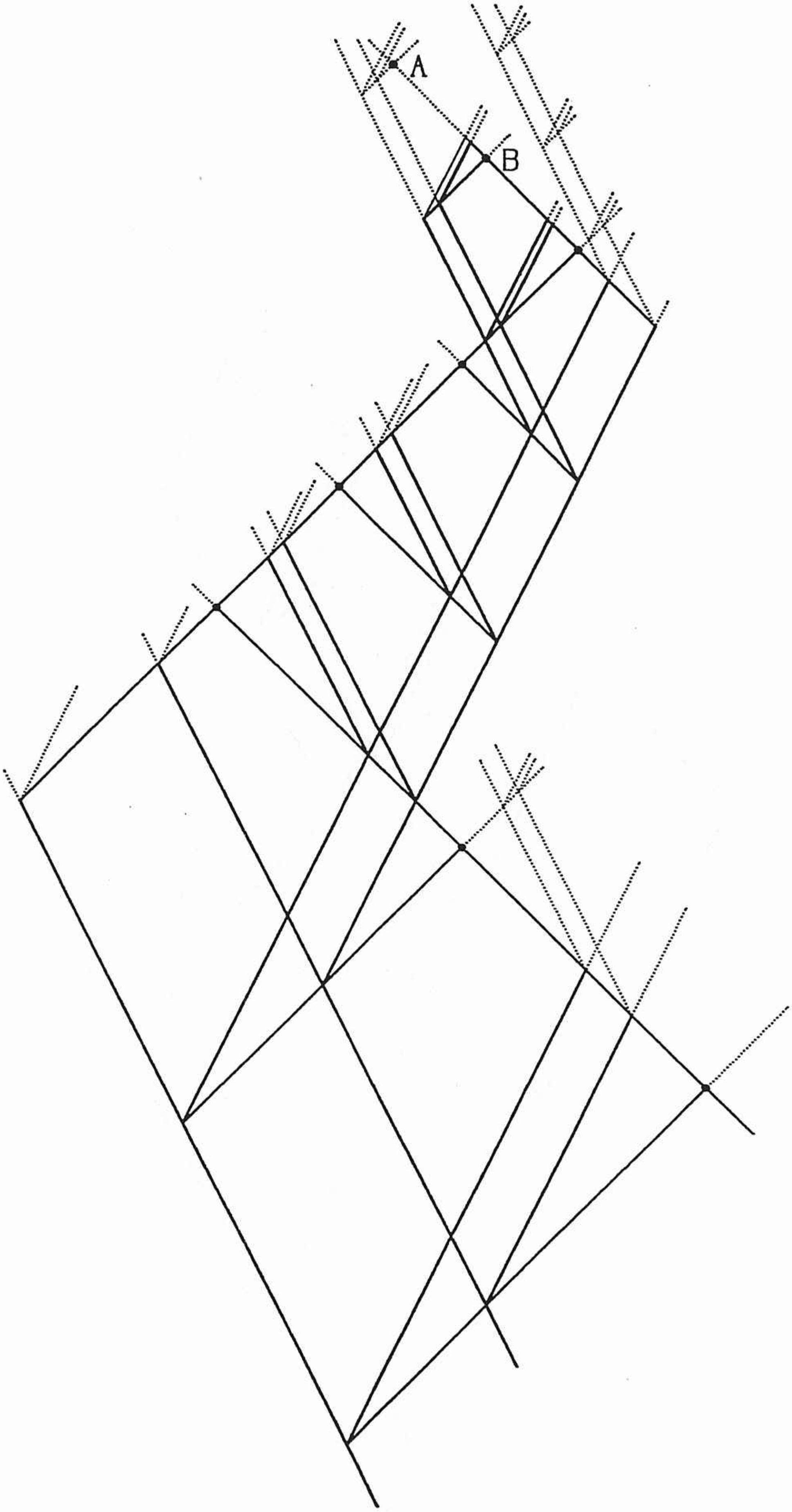
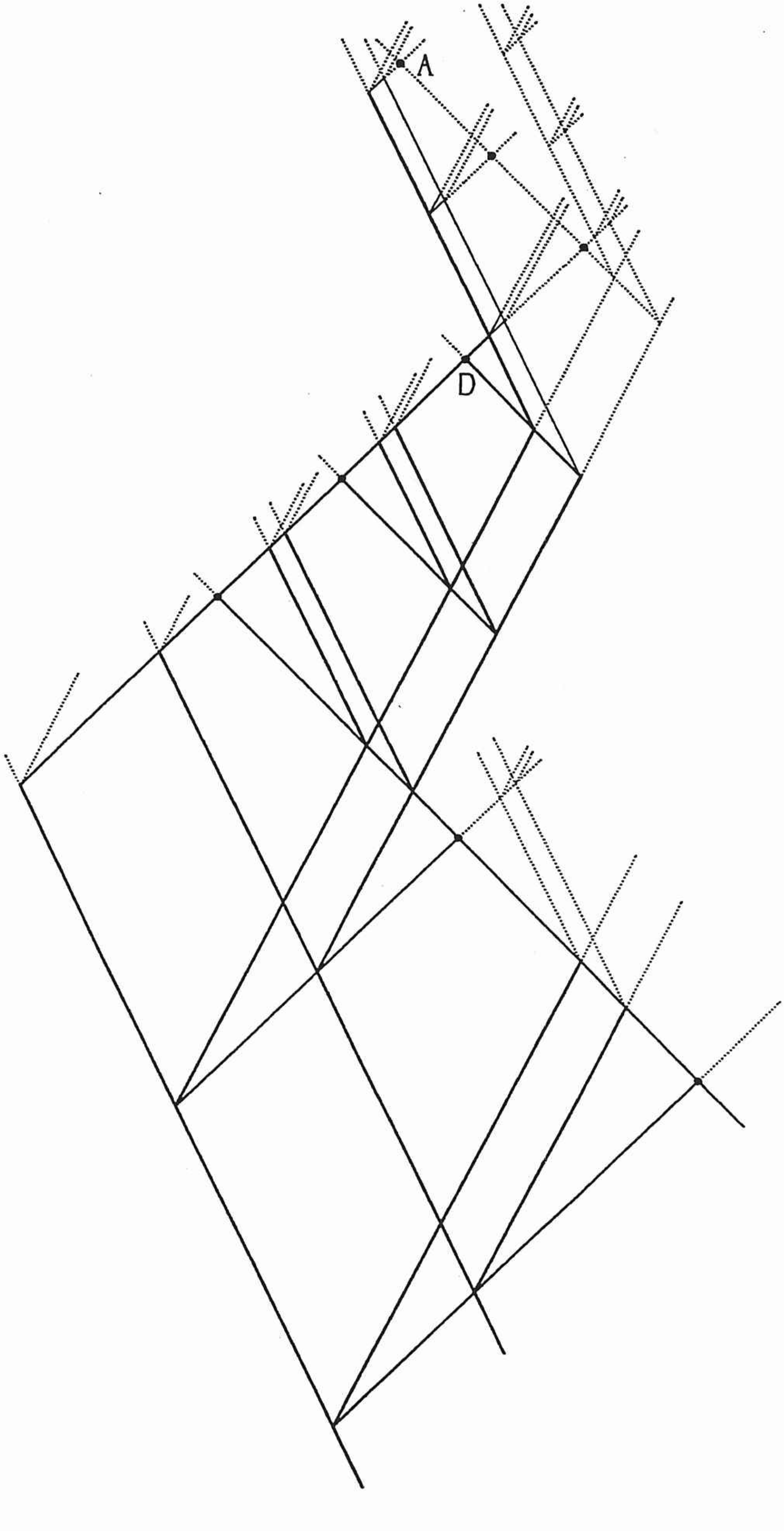
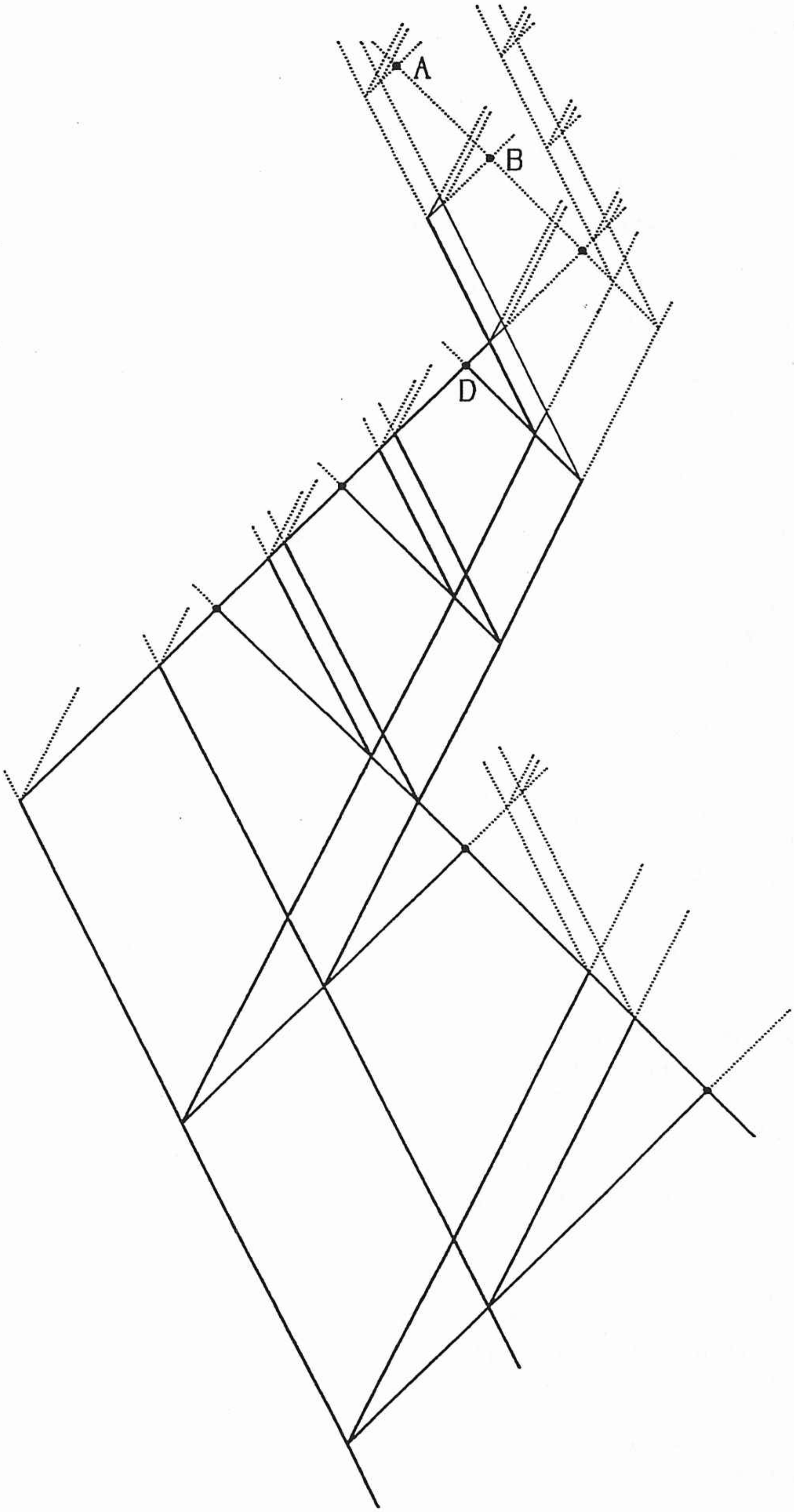

A


A
B


A
D


A
B
D

The calculation of the enlargement-paths:

We have already mentioned that the calculation procedures are often cumbersome when manually performed, and hence the desirability of performing these calculations with the aid of a suitable computer program.

The calculation procedures are based on the various important properties of the component-tables, which are as follows:

(1). A component-table can contain either *one*, *two*, or *three* different parts/bights specifications.
(2). All components with identical parts/bights specifications are in the component-table adjacent. This means that their transition-points of string-entry are adjacent.
(3). The g.c.d. (greatest common divisor) of the parts-value and bights-value of any end-keeper on the enlargement-path must be $\leq x$ (equal or less than the number of transition-points).
(4). The nodes in the RKT, representing the different parts/bights specifications of the components in the component-table, all lie on the same evolution-path from the trunk-node 1/1 to the node which represents the highest parts/bights specification.
(5). When a component-table contains two different parts/bights specifications, then the nodes in the RKT representing them are either:
(i) the two final nodes on the final branch of the evolution-path in the RKT, or
(ii) the lower node is the penultimate node on the penultimate branch of the evolution-path in the RKT.

Let the parts/bights specifications of the first set of components in the component-table be p_1/b_1 and of the second set be p_2/b_2. Then:
(a) For (5i), when the final branch of the evolution-path in the RKT is a left-branch, then p_1/b_1 is higher in the tree than p_2/b_2.
(b) For (5i), when the final branch of the evolution-path in the RKT is a right-branch, then p_2/b_2 is higher in the tree than p_1/b_1.
(c) For (5ii), when the final branch of the evolution-path in the RKT is a left-branch, then p_2/b_2 is higher in the tree than p_1/b_1.
(d) For (5ii), when the final branch of the evolution-path in the RKT is a right-branch, then p_1/b_1 is higher in the tree than p_2/b_2.
(6). When a component-table contains three different parts/bights specifications, then the nodes in the RKT representing the two highest parts/bights specifications are the final two nodes on the final branch of the evolution-path, and the node which represents the lowest parts/bights specification is the penultimate node on the penultimate branch of the evolution-path.

Let the parts/bights specifications for the first set of components be p_1/b_1, for the second set be p_2/b_2, and for the third set be p_3/b_3. Then $p_2 = (p_1 + p_3)$ and $b_2 = (b_1 + b_3)$. Furthermore:
(a) When the final branch of the evolution-path in the RKT is a left-branch, then p_3/b_3 is higher in the tree than p_1/b_1.
(b) When the final branch of the evolution-path in the RKT is a right-branch, then p_1/b_1 is higher in the tree than p_3/b_3.
(7). Only the components which exit in transition index-numbers 0 and $(x-1)$ can be enlarged.
(8). The component which exits in transition index-number 0 can be enlarged by an Enlargement Method I only.

(9). The component which exits in transition index-number $(x-1)$ can be enlarged by an Enlargement Method II only.

(10). The component which exits in transition index-number $(x-1)$ will always be in the first set of parts/bights specification types.

When there is only one set (all components have the same parts/bights specification), then the component which exits in transition index-number $(x-1)$ will always be the last component in the component-table.

When there are two or three sets, then the component which exits in transition index-number $(x-1)$ will always be the last component in the first set of components in the component-table.

(11). The component which exits in transition index-number 0 will always be in the last set of parts/bights specification types. Hence in the first set when there is only one set, in the second set when there are two sets, or in the third set when there are three sets.

When there is only one set (all components have the same parts/bights specification), then the component which exits in transition index-number 0 will always be the first component in the component-table.

When there are two or three sets, then the component which exits in transition index-number 0 will always be the first component in the last set of components in the component-table.

From the above rules the following rules become self evident:

(A). When for an end-keeper the parts/bights specifications of all the components in its component-table are identical, say p_1/b_1, then this end-keeper is either enlarged from:

(i) an end-keeper of which the first component in its component-table has a parts/bights specification of $(p_1 - \{\Delta p_1\}_{II})/(b_1 - \{\Delta b_1\}_{II})$, and all other components in its component-table have the parts/bights specification p_1/b_1, or

(ii) an end-keeper of which the last component in its component-table has a parts/bights specification of $(p_1 - \{\Delta p_1\}_I)/(b_1 - \{\Delta b_1\}_I)$, and all other components have the parts/bights specification p_1/b_1.

(B). When an end-keeper has two different parts/bights specifications, say p_1/b_1 and p_2/b_2, where p_1/b_1 and p_2/b_2 are respectively the component specifications of the first and second set of components in its component-table, then:

(1) When $p_1/b_1 > p_2/b_2$, this end-keeper is either enlarged from:

(i) an end-keeper of which the first component in its component-table has a parts/bights specification of $(p_1 - p_2)/(b_1 - b_2)$, while the parts/bights specifications of the other components in its component-table are identical to those in the component-table of the enlarged end-keeper, or

(ii) an end-keeper of which the last component in the first set of components in its component-table has a parts/bights specification of p_2/b_2, while the parts/bights specifications of the other components are identical to those in the component-table of the enlarged end-keeper.

There is a special case here when the last component in the first set of components is the first component of this set. In this case this enlargement process only exists when p_2/b_2 is associated with the penultimate node on the evolution-path from the trunk-node 1/1 to the node p_1/b_1 in the RKT. If this is not the case, then only the process under (i) exists.

(2) When $p_2/b_2 > p_1/b_1$, this end-keeper is either enlarged from:
(i) an end-keeper of which the last component in the second set of components in its component-table has a parts/bights specification of $(p_2 - p_1)/(b_2 - b_1)$, while the parts/bights specifications of the other components are identical to those in the component-table of the enlarged end-keeper, or
(ii) an end-keeper of which the first component of the second set of components in its component-table has a parts/bights specification of p_1/b_1, while the parts/bights specifications of the other components are identical to those in the component-table of the enlarged end-keeper.

There is a special case here when the first component in the second set of components is the last component of this set. In this case this enlargement process only exists when p_1/b_1 is associated with the penultimate node on the evolution-path from the trunk-node 1/1 to the node p_2/b_2 in the RKT. If this is not the case, then only the process under (i) exists.

(C). When an end-keeper has three different parts/bights specifications, say p_1/b_1, p_2/b_2 and p_3/b_3, where p_1/b_1, p_2/b_2 and p_3/b_3 are respectively the component specifications of the first, second and third set of components in its component-table, then this end-keeper is either enlarged from:
(i) an end-keeper of which the first component of the second set of components in its component-table has a parts/bights specification of p_1/b_1, while the parts/bights specifications of the other components are identical to those in the component-table of the enlarged end-keeper, or
(ii) an end-keeper in which the last component of the second set of components in its component-table has a parts/bights specification of p_3/b_3, while the parts/bights specifications of the other components are identical to those in the component-table of the enlarged end-keeper.

In order to obtain a clear overview of the above rules A, B, and C, we have presented them in component-table form on pg. 111.

All the necessary calculations for establishing the various possible enlargement-paths from the trunk-node x/x to the end-keeper node p/b can readily be performed with the aid of these rules A, B, and C.

An Example of such a calculation-process follows:

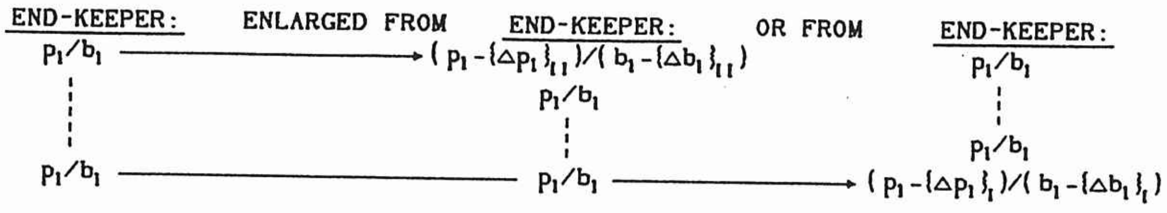
Example: An end-keeper with $p/b = 36/51$, and $x = 4$.

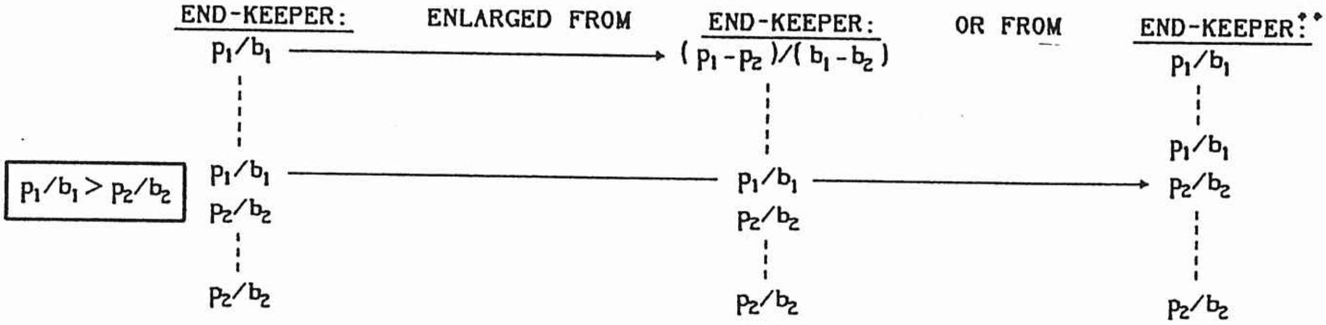
First we have to calculate the component-table of this end-keeper:

36/51-ENDKEEPER — x=4

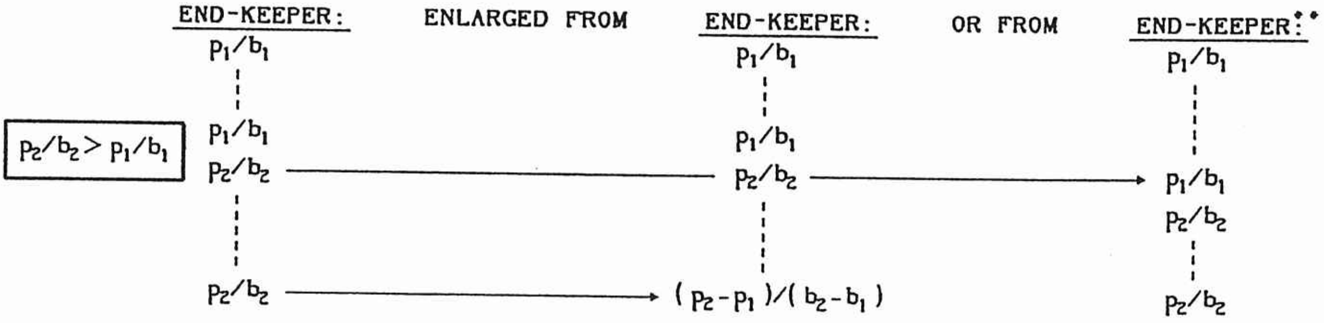
0-36-21-6-42-27-12-48-33-18-3	7/10
1-37-22-7-43-28-13-49-34-19-4-40-25-10-46-31-16-1	12/17
2-38-23-8-44-29-14-50-35-20-5-41-26-11-47-32-17-2	12/17
3-39-24-9-45-30-15-0	5/7

Then we calculate with the aid of Euclid's Algorithm the evolution-vector (path-formula) of the component with the largest parts/bights specification. This evolution-vector defines the evolution-path in the RKT from the trunk-node 1/1 to the node which represents the parts/bights specification in question.

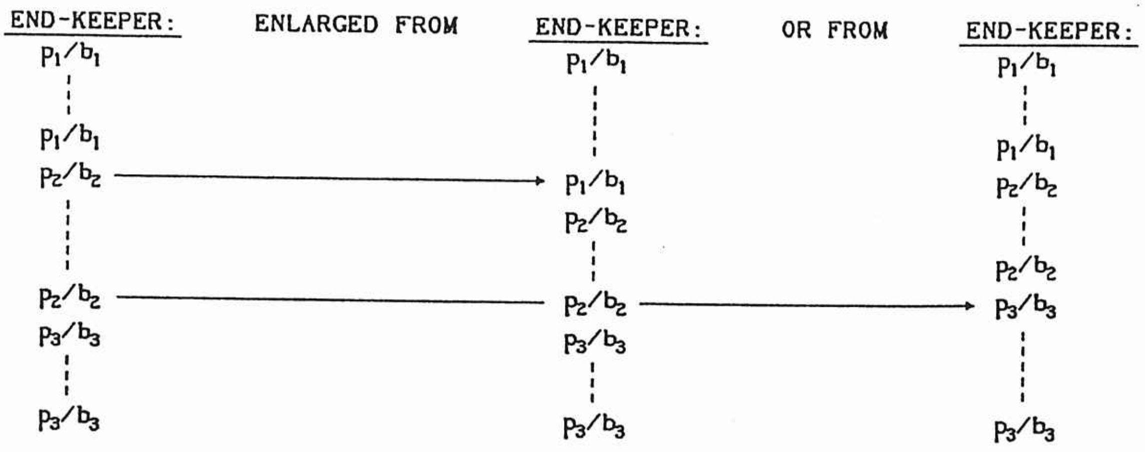


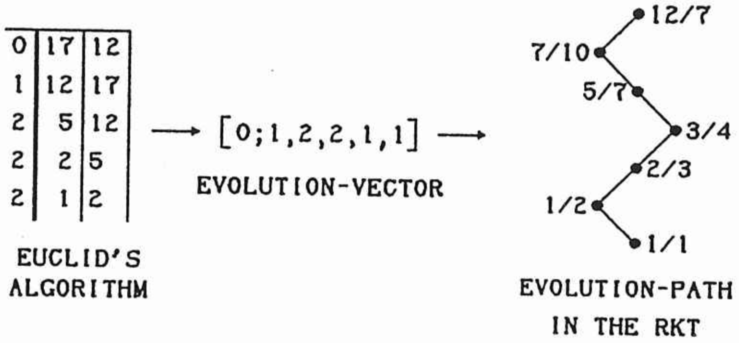


** WHEN ONLY THE FIRST COMPONENT HAS THE PARTS/BIGHTS SPECIFICATION p_1/b_1 THIS CASE ONLY EXISTS WHEN p_2/b_2 IS ASSOCIATED WITH THE PENULTIMATE NODE ON THE EVOLUTION-PATH FROM THE TRUNK-NODE 1/1 TO THE NODE p_1/b_1 IN THE RKT.



** WHEN ONLY THE LAST COMPONENT HAS THE PARTS/BIGHTS SPECIFICATION p_2/b_2 THIS CASE ONLY EXISTS WHEN p_1/b_1 IS ASSOCIATED WITH THE PENULTIMATE NODE ON THE EVOLUTION-PATH FROM THE TRUNK-NODE 1/1 TO THE NODE p_2/b_2 IN THE RKT.





According to the rules A, B, and C, the 36/51-endkeeper with its parts/bights specifications

$$\underbrace{\begin{matrix}7/10\\12/17\\12/17\\5/7\end{matrix}}_{36/51}$$

has been enlarged from end-keepers with the parts/bights specifications:

$$\underbrace{\begin{matrix}7/10\\7/10\\12/17\\5/7\end{matrix}}_{31/44}\qquad\underbrace{\begin{matrix}7/10\\12/17\\5/7\\5/7\end{matrix}}_{29/41}$$

These in turn have been enlarged from:

$$\overbrace{\underbrace{\begin{matrix}7/10\\7/10\\7/10\\5/7\end{matrix}}_{26/37}\quad\underbrace{\begin{matrix}7/10\\7/10\\5/7\\5/7\end{matrix}}_{24/34}}^{31/44}\qquad\overbrace{\underbrace{\begin{matrix}7/10\\7/10\\5/7\\5/7\end{matrix}}_{24/34}\quad\underbrace{\begin{matrix}7/10\\5/7\\5/7\\5/7\end{matrix}}_{22/31}}^{29/41}$$

These in turn have been enlarged from:

$$\overbrace{\underbrace{\begin{matrix}7/10\\7/10\\5/7\\5/7\end{matrix}}_{24/34}\quad\underbrace{\begin{matrix}2/3\\7/10\\7/10\\5/7\end{matrix}}_{21/30}}^{26/37}\qquad\overbrace{\underbrace{\begin{matrix}7/10\\5/7\\5/7\\5/7\end{matrix}}_{22/31}\quad\underbrace{\begin{matrix}2/3\\7/10\\5/7\\5/7\end{matrix}}_{19/27}}^{24/34}\qquad\overbrace{\underbrace{\begin{matrix}5/7\\5/7\\5/7\\5/7\end{matrix}}_{20/28}\quad\underbrace{\begin{matrix}2/3\\5/7\\5/7\\5/7\end{matrix}}_{17/24}}^{22/31}$$

These have been enlarged from previous and the following ones:

$$\overbrace{\underbrace{\begin{matrix}2/3\\7/10\\5/7\\5/7\end{matrix}}_{19/27}\quad\underbrace{\begin{matrix}2/3\\2/3\\7/10\\5/7\end{matrix}}_{16/23}}^{21/30}\qquad\overbrace{\underbrace{\begin{matrix}2/3\\5/7\\5/7\\5/7\end{matrix}}_{17/24}\quad\underbrace{\begin{matrix}2/3\\2/3\\5/7\\5/7\end{matrix}}_{14/20}}^{19/27}\qquad\overbrace{\underbrace{\begin{matrix}2/3\\5/7\\5/7\\3/4\end{matrix}}_{15/21}\quad\underbrace{\begin{matrix}2/3\\2/3\\5/7\\5/7\end{matrix}}_{14/20}}^{17/24}\qquad\overbrace{\underbrace{\begin{matrix}5/7\\5/7\\5/7\\3/4\end{matrix}}_{18/25}\quad\underbrace{\begin{matrix}2/3\\5/7\\5/7\\5/7\end{matrix}}_{17/24}}^{20/28}$$

These have been enlarged from previous and the following ones:

$$\overbrace{\underbrace{\begin{matrix}2/3\\2/3\\5/7\\5/7\end{matrix}}_{14/20}\quad\underbrace{\begin{matrix}2/3\\2/3\\2/3\\5/7\end{matrix}}_{11/16}}^{16/23}\qquad\overbrace{\underbrace{\begin{matrix}2/3\\2/3\\5/7\\3/4\end{matrix}}_{12/17}\quad\underbrace{\begin{matrix}2/3\\2/3\\2/3\\5/7\end{matrix}}_{11/16}}^{14/20}\qquad\overbrace{\underbrace{\begin{matrix}2/3\\5/7\\3/4\\3/4\end{matrix}}_{13/18}\quad\underbrace{\begin{matrix}2/3\\2/3\\5/7\\3/4\end{matrix}}_{12/17}}^{15/21}\qquad\overbrace{\underbrace{\begin{matrix}5/7\\5/7\\3/4\\3/4\end{matrix}}_{16/22}\quad\underbrace{\begin{matrix}2/3\\5/7\\5/7\\3/4\end{matrix}}_{15/21}}^{18/25}$$

These have been enlarged from previous and the following ones:

$$\overbrace{\underbrace{\begin{matrix}2/3\\2/3\\2/3\\3/4\end{matrix}}_{9/13}}^{11/16}\qquad\overbrace{\underbrace{\begin{matrix}2/3\\2/3\\3/4\\3/4\end{matrix}}_{10/14}\quad\underbrace{\begin{matrix}2/3\\2/3\\2/3\\3/4\end{matrix}}_{9/13}}^{12/17}\qquad\overbrace{\underbrace{\begin{matrix}2/3\\3/4\\3/4\\3/4\end{matrix}}_{11/15}\quad\underbrace{\begin{matrix}2/3\\2/3\\3/4\\3/4\end{matrix}}_{10/14}}^{13/18}\qquad\overbrace{\underbrace{\begin{matrix}5/7\\3/4\\3/4\\3/4\end{matrix}}_{14/19}\quad\underbrace{\begin{matrix}2/3\\5/7\\3/4\\3/4\end{matrix}}_{13/18}}^{16/22}$$

These have been enlarged from previous and the following ones:

$$\overbrace{\underbrace{\begin{matrix}2/3\\2/3\\2/3\\2/3\end{matrix}}_{8/12}\quad\underbrace{\begin{matrix}2/3\\2/3\\2/3\\1/1\end{matrix}}_{7/10}}^{9/13}\qquad\overbrace{\underbrace{\begin{matrix}2/3\\2/3\\2/3\\3/4\end{matrix}}_{9/13}\quad\underbrace{\begin{matrix}2/3\\2/3\\3/4\\1/1\end{matrix}}_{8/11}}^{10/14}\qquad\overbrace{\underbrace{\begin{matrix}2/3\\2/3\\3/4\\3/4\end{matrix}}_{10/14}\quad\underbrace{\begin{matrix}2/3\\3/4\\3/4\\1/1\end{matrix}}_{9/12}}^{11/15}\qquad\overbrace{\underbrace{\begin{matrix}3/4\\3/4\\3/4\\3/4\end{matrix}}_{12/16}\quad\underbrace{\begin{matrix}2/3\\3/4\\3/4\\3/4\end{matrix}}_{11/15}}^{14/19}$$

These have been enlarged from previous and the following ones:

$$\overbrace{\underbrace{\begin{matrix}1/2\\2/3\\2/3\\2/3\end{matrix}}_{7/11}\quad\underbrace{\begin{matrix}2/3\\2/3\\2/3\\1/1\end{matrix}}_{7/10}}^{8/12}\qquad\overbrace{\underbrace{\begin{matrix}1/2\\2/3\\2/3\\1/1\end{matrix}}_{6/9}\quad\underbrace{\begin{matrix}2/3\\2/3\\1/1\\1/1\end{matrix}}_{6/8}}^{7/10}\qquad\overbrace{\underbrace{\begin{matrix}2/3\\2/3\\2/3\\1/1\end{matrix}}_{7/10}\quad\underbrace{\begin{matrix}2/3\\2/3\\1/1\\1/1\end{matrix}}_{6/8}}^{8/11}\qquad\overbrace{\underbrace{\begin{matrix}2/3\\2/3\\3/4\\1/1\end{matrix}}_{8/11}\quad\underbrace{\begin{matrix}2/3\\3/4\\1/1\\1/1\end{matrix}}_{7/9}}^{9/12}\qquad\overbrace{\underbrace{\begin{matrix}2/3\\3/4\\3/4\\3/4\end{matrix}}_{11/15}\quad\underbrace{\begin{matrix}3/4\\3/4\\3/4\\1/1\end{matrix}}_{10/13}}^{12/16}$$

These have been enlarged from previous and the following ones:

$$\overbrace{\underbrace{\begin{matrix}1/2\\1/2\\2/3\\2/3\end{matrix}}_{6/10}\quad\underbrace{\begin{matrix}1/2\\2/3\\2/3\\1/1\end{matrix}}_{6/9}}^{7/11}\qquad\overbrace{\underbrace{\begin{matrix}1/2\\1/2\\2/3\\1/1\end{matrix}}_{5/8}\quad\underbrace{\begin{matrix}1/2\\2/3\\1/1\\1/1\end{matrix}}_{5/7}}^{6/9}\qquad\overbrace{\underbrace{\begin{matrix}1/2\\2/3\\1/1\\1/1\end{matrix}}_{5/7}\quad\underbrace{\begin{matrix}2/3\\1/1\\1/1\\1/1\end{matrix}}_{5/6}}^{6/8}\qquad\overbrace{\underbrace{\begin{matrix}2/3\\2/3\\1/1\\1/1\end{matrix}}_{6/8}\quad\underbrace{\begin{matrix}2/3\\1/1\\1/1\\1/1\end{matrix}}_{5/6}}^{7/9}\qquad\overbrace{\underbrace{\begin{matrix}2/3\\3/4\\3/4\\1/1\end{matrix}}_{9/12}\quad\underbrace{\begin{matrix}3/4\\3/4\\1/1\\1/1\end{matrix}}_{8/10}}^{10/13}$$

These have been enlarged from previous and the following ones:

$$\overbrace{\underbrace{\begin{matrix}1/2\\1/2\\1/2\\2/3\end{matrix}}_{5/9}\quad\underbrace{\begin{matrix}1/2\\1/2\\2/3\\1/1\end{matrix}}_{5/8}}^{6/10}\qquad\overbrace{\underbrace{\begin{matrix}1/2\\1/2\\1/2\\1/1\end{matrix}}_{4/7}\quad\underbrace{\begin{matrix}1/2\\1/2\\1/1\\1/1\end{matrix}}_{4/6}}^{5/8}\qquad\overbrace{\underbrace{\begin{matrix}1/2\\1/2\\1/1\\1/1\end{matrix}}_{4/6}\quad\underbrace{\begin{matrix}1/2\\1/1\\1/1\\1/1\end{matrix}}_{4/5}}^{5/7}\qquad\overbrace{\underbrace{\begin{matrix}1/2\\1/1\\1/1\\1/1\end{matrix}}_{4/5}}^{5/6}\qquad\overbrace{\underbrace{\begin{matrix}2/3\\3/4\\1/1\\1/1\end{matrix}}_{7/9}\quad\underbrace{\begin{matrix}3/4\\1/1\\1/1\\1/1\end{matrix}}_{6/7}}^{8/10}$$

These have been enlarged from previous and the following ones:

$$\overbrace{\underbrace{\begin{matrix}1/2\\1/2\\1/2\\1/2\end{matrix}}_{4/8}\quad\underbrace{\begin{matrix}1/2\\1/2\\1/2\\1/1\end{matrix}}_{4/7}}^{5/9}\qquad\overbrace{\underbrace{\begin{matrix}1/2\\1/2\\1/1\\1/1\end{matrix}}_{4/6}\quad\underbrace{\begin{matrix}0/1\\1/2\\1/2\\1/1\end{matrix}}_{3/6}}^{4/7}\qquad\overbrace{\underbrace{\begin{matrix}1/2\\1/1\\1/1\\1/1\end{matrix}}_{4/5}\quad\underbrace{\begin{matrix}0/1\\1/2\\1/1\\1/1\end{matrix}}_{3/5}}^{4/6}\qquad\overbrace{\underbrace{\begin{matrix}1/1\\1/1\\1/1\\1/1\end{matrix}}_{4/4}\quad\underbrace{\begin{matrix}0/1\\1/1\\1/1\\1/1\end{matrix}}_{3/4}}^{4/5}\qquad\overbrace{\underbrace{\begin{matrix}2/3\\1/1\\1/1\\1/1\end{matrix}}_{5/6}}^{6/7}$$

These have been enlarged from previous and the following ones:

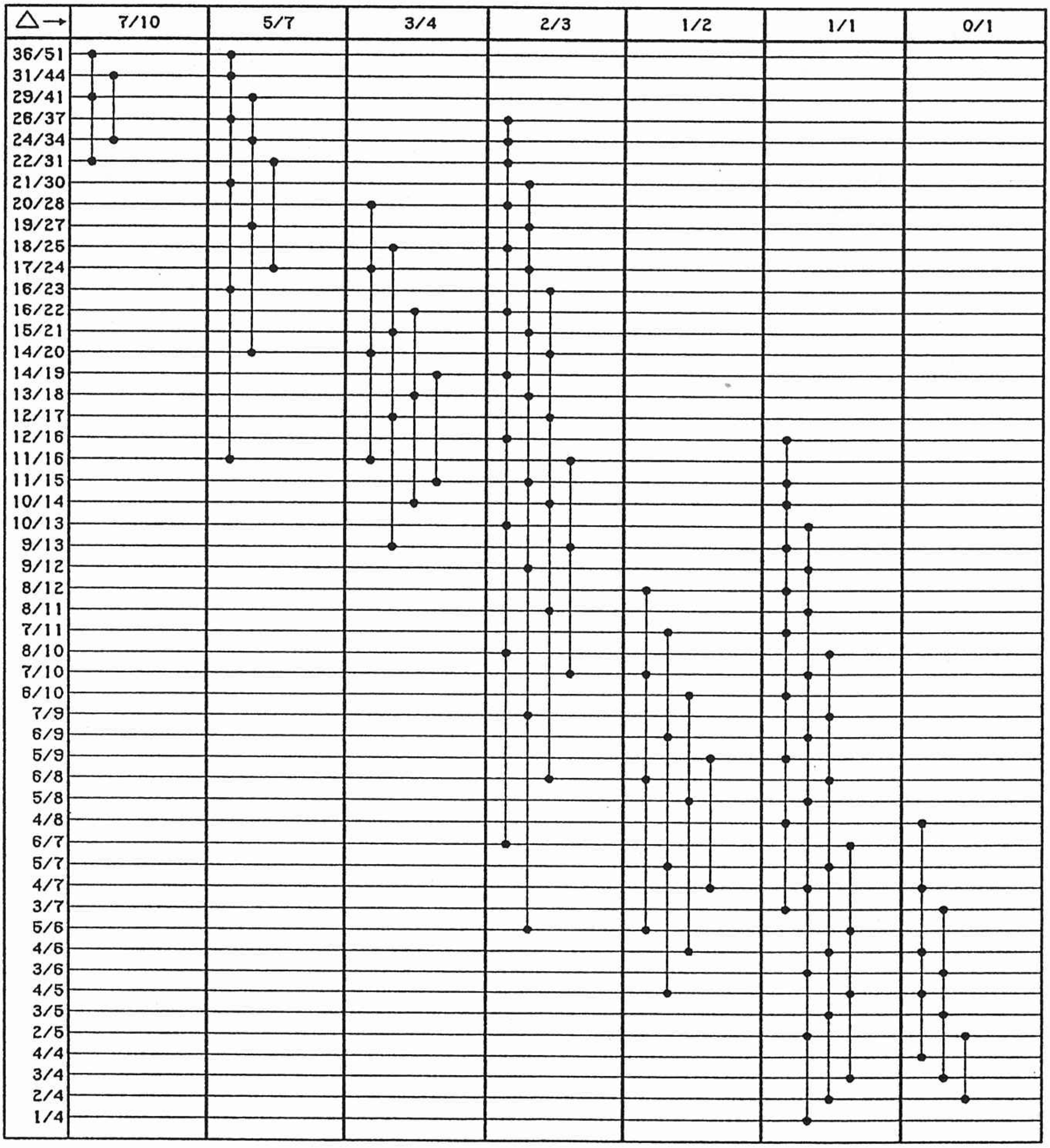
$$\overbrace{\underbrace{\begin{matrix}1/2\\1/2\\1/2\\1/1\end{matrix}}_{4/7}\quad\underbrace{\begin{matrix}0/1\\1/2\\1/2\\1/2\end{matrix}}_{3/7}}^{4/8}\qquad\overbrace{\underbrace{\begin{matrix}0/1\\1/2\\1/1\\1/1\end{matrix}}_{3/5}\quad\underbrace{\begin{matrix}0/1\\0/1\\1/2\\1/1\end{matrix}}_{2/5}}^{3/6}\qquad\overbrace{\underbrace{\begin{matrix}0/1\\1/1\\1/1\\1/1\end{matrix}}_{3/4}\quad\underbrace{\begin{matrix}0/1\\0/1\\1/1\\1/1\end{matrix}}_{2/4}}^{3/5}$$

These have been enlarged from previous and the following ones:

$$\overbrace{\underbrace{\begin{matrix}0/1\\1/2\\1/2\\1/1\end{matrix}}_{3/6}}^{3/7}\qquad\overbrace{\underbrace{\begin{matrix}0/1\\0/1\\1/1\\1/1\end{matrix}}_{2/4}\quad\underbrace{\begin{matrix}0/1\\0/1\\0/1\\1/1\end{matrix}}_{1/4}}^{2/5}$$

A tabulated overview of the various enlargement-paths:

Whether the various enlargement-paths are obtained from the appropriate part of an Enlargement Graph or by calculation, we still have no proper overview of these paths. Such an overview is however essential before it is possible to decide on the best way in which to braid the end-keeper concerned by means of successive enlargements. In order for the overview to be easily comprehensible, it should be in a graphical tabulated form, for example as presented below for the 36/51-endkeeper ($x = 4$).



Graphical overview of the enlargement-paths associated with a 36/51-endkeeper ($x = 4$).

Although it is generally easier to draw the appropriate part of the Enlargement Graph than to calculate the various enlargement-paths, it is easier to assemble the information for a graphical overview from the calculation results rather than from the appropriate part of the Enlargement Graph. The best, and hence most convenient method is to calculate and graph the data concerned with the aid of a computer by means of suitable software. Such a computer program can be enhanced by a function which finds and records all possible paths from the end-keeper in question with a given coding to the smallest, coding-wise compatible, end-keeper with x transition points. A further analyzation function can be incorporated, which analyzes these various suitable enlargement-paths with respect to time-wise braiding efficiency. Since in general the braiding of end-keepers via enlargements is only of practical value when the end-keeper has a Casa-coding, these computer aided analyzation processes can be limited to such a coding arrangement.

Let's assume that this 36/51-endkeeper with $x = 4$ has a Casa-coding, and that we intend to braid this end-keeper by means of the common Casa enlargement procedures of "track-laying" and "track-splitting". It will readily be seen from the Enlargement-path table, that the smallest suitable end-keeper with which to start the enlargement processes is the 2/5 Casa-coded end-keeper. We can also start with the 4/5 Casa-coded end-keeper. There are no 4-bights, 6-bights, or other 5-bights Casa-coded end-keepers (with $x = 4$ transition-points) which can be enlarged by the common Casa enlargement procedures to the 36/51 Casa-coded end-keeper.

REGULAR MID-KEEPERS

A **regular mid-keeper** is a Regular Cylindrical Braid with p-parts and b-bights, which at both bight-boundaries goes over into Flat Braids, whereby the flat braids are in line and of identical size with respect to their parts. If the Flat Braids have $2x$ parts each (hence are $2x$ lead Flat Braids), then the transitions between the Regular Cylindrical Braid and the Flat Braids are at x **adjacent** "bight-points" of each of the two bight-boundaries of the Regular Cylindrical Braid; the two sets of x "bight-points" are in line.

There are several methods by which regular mid-keepers can be braided, but before braiding can commence we have to know the valid numerical values of the variables x and p/b of the regular cylindrical braid involved. For convenience we have provided on pgs. 150 — 165 general string-run diagrams in which these numerical values are indicated for the more often encountered mid-keepers. A simple calculation method by which we can obtain all valid p/b-values associated with a set x-value will be discussed later. First we shall, by means of Examples, illustrate some of the several ways in which mid-keepers can be braided.

The grid-diagrams on pg. 118 illustrate the construction steps of a Casa-coded 4/7-midkeeper with $x = 3$. The associated pictorial drawings are presented on pg. 119.

The grid-diagrams on pg. 120 illustrate the construction steps of a Casa-coded 10/7-midkeeper with $x = 3$. The associated pictorial drawings are presented on pg. 121.

The grid-diagrams on pg. 122 illustrate the construction steps of a Casa-coded 12/7-midkeeper with $x = 3$. The associated pictorial drawings are presented on pg. 123.

The grid-diagrams on pgs. 124 & 125 illustrate the construction steps of a Casa-coded 12/9-midkeeper with $x = 3$.

The grid-diagrams on pg. 126 illustrate the construction steps of a Casa-coded 4/8-midkeeper with $x = 4$. The associated pictorial drawings are presented on pg. 127.

The grid-diagrams on pg. 128 illustrate the construction steps of a Casa-coded 12/8-midkeeper with $x = 4$. The associated pictorial drawings are presented on pg. 129.

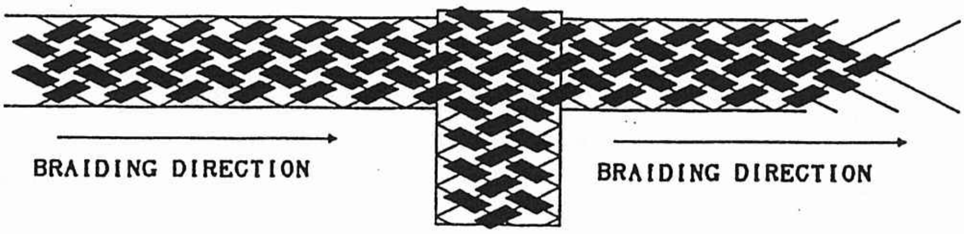
The grid-diagrams on pg. 130 illustrate the construction steps of a Casa-coded 6/10-midkeeper with $x = 4$. The associated pictorial drawings are presented on pg. 131.

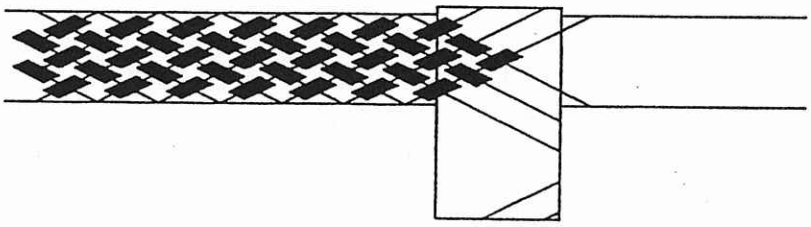
The grid-diagrams on pg. 132 illustrate the construction steps of a Casa-coded 4/13-midkeeper with $x = 5$. The associated pictorial drawings are presented on pg. 133.

The grid-diagrams on pg. 134 illustrate the construction steps of a Casa-coded 6/11-midkeeper with $x = 5$. The associated pictorial drawings are presented on pg. 135.

The grid-diagrams on pg. 136 illustrate the construction steps of a Casa-coded 6/12-midkeeper with $x = 6$. The associated pictorial drawings are presented on pg. 137.

In some applications, where the mid-keeper is centrally placed with respect to the ends of the flat braids, it is much more convenient to start the braiding process from the centre of the mid-keeper. An example of this is given on pgs. 138 and 139, where for a 20/8-midkeeper the grid-diagrams show the consecutive braiding steps. The associated pictorial drawings are presented on pgs. 140 and 141.

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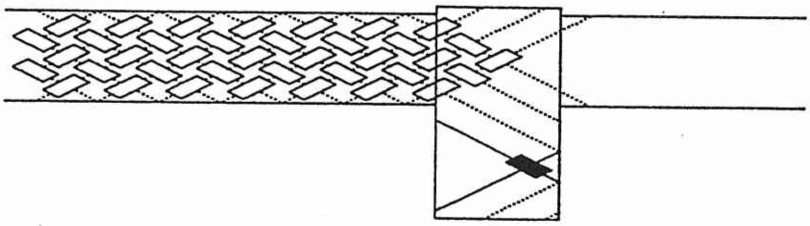
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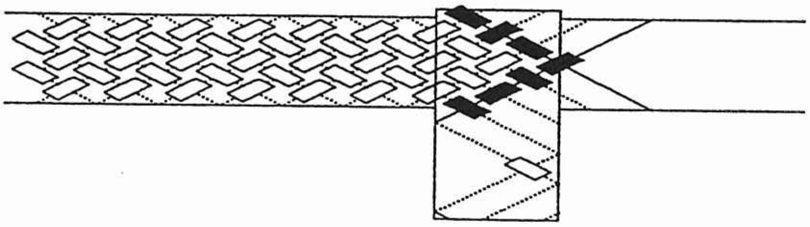
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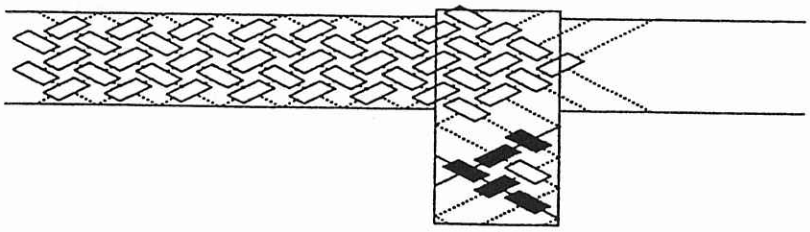
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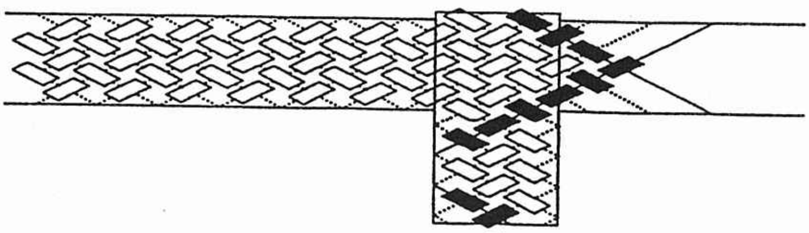
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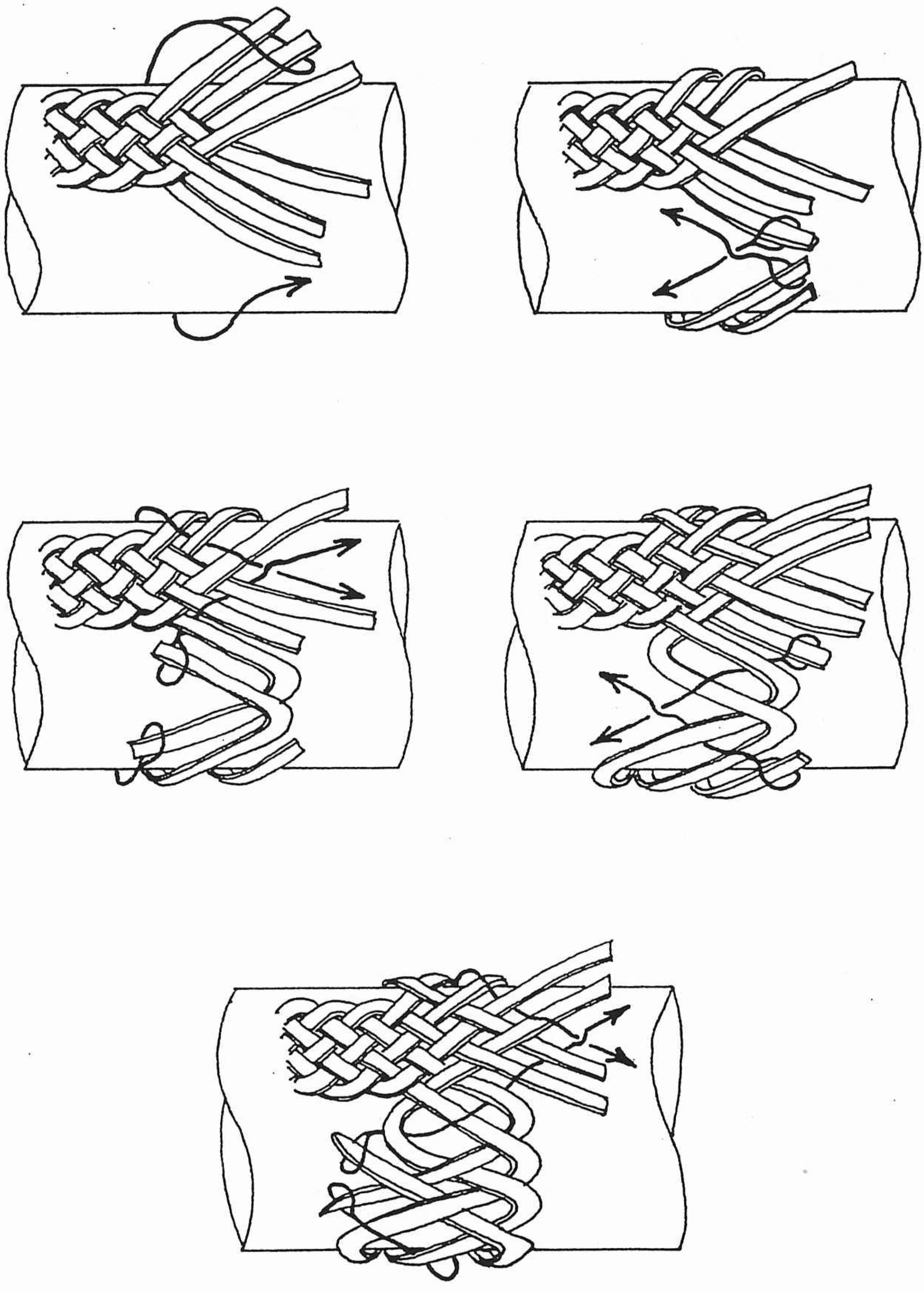
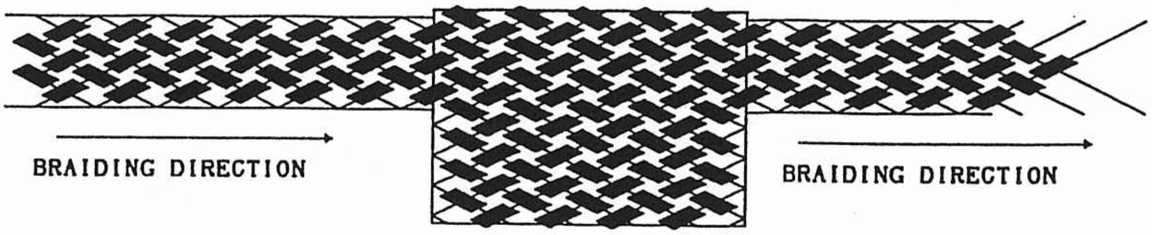
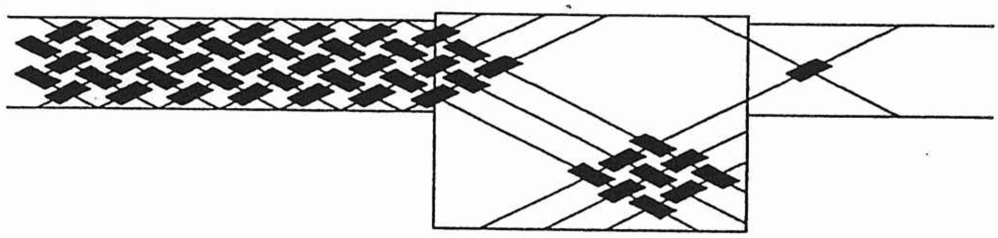


Pictorial drawings associated with pg. 118.

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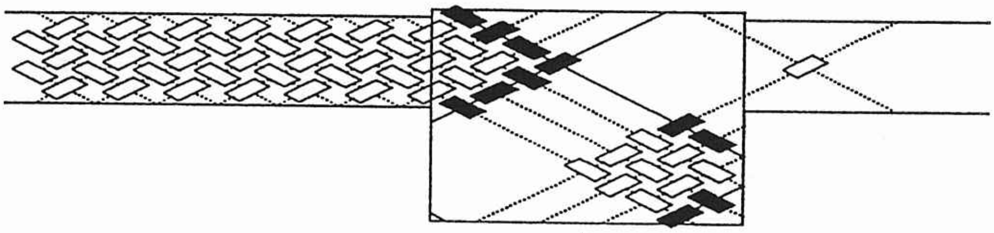
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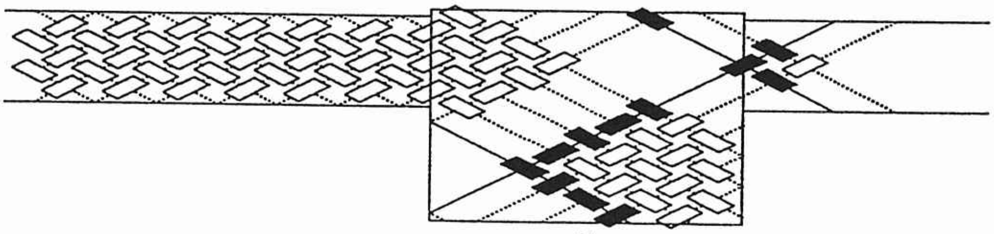
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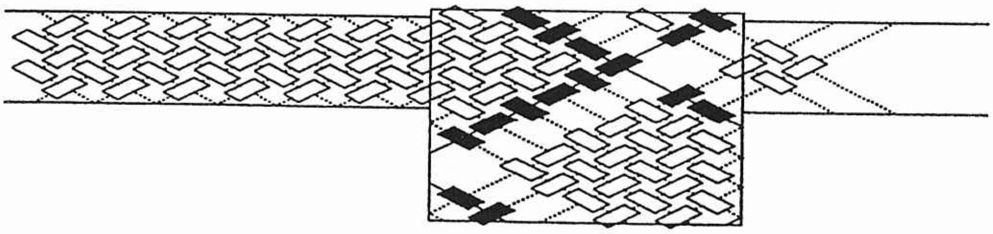
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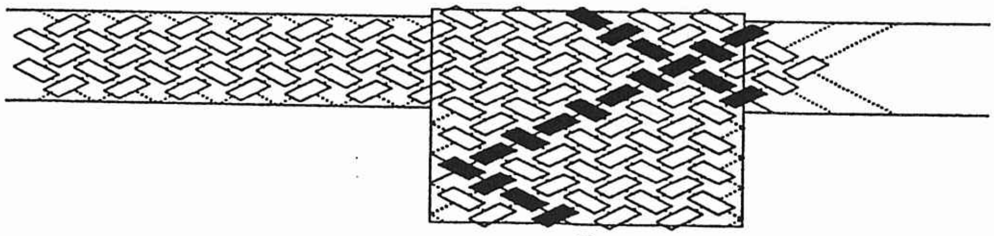
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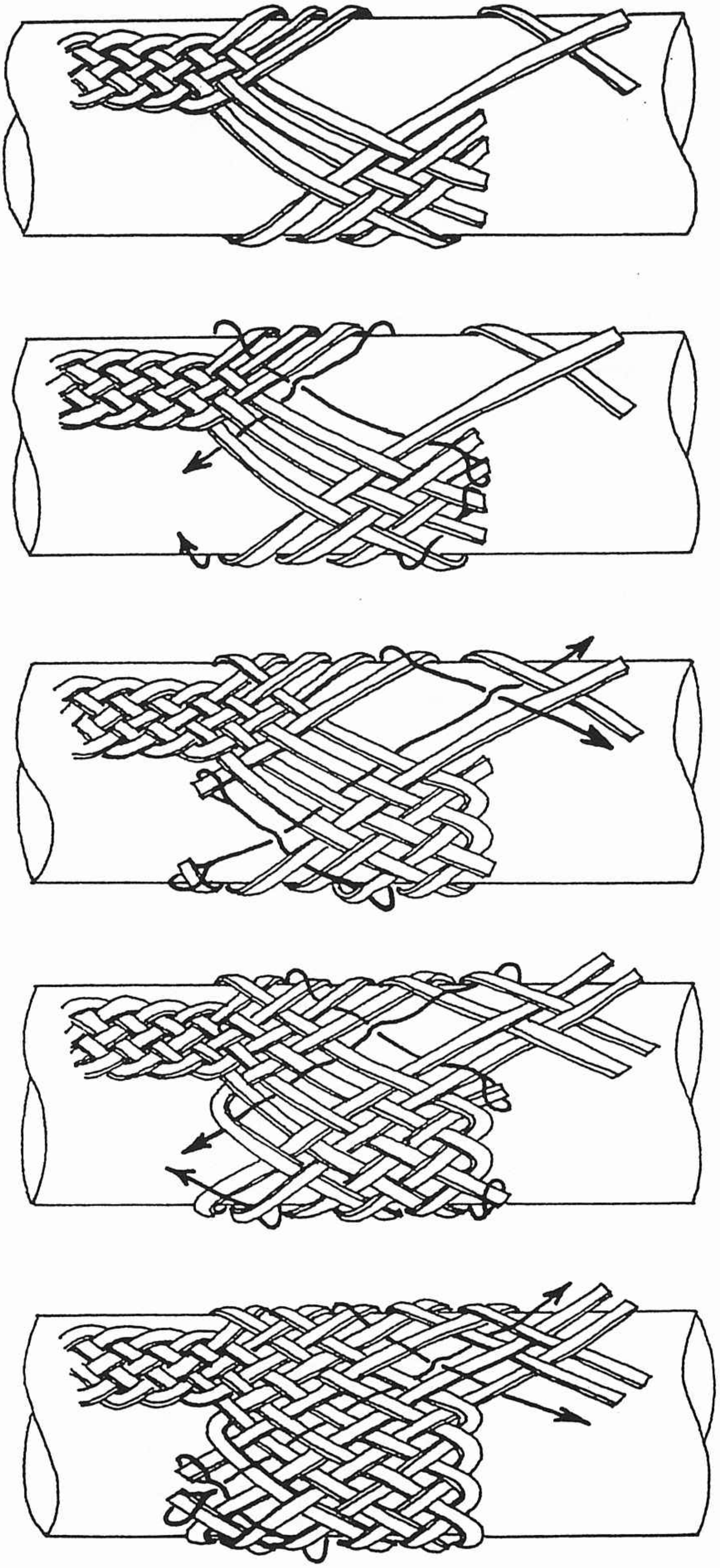
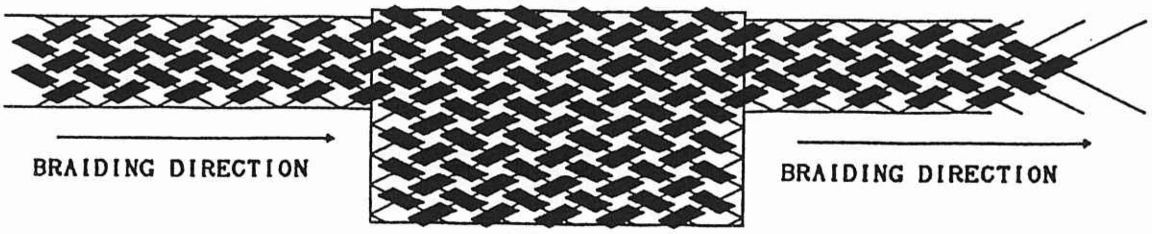
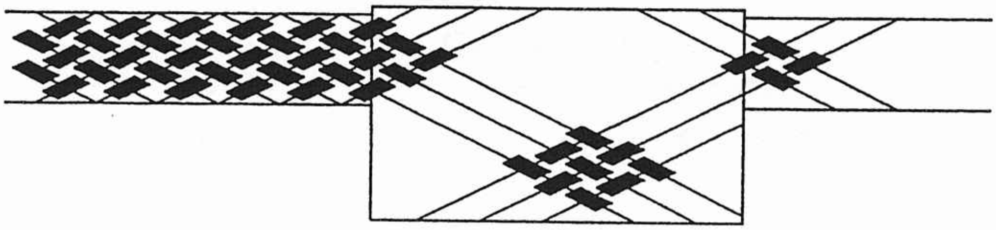


Pictorial drawings associated with pg. 120.

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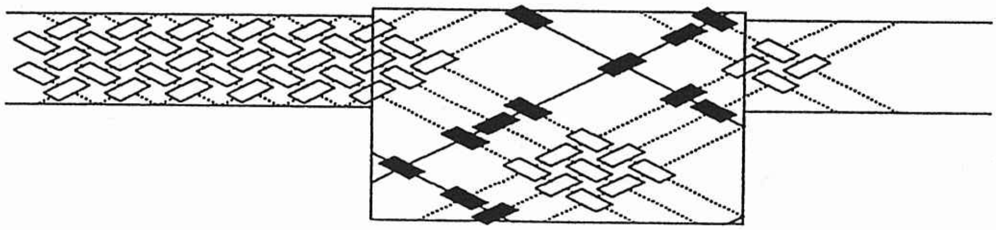
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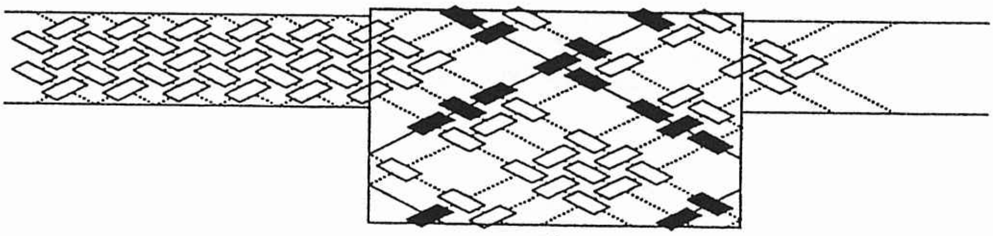
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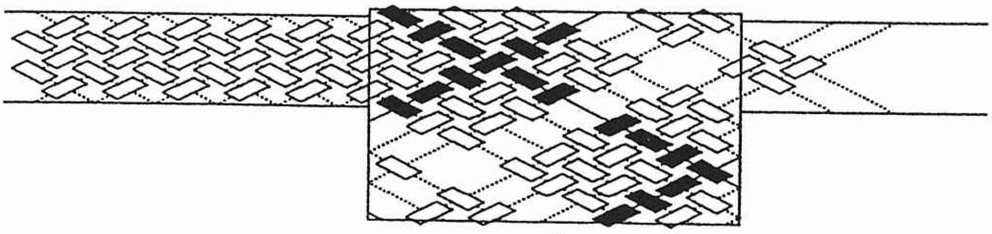
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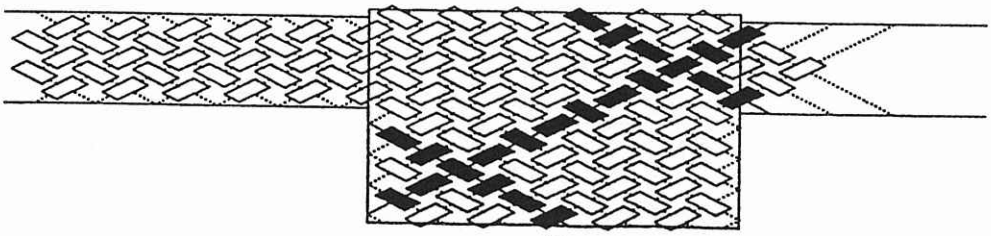
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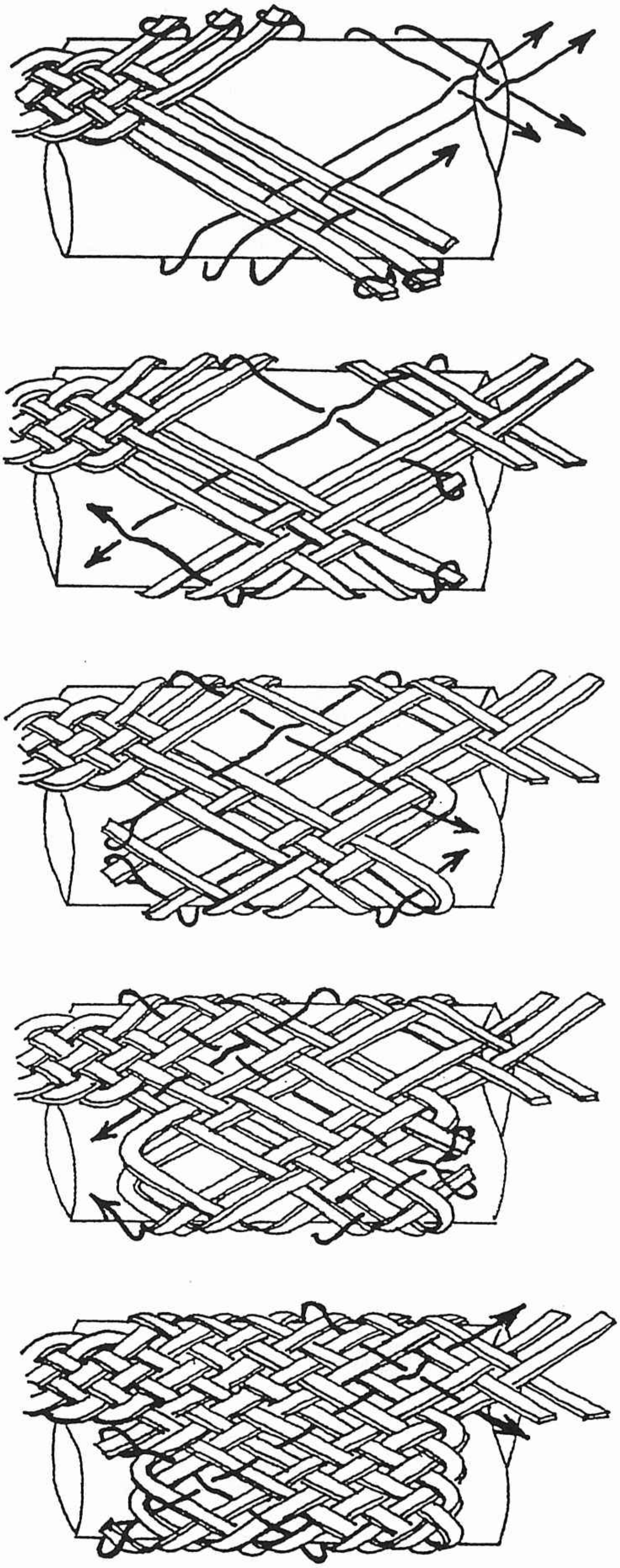
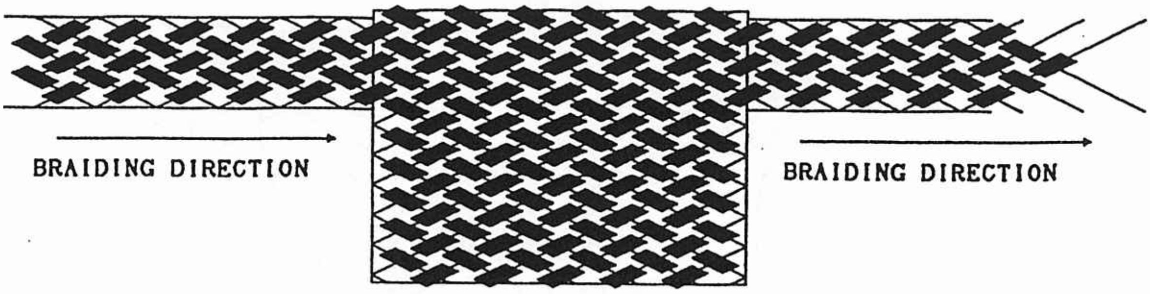
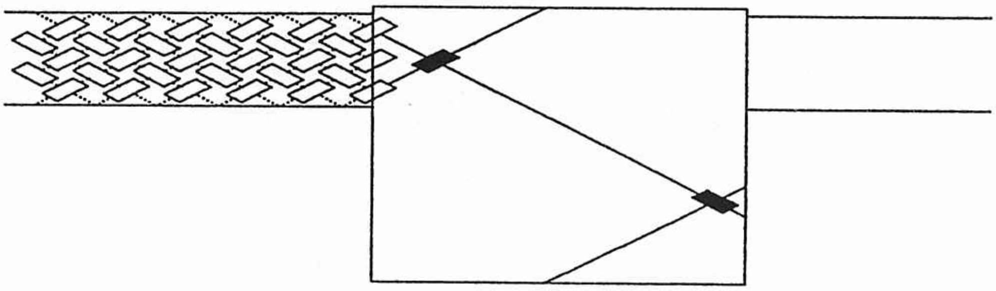


Pictorial drawings associated with pg. 122.

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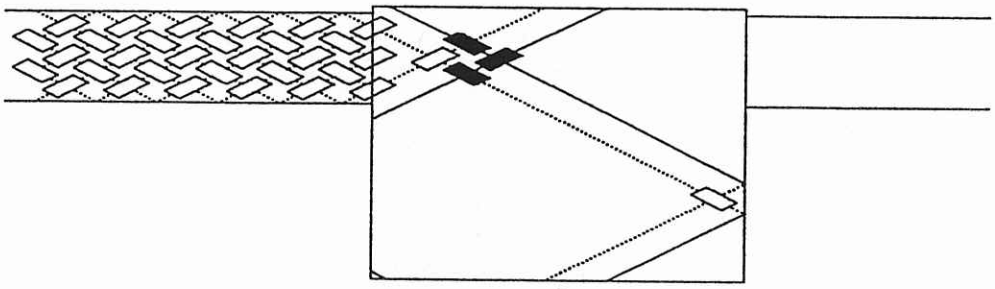
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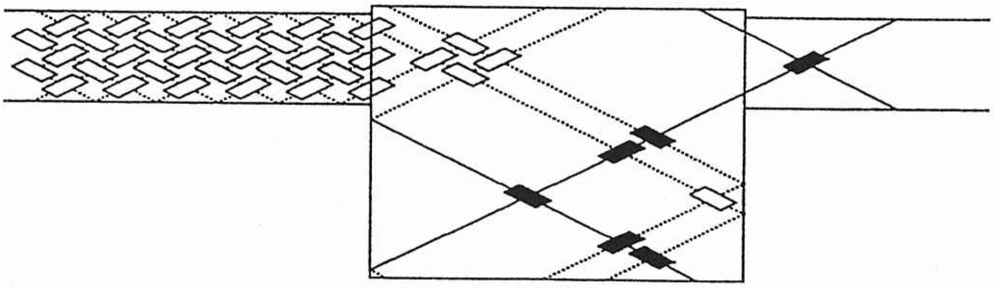
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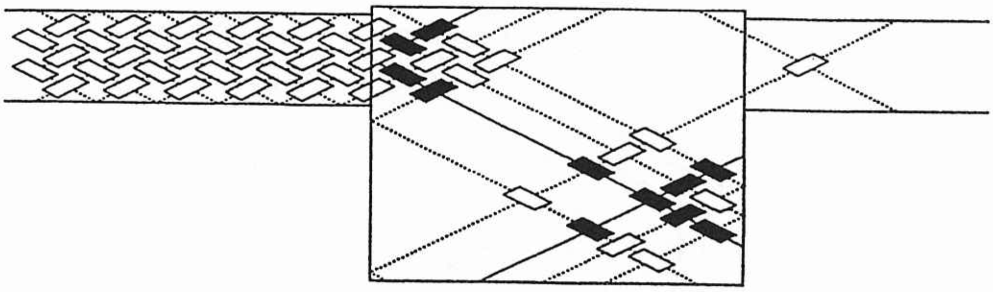
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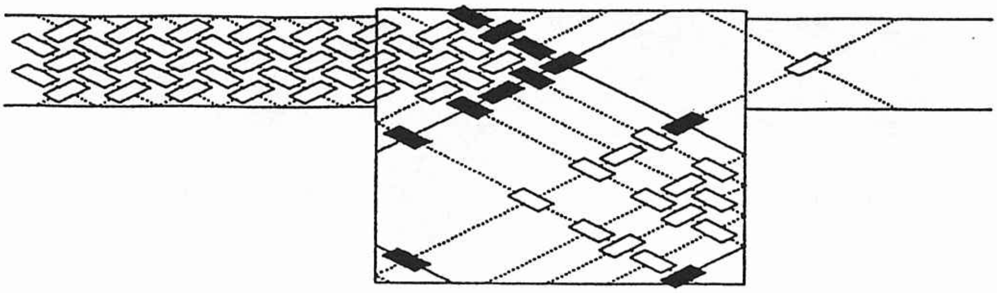

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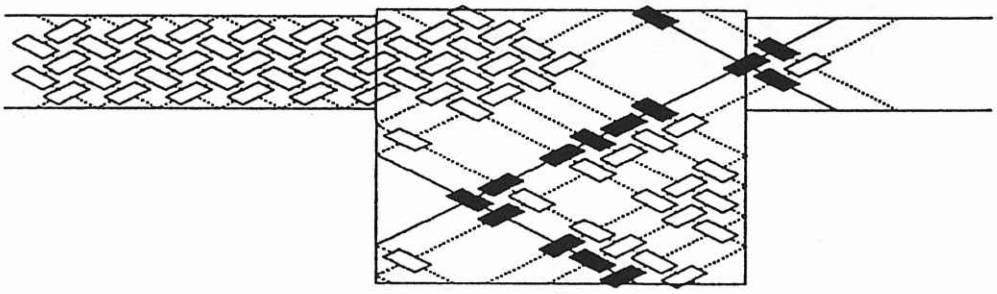

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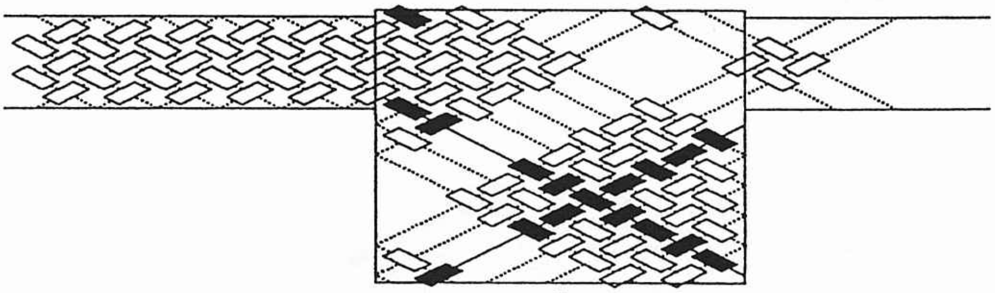

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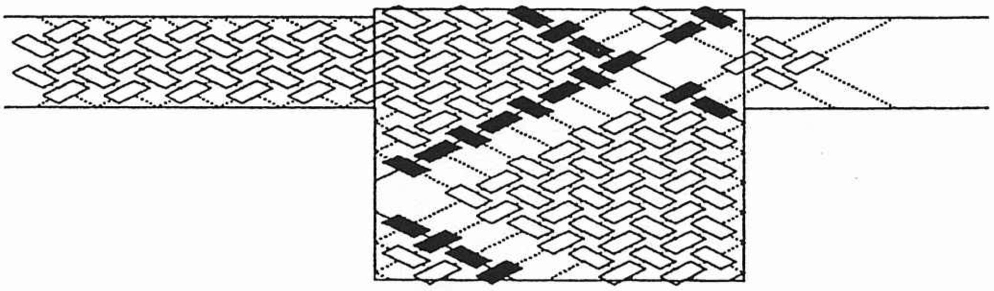

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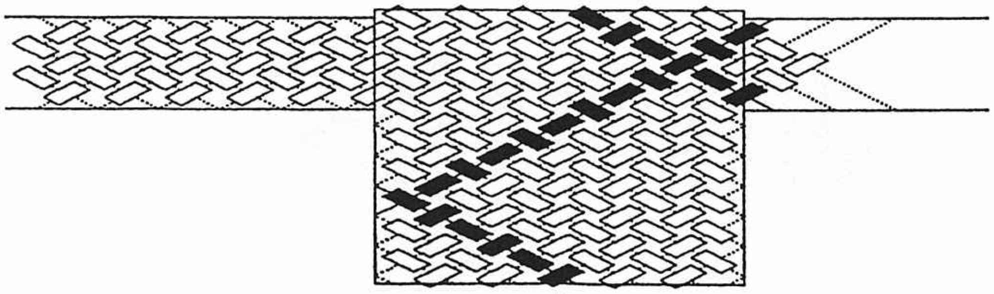

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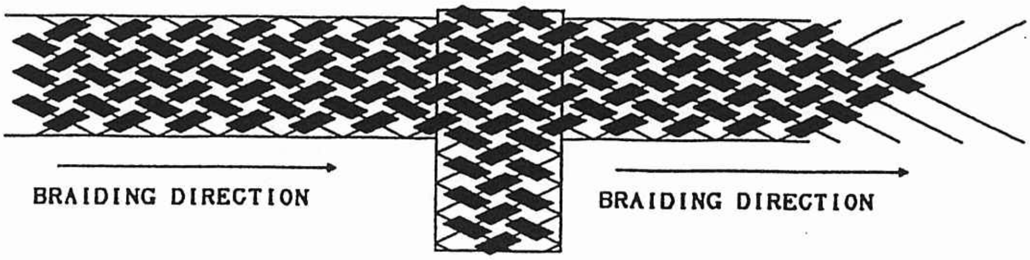

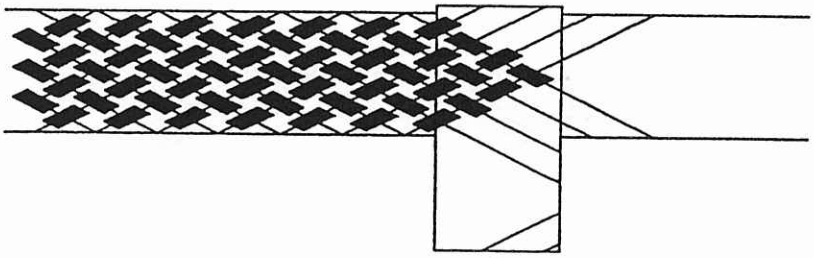
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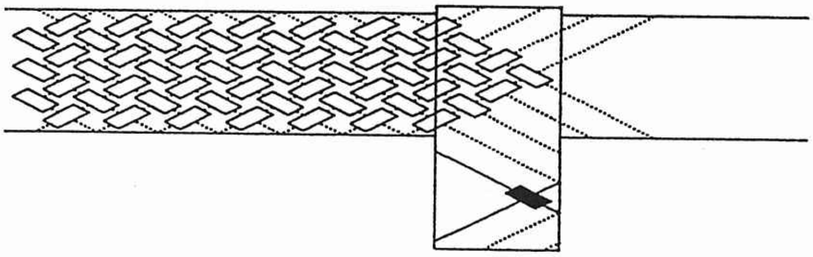
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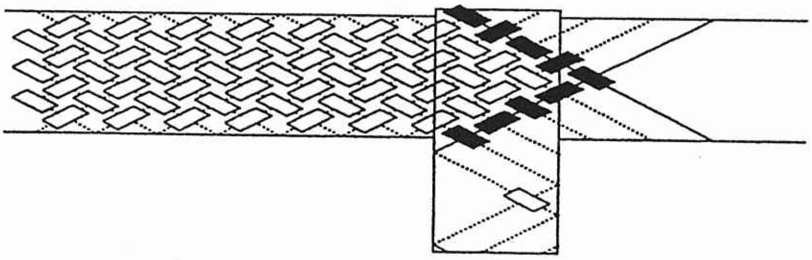
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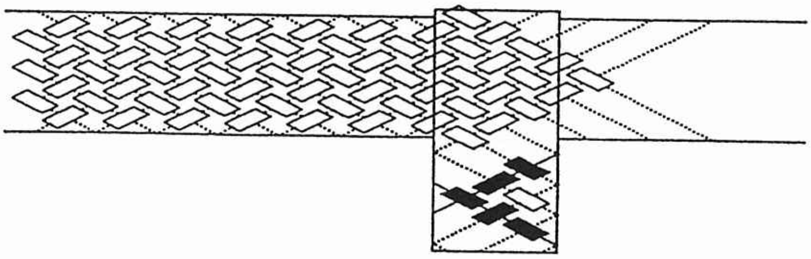
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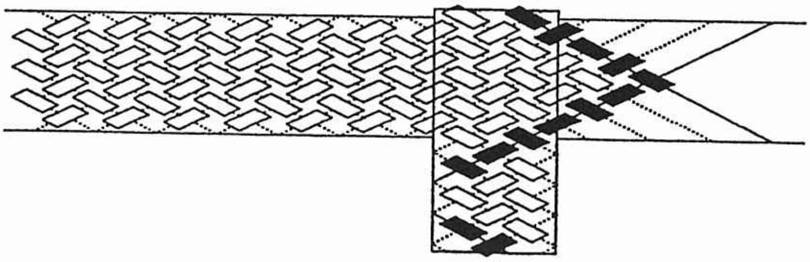
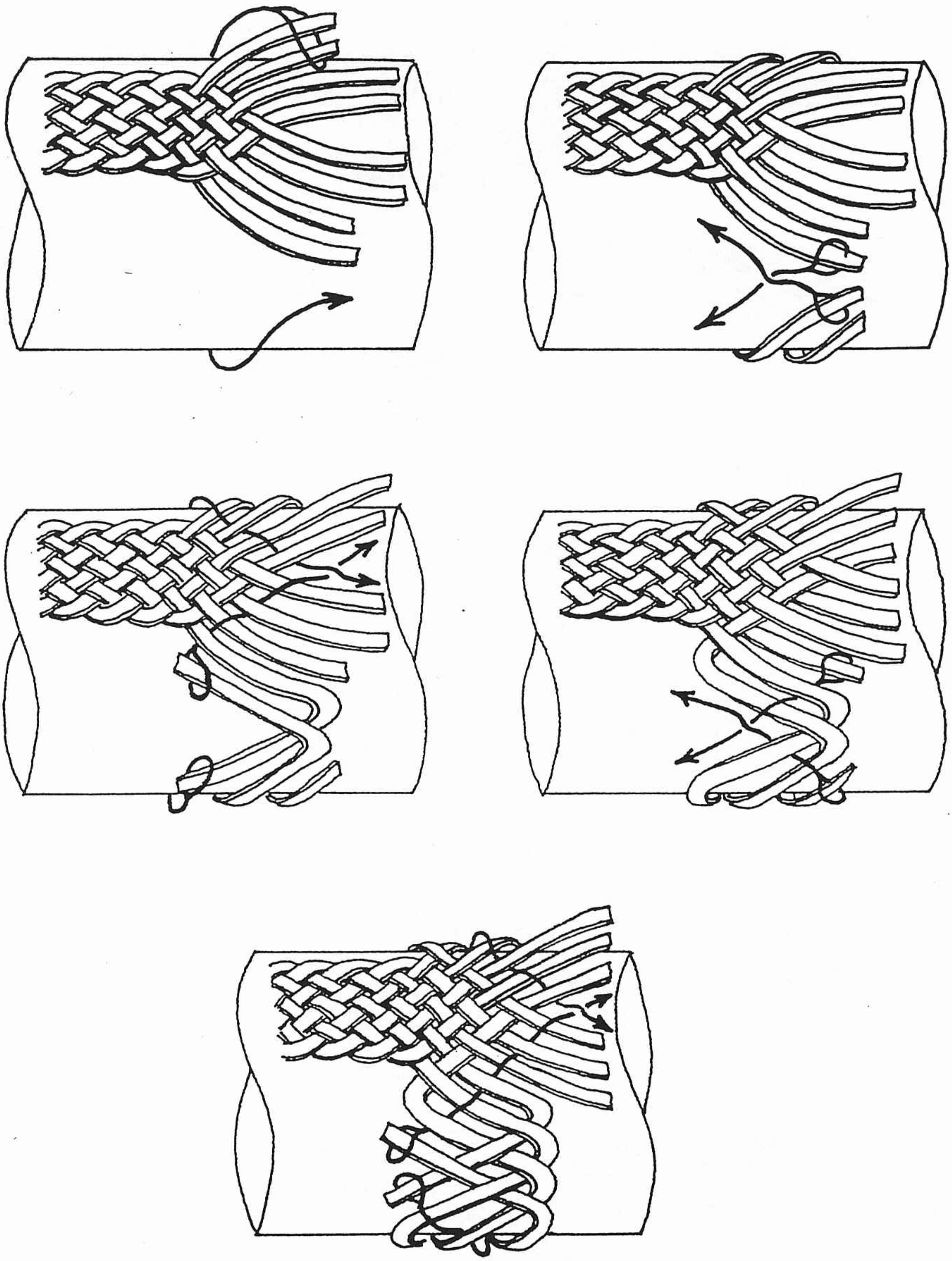
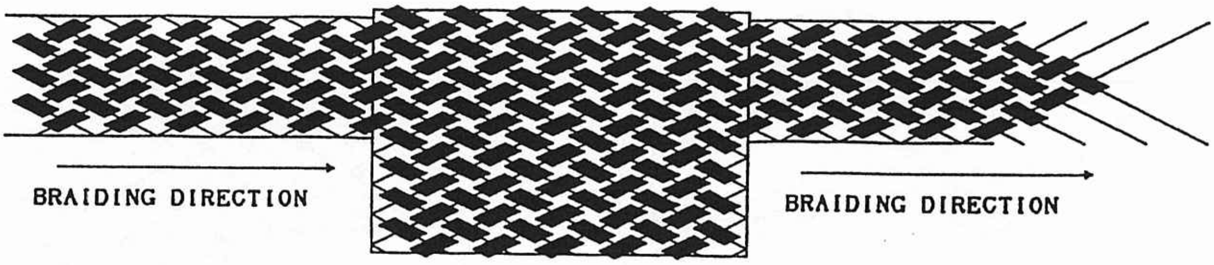
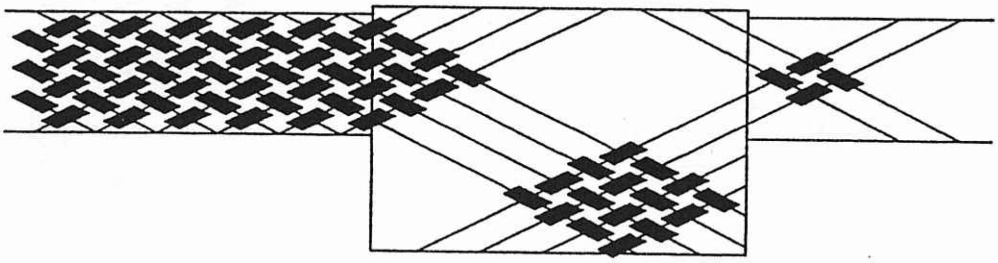

5



Pictorial drawings associated with pg. 126.

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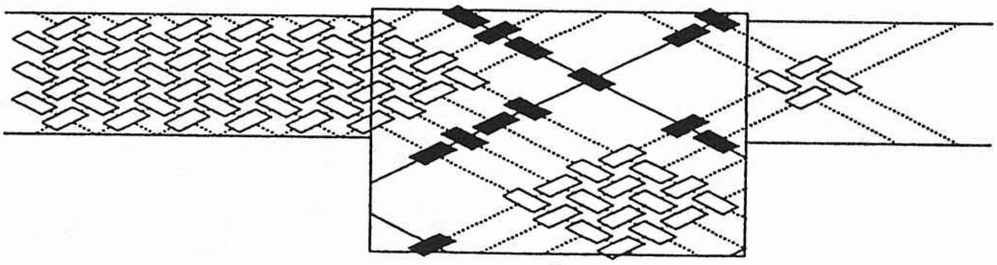
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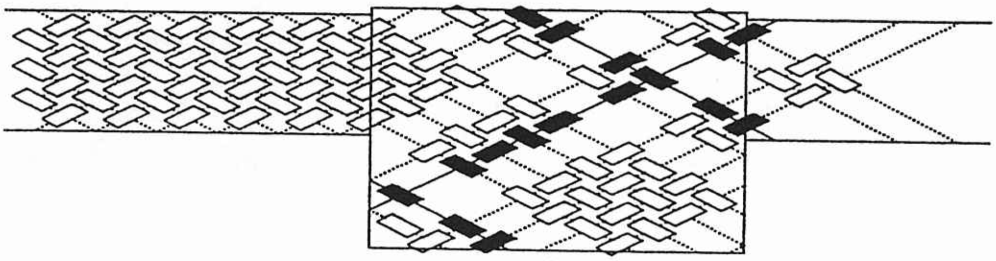
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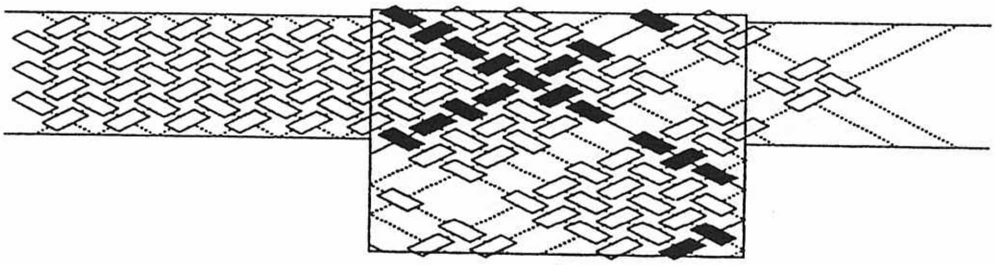
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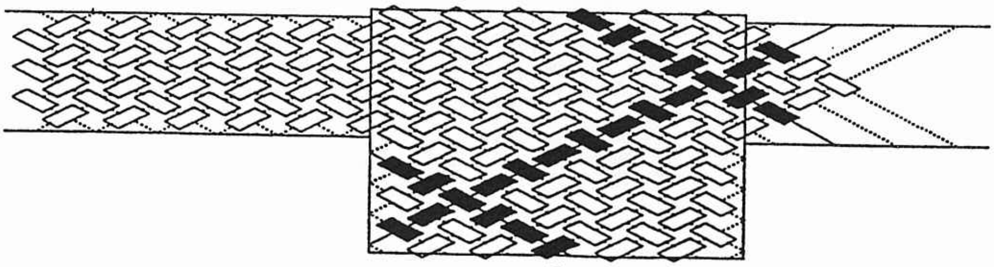
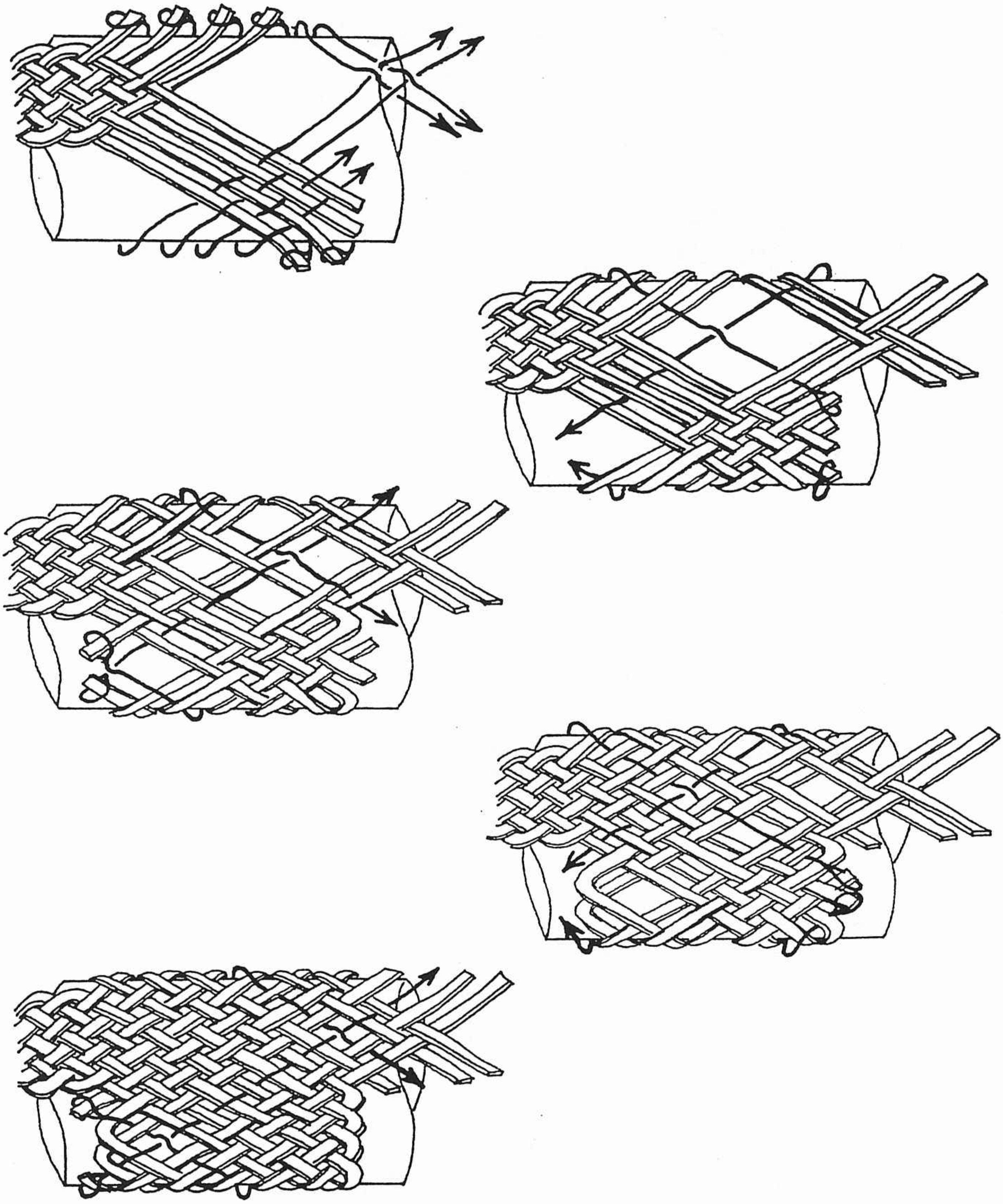
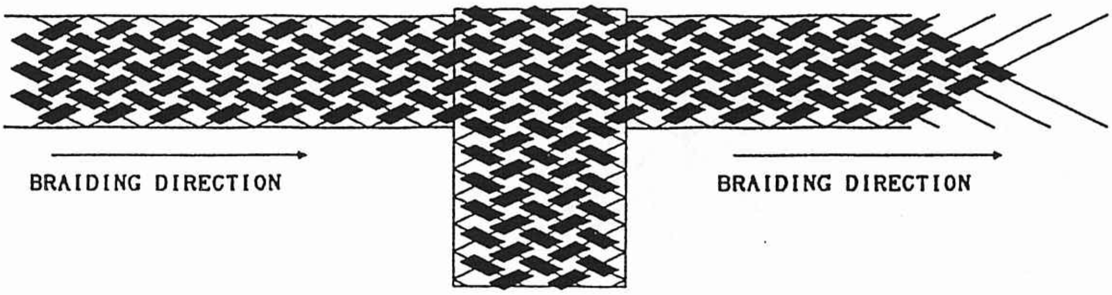
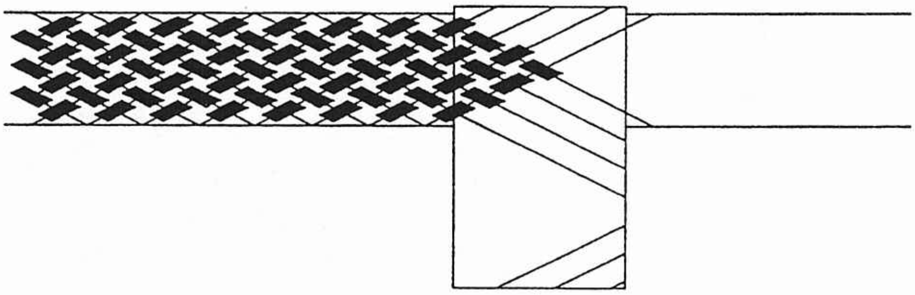

5



Pictorial drawings associated with pg. 128.

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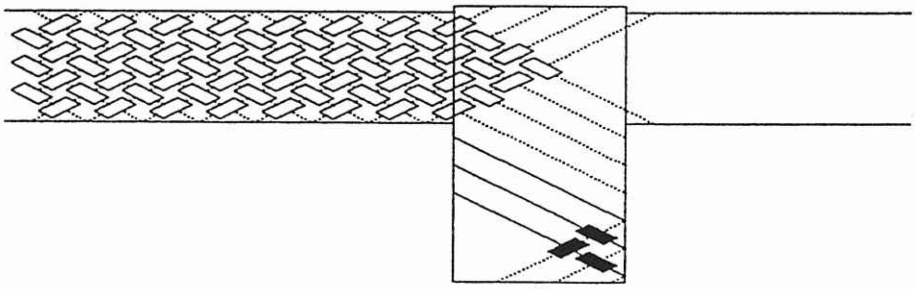
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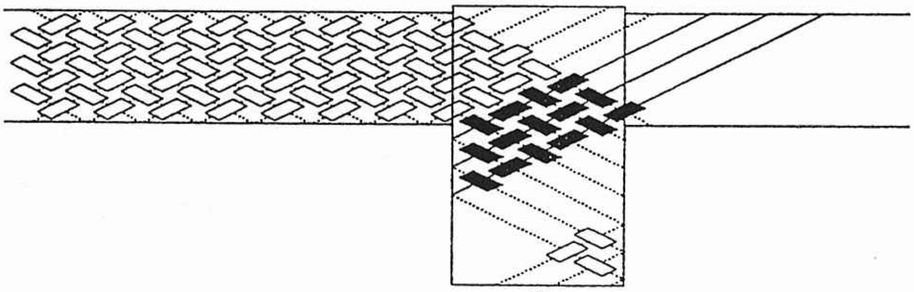
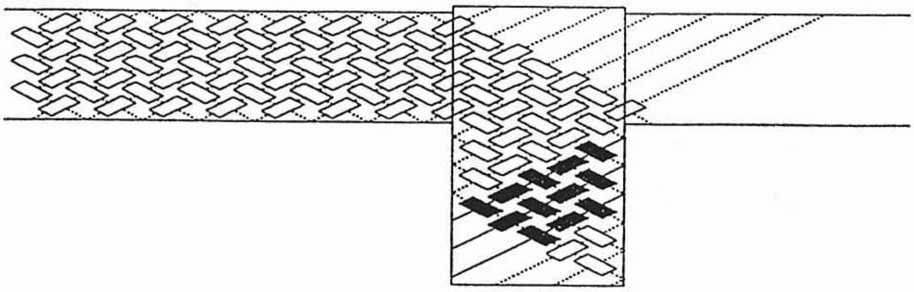

3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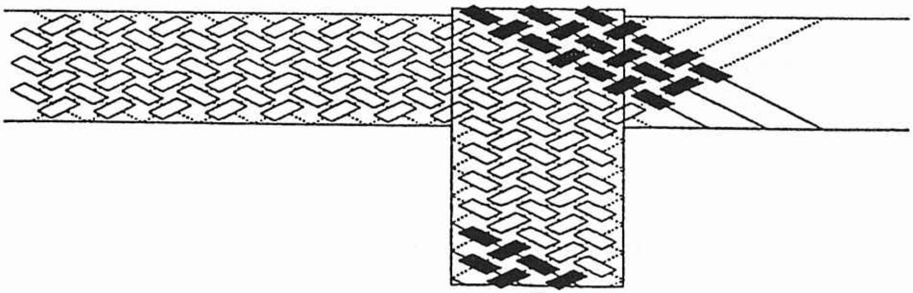
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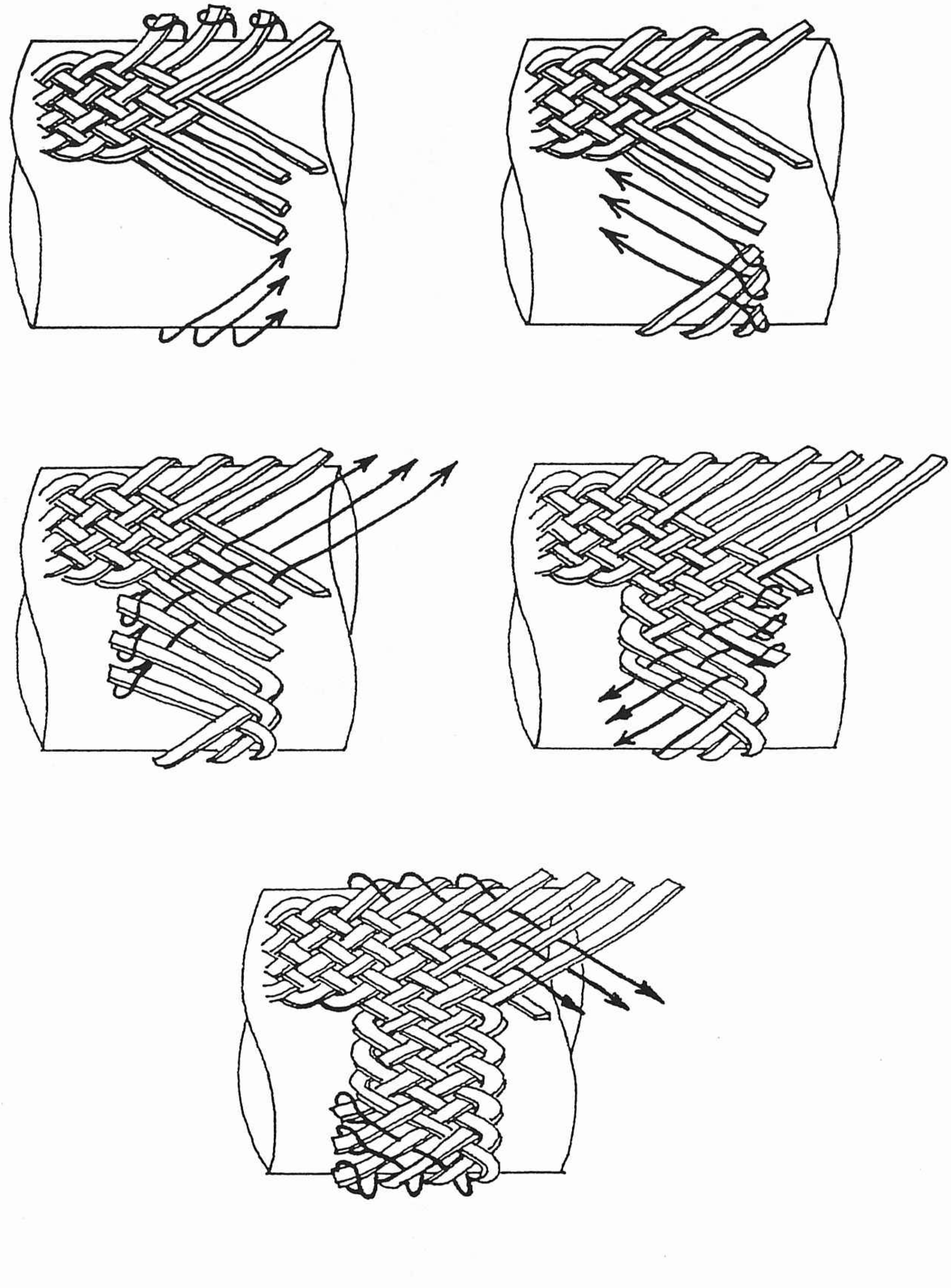
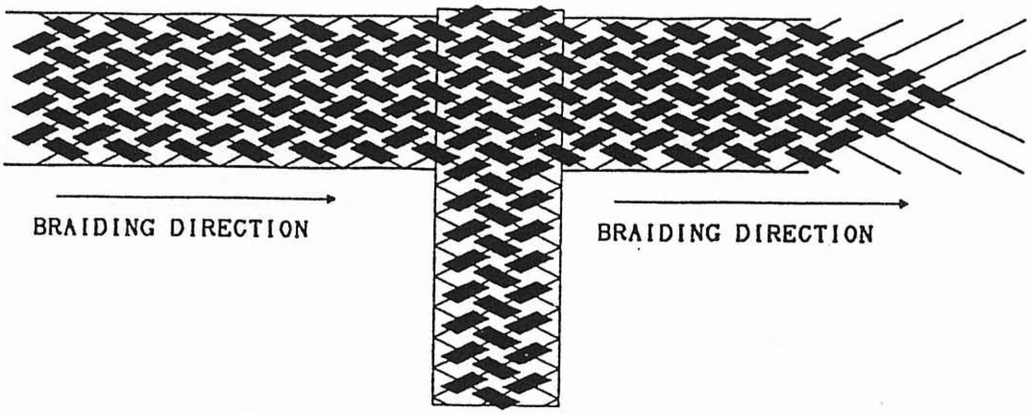
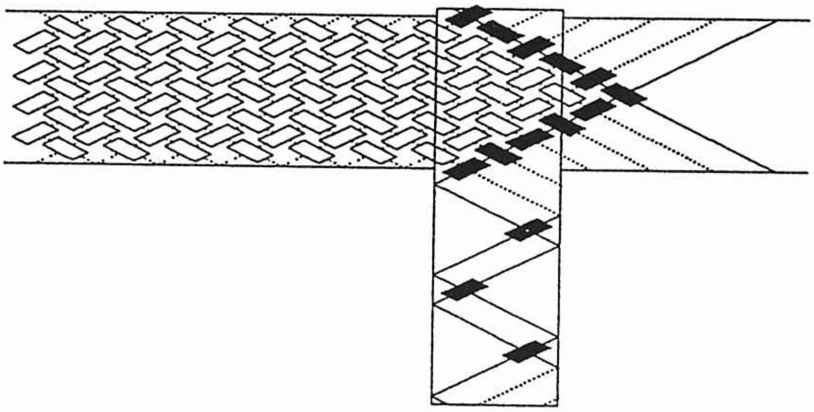


Pictorial drawings associated with pg. 130.

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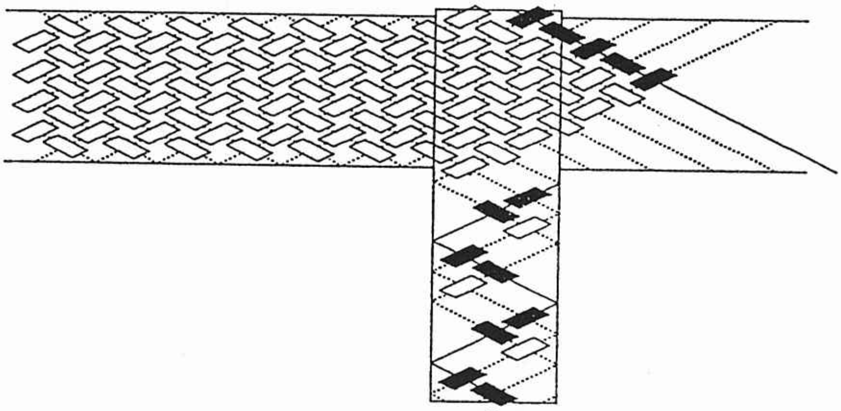
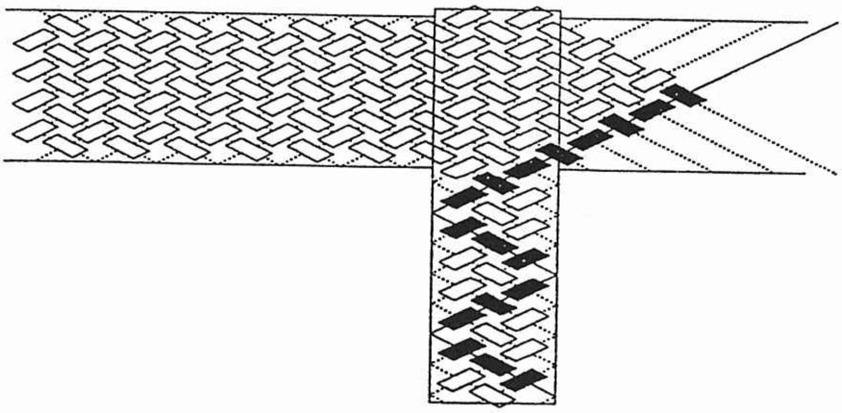
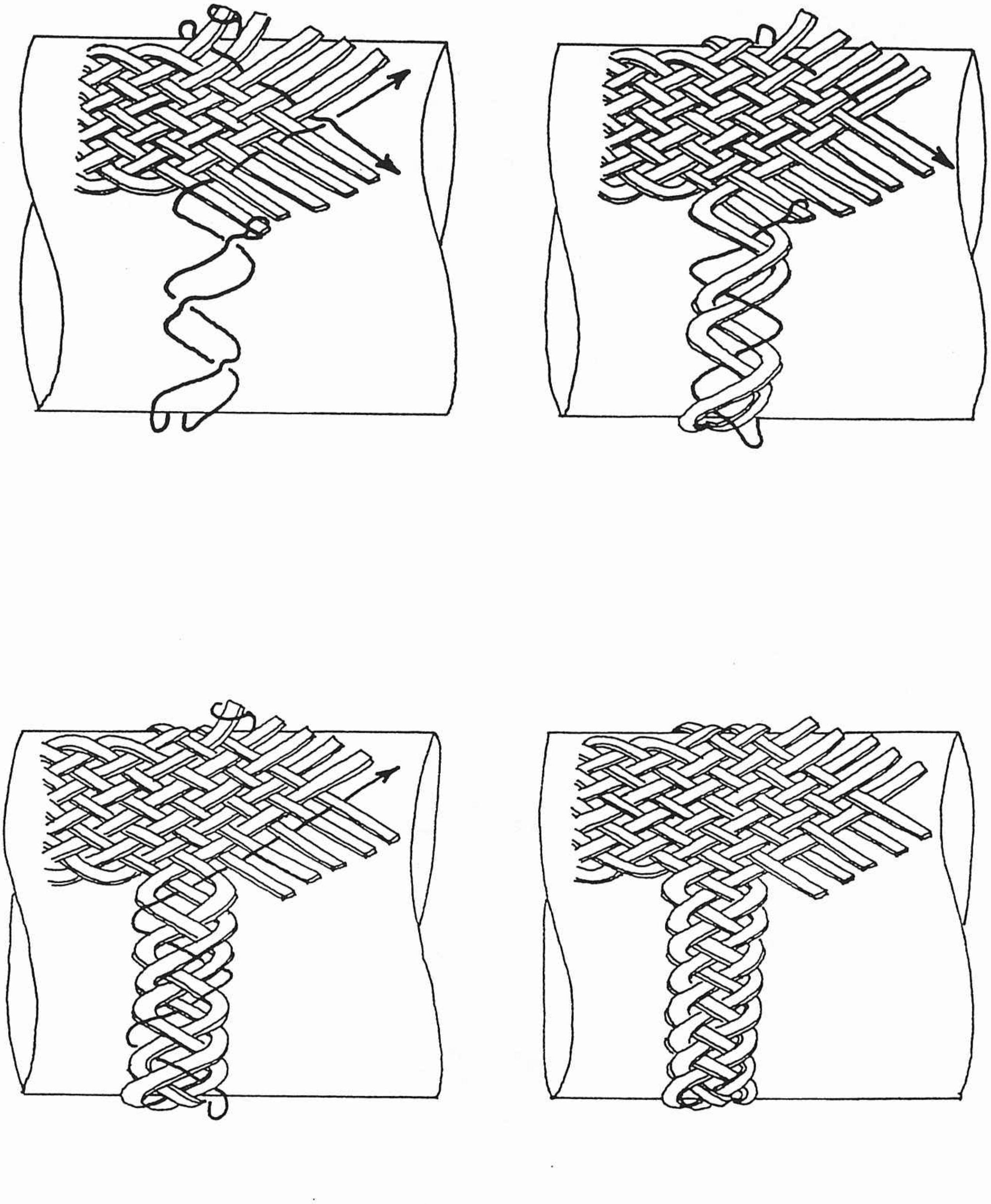
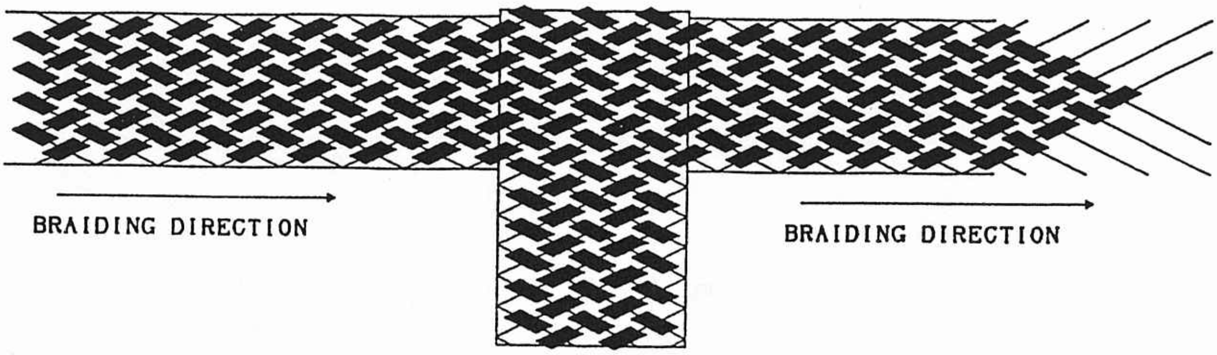
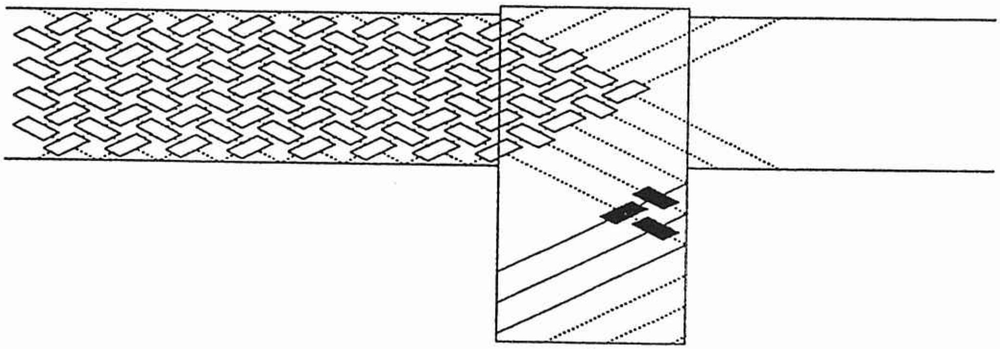

2


3



Pictorial drawings associated with pg. 132.

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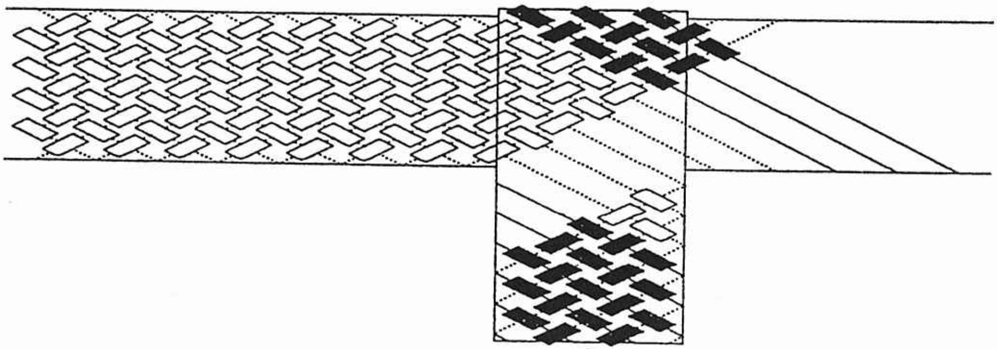
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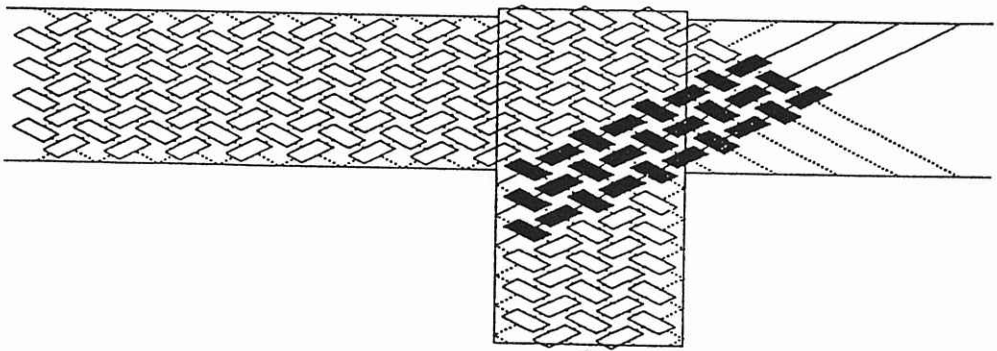
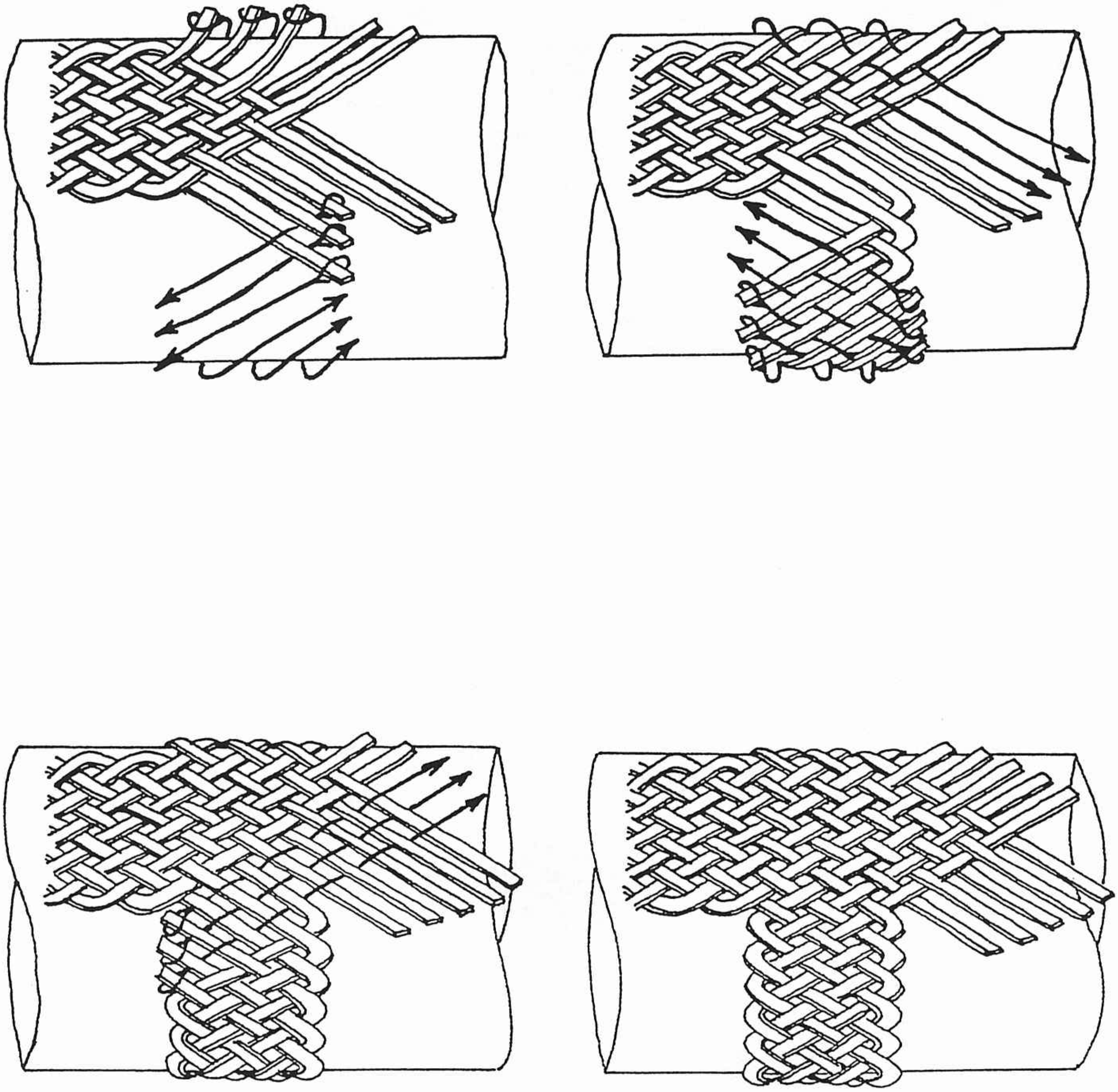
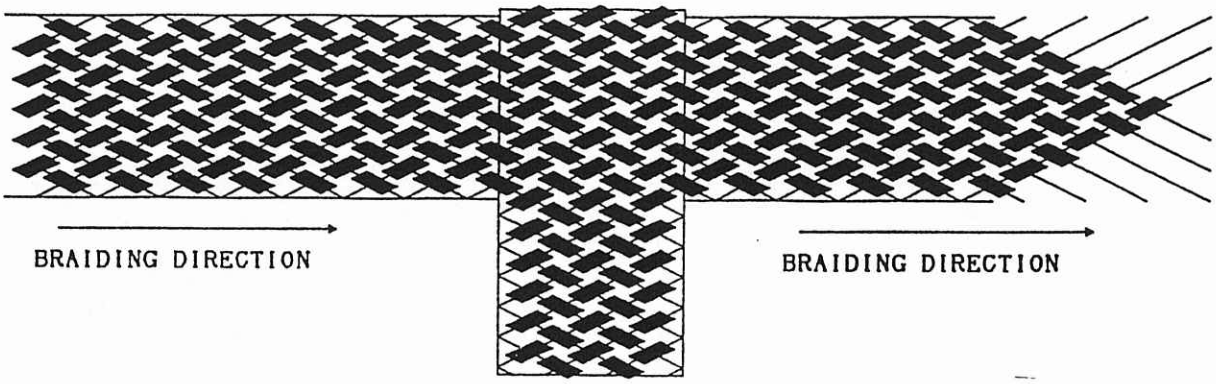
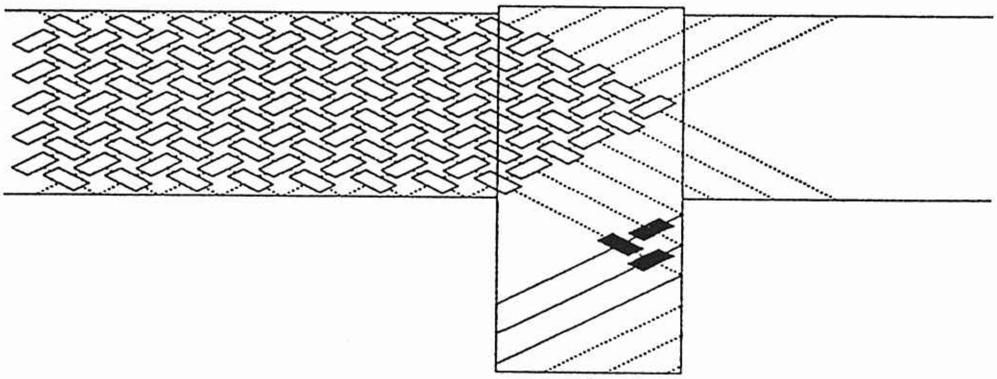

3

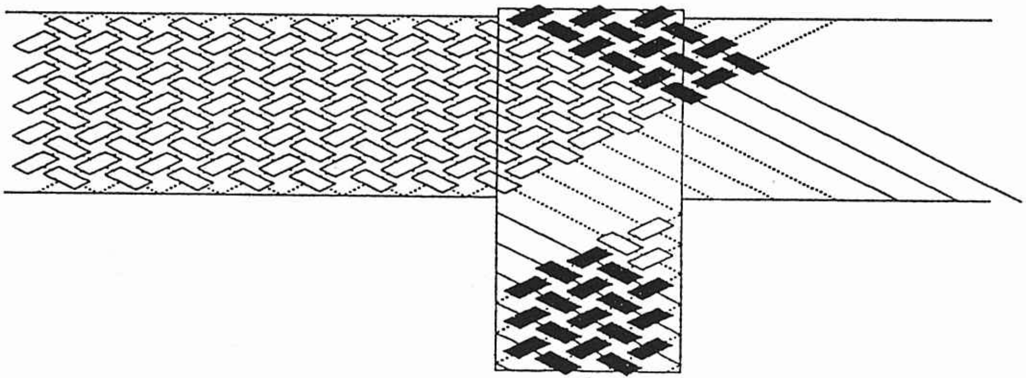


Pictorial drawings associated with pg. 134.

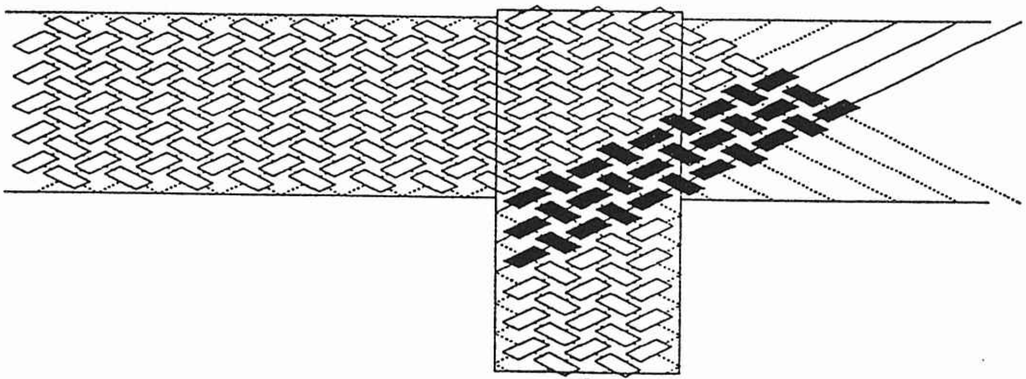




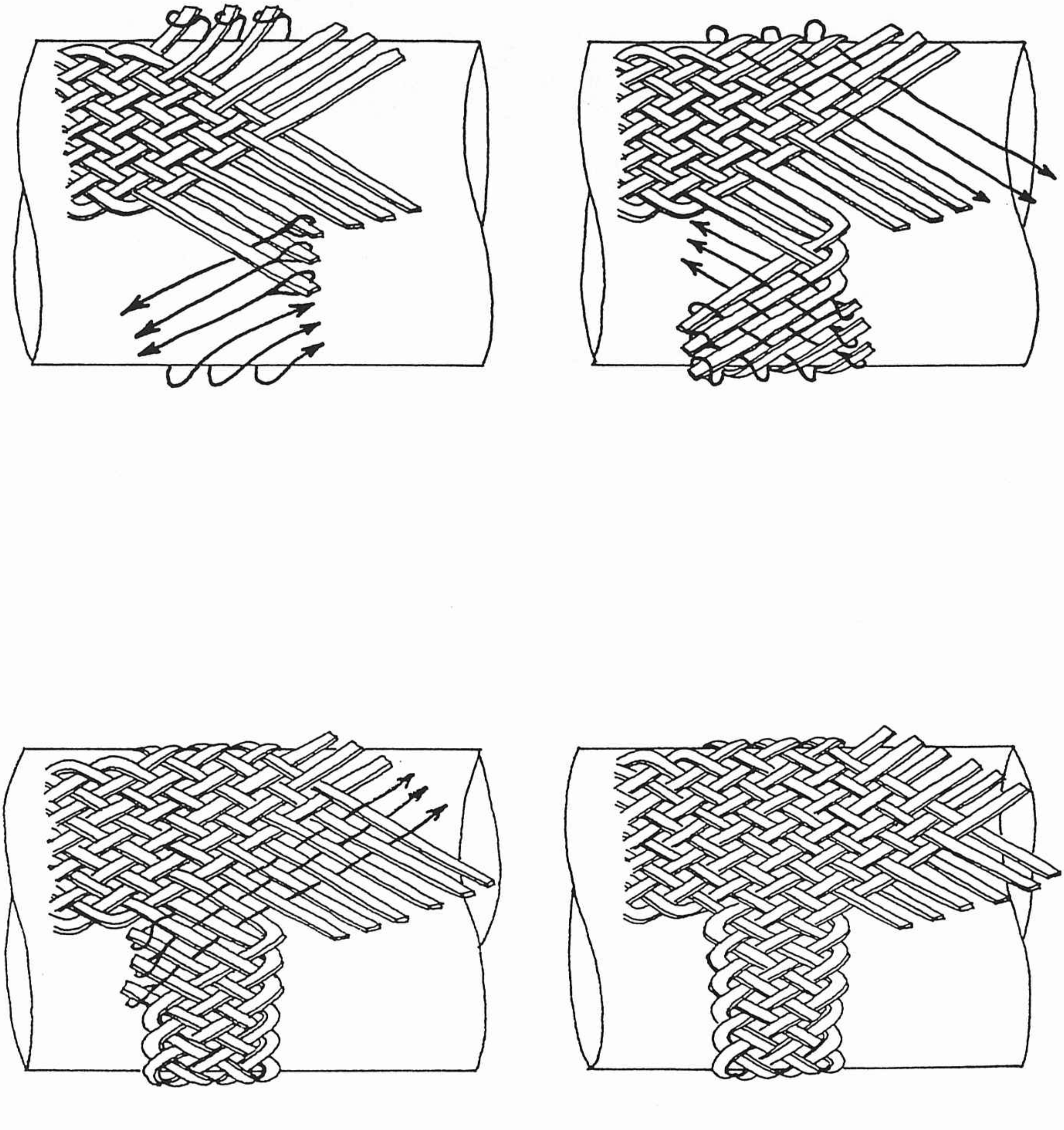
1



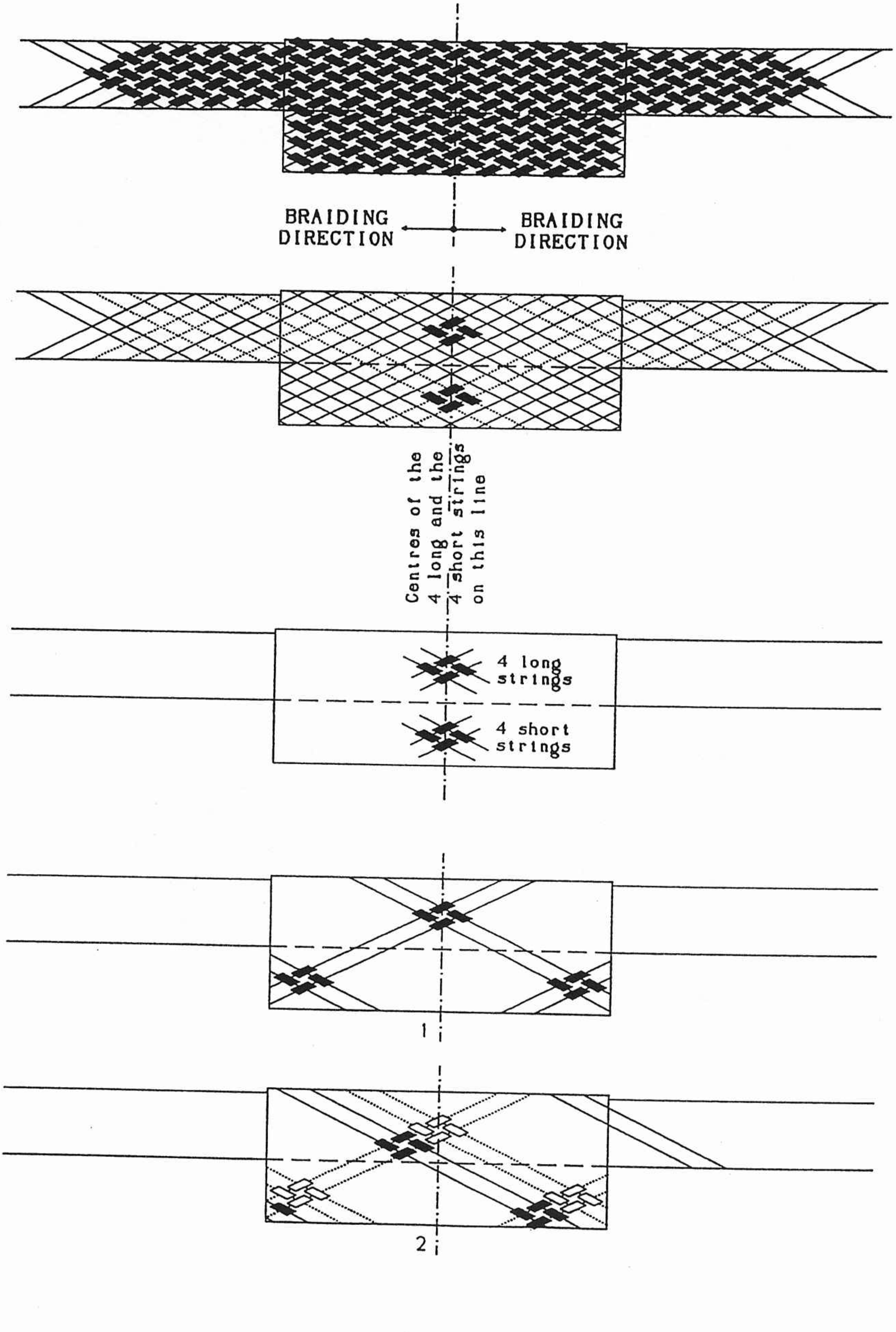
2

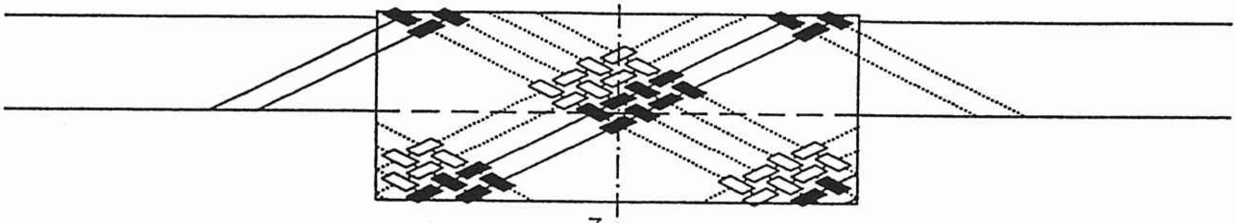


3



Pictorial drawings associated with pg. 136.


BRAIDING DIRECTION
BRAIDING DIRECTION
Centres of the 4 long and the 4 short strings on this line
4 long strings
4 short strings
1
2



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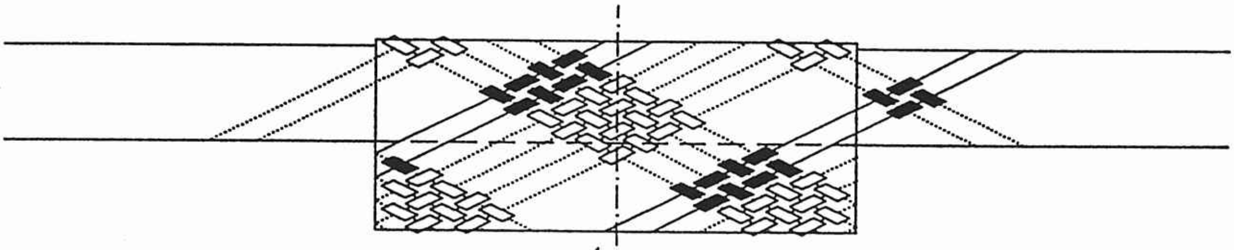
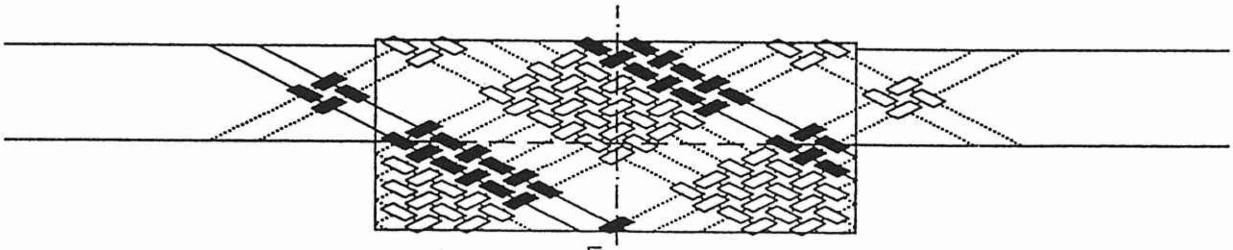
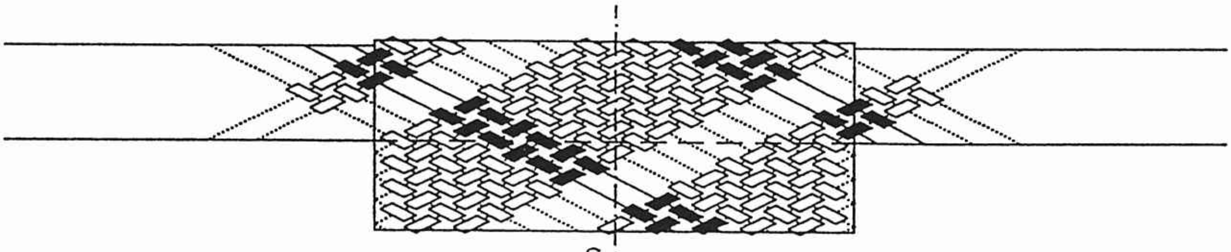


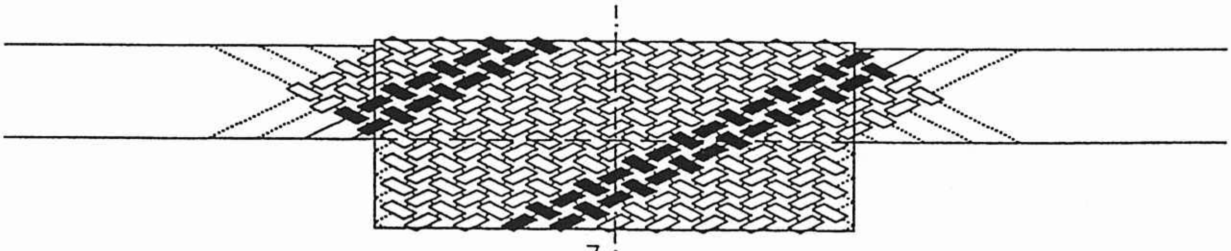
4



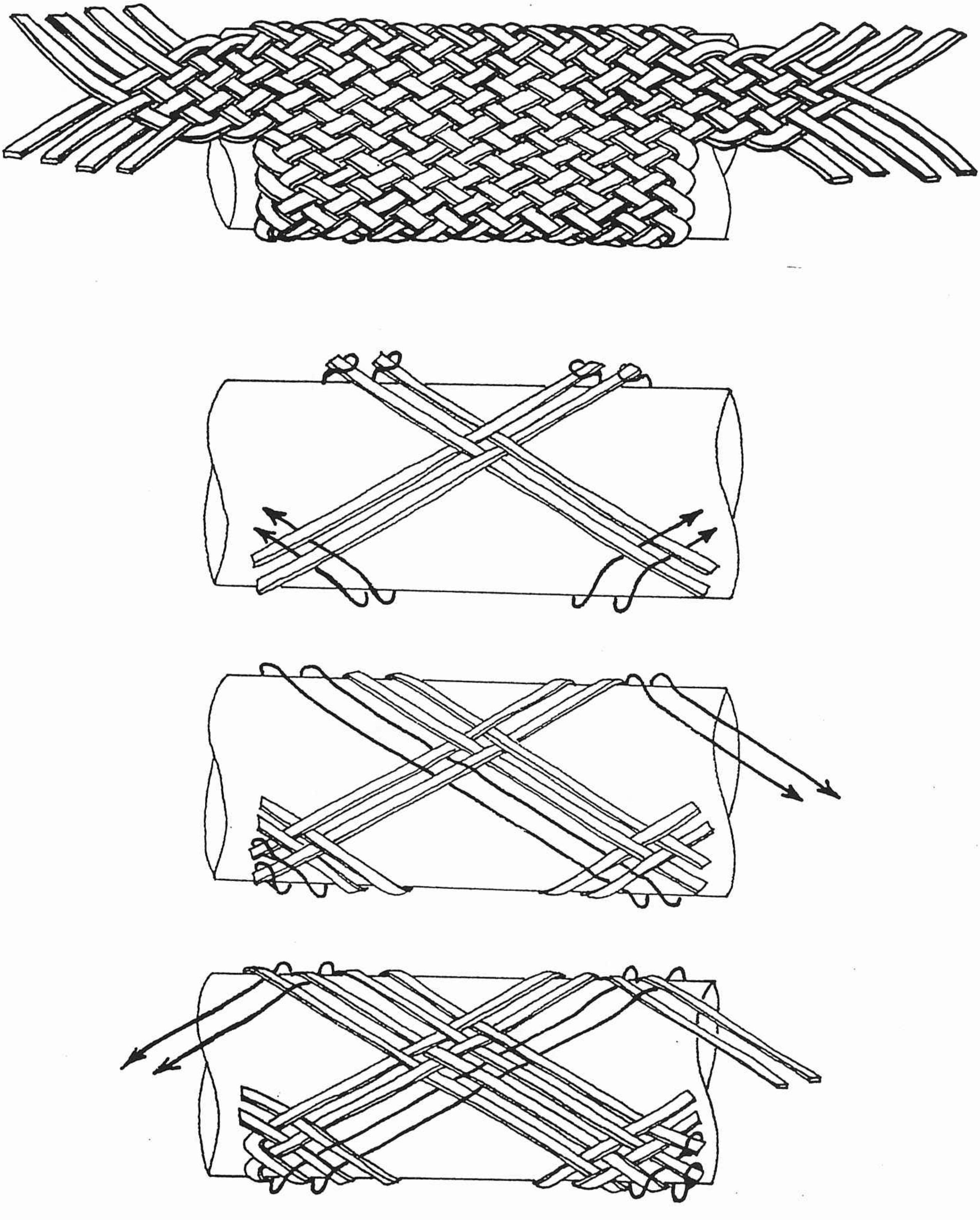
5



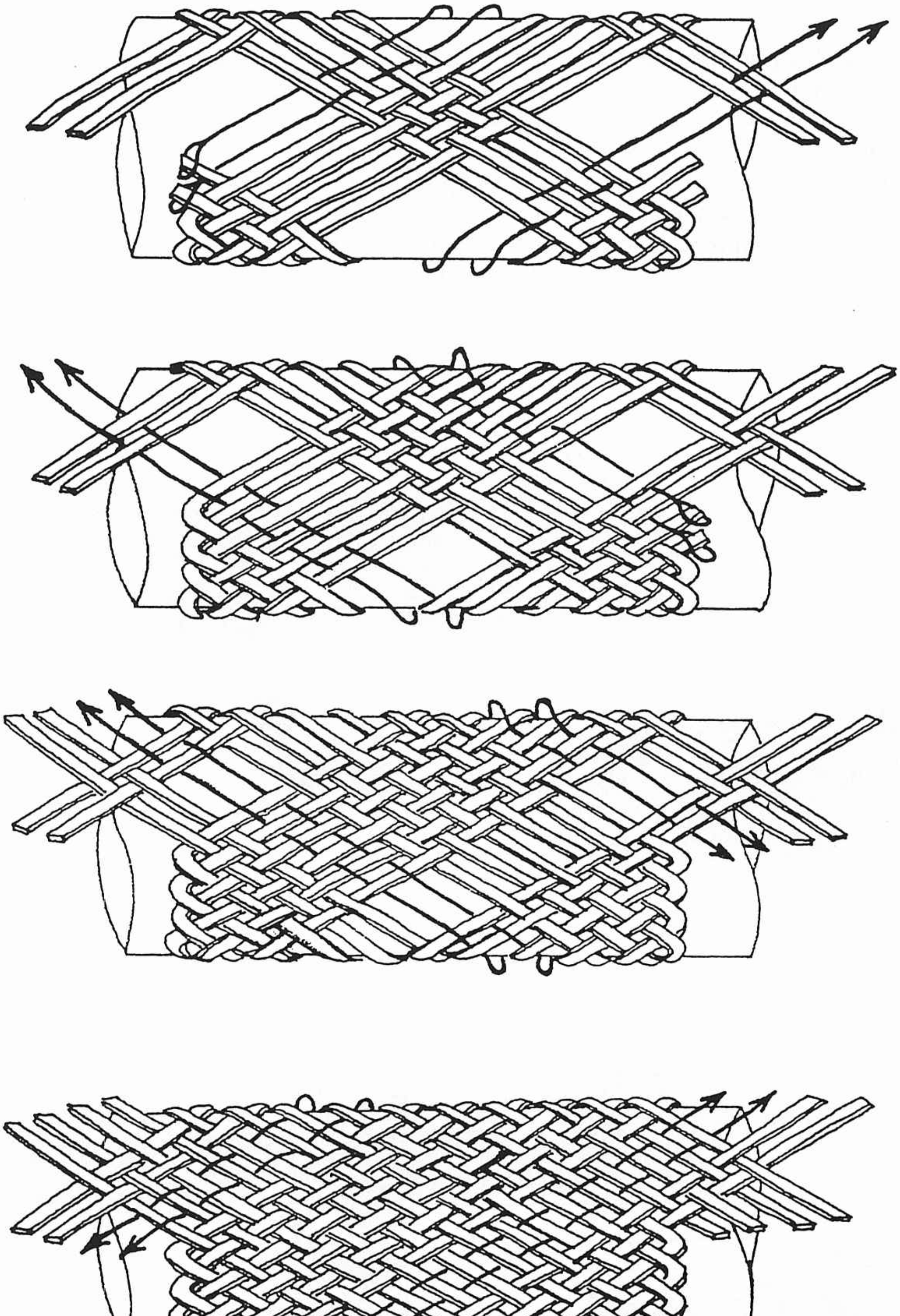
6



7



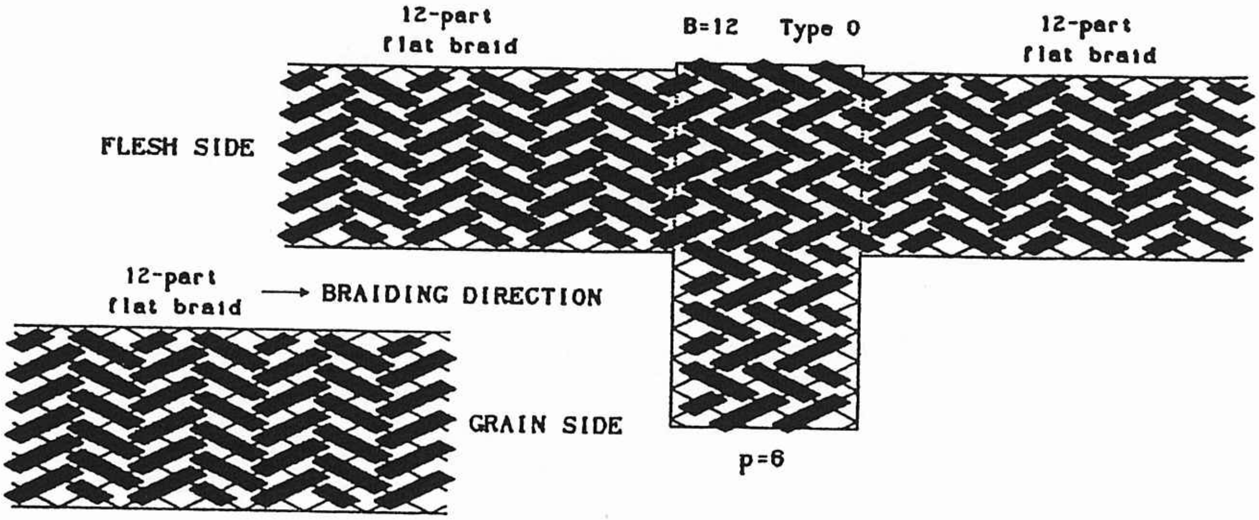
Pictorial drawings associated with pgs. 138 & 139.

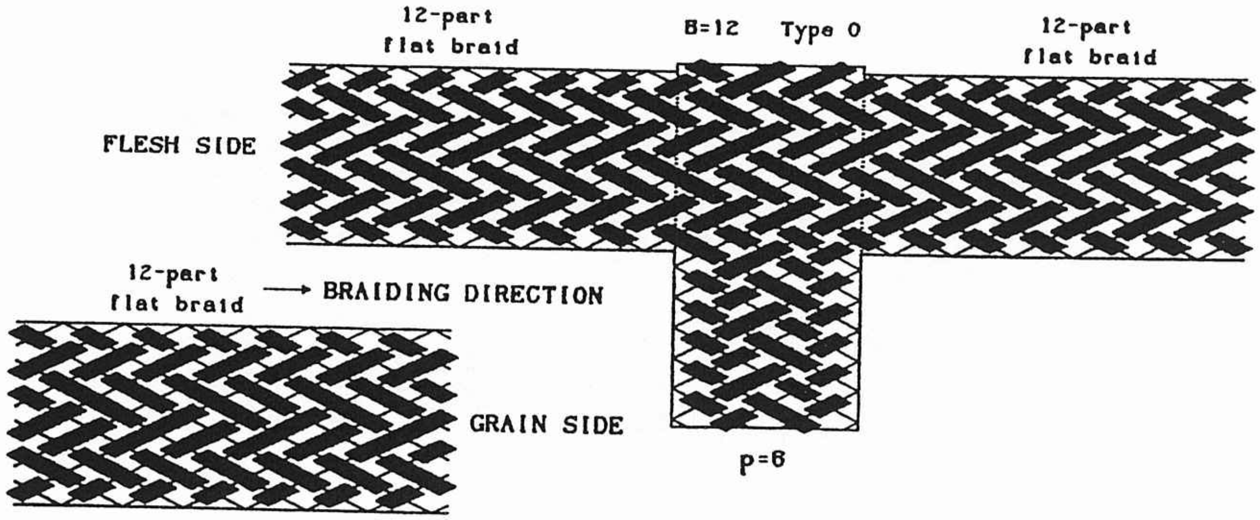


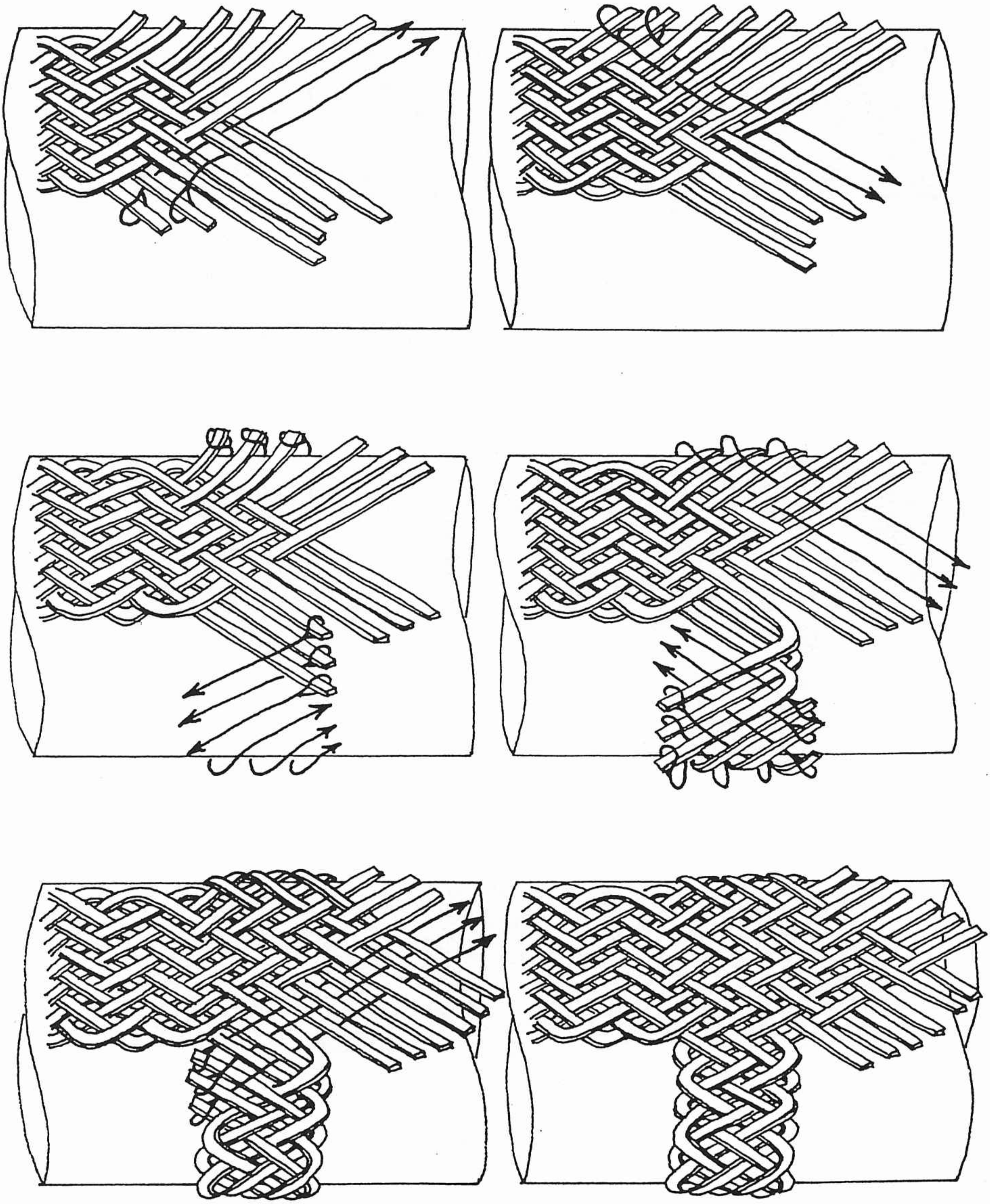
Pictorial drawings associated with pgs. 138 & 139.

When the coding of the flat braids and the mid-keeper differ from a Casa-coding, special attention should be given to the design of an acceptable coding combination for the flat braids and the mid-keeper. Such a design can best be accomplished with the aid of a grid-diagram, since it is a simple matter to superimpose a coding on the string-run involved. For an aesthetically pleasing flat braid — mid-keeper combination, we will often require coding-transitions between them. These coding-transitions should be hidden from view, and hence should be placed in the mid-keeper between the flat braids. In contrast to the most common applications of end-keepers, we should be aware that in the application of mid-keepers the grain-side of the flat braid and the flesh-side of the cylindrical braid (or vice versa) are both in view.

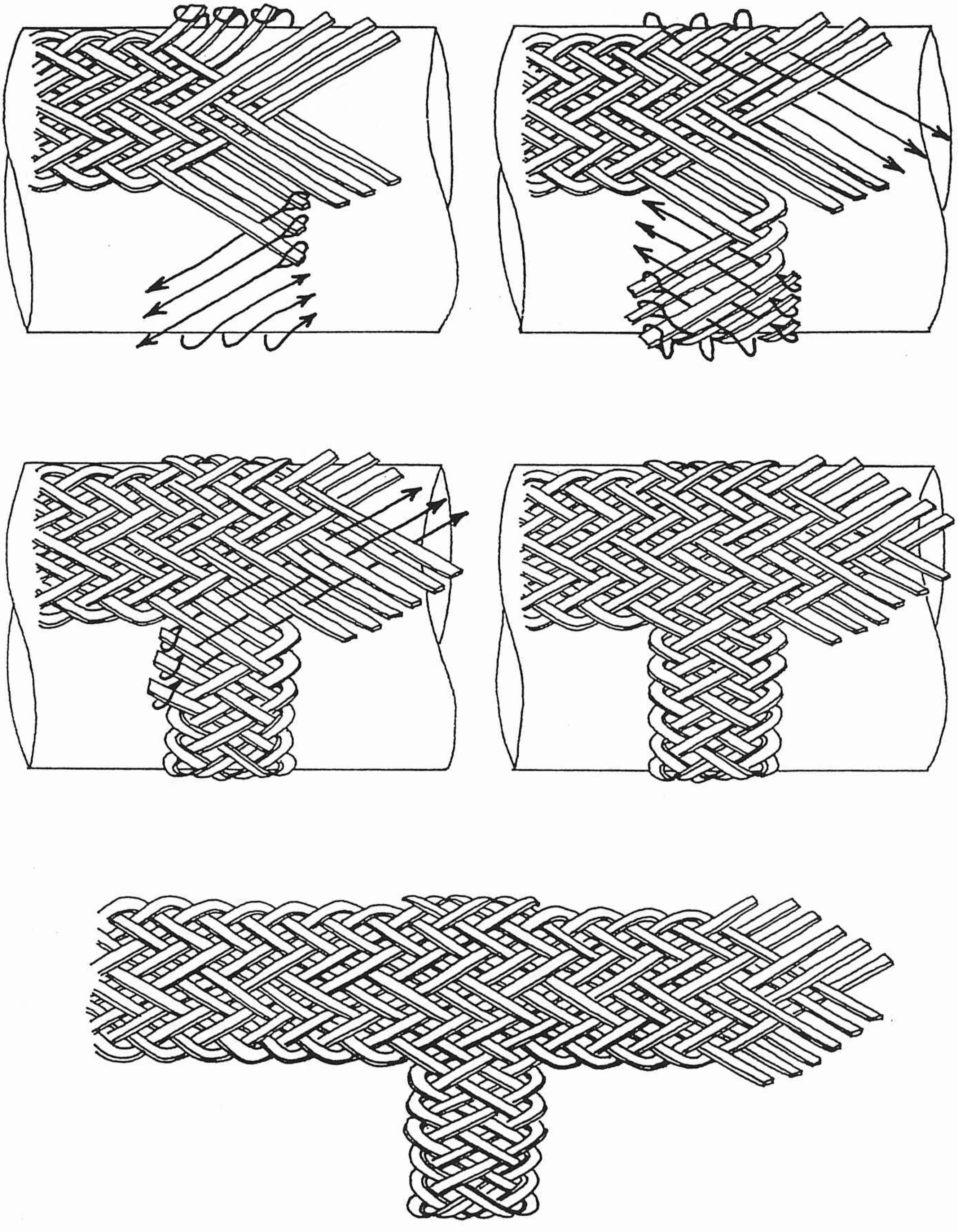
Below we have presented two Examples of good coding combinations with good coding-transitions for 6/12-midkeepers ($x = 6$). Their associated pictorial drawings outlining the consecutive braiding steps are presented on pgs. 143 and 144 respectively.





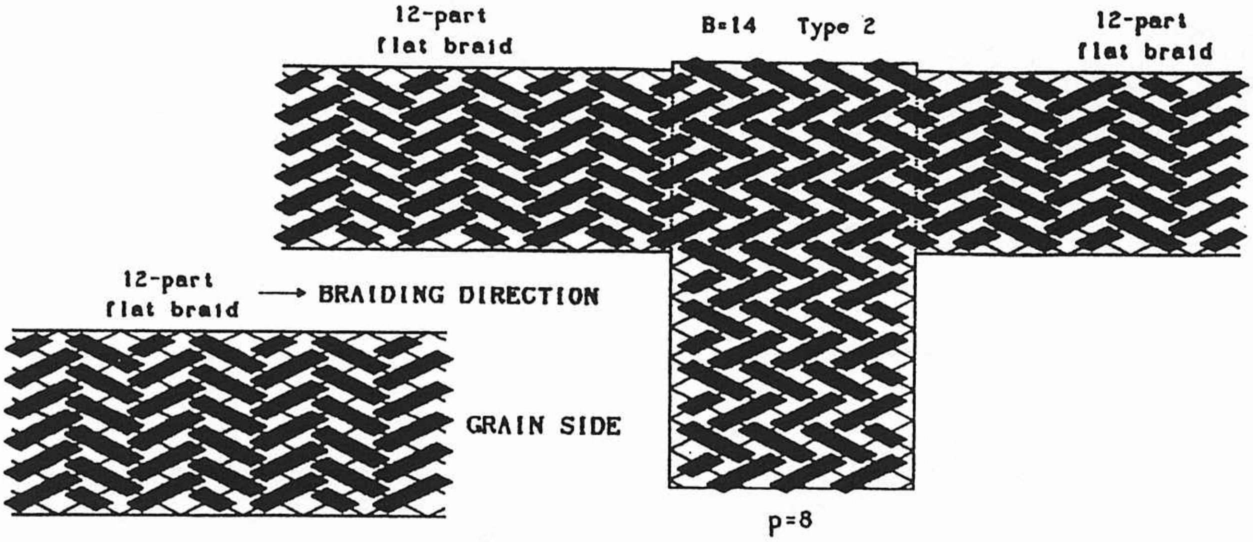


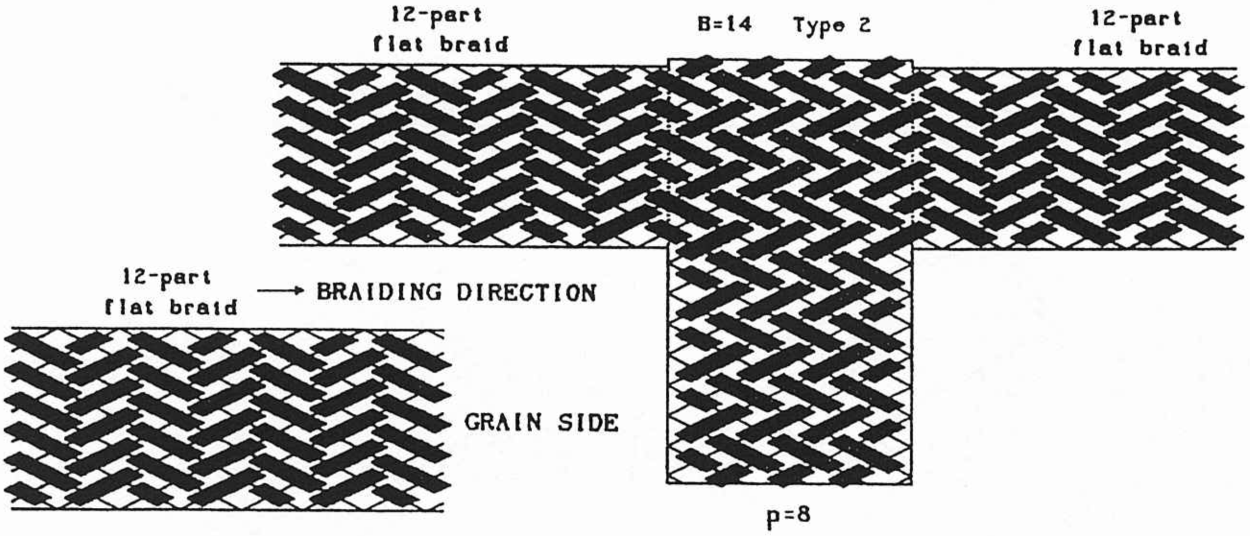
Pictorial drawings associated with pg. 142.



Pictorial drawings associated with pg. 142.

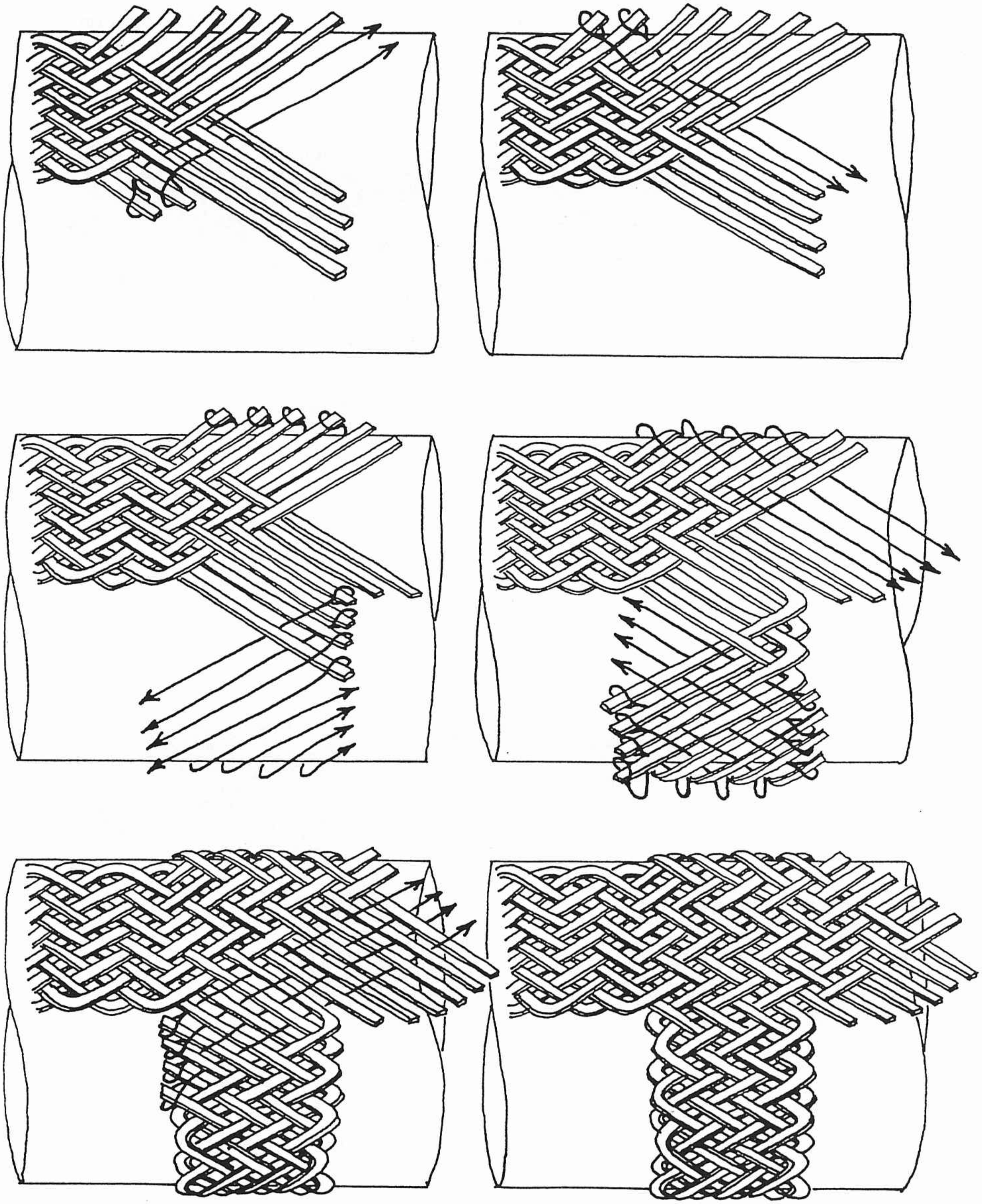
A further two Examples of good coding combinations together with good coding-transitions for 8/14-midkeepers ($x = 6$) are shown below. Their associated pictorial drawings which outline the consecutive braiding steps are presented on pgs. 146 and 147 respectively.



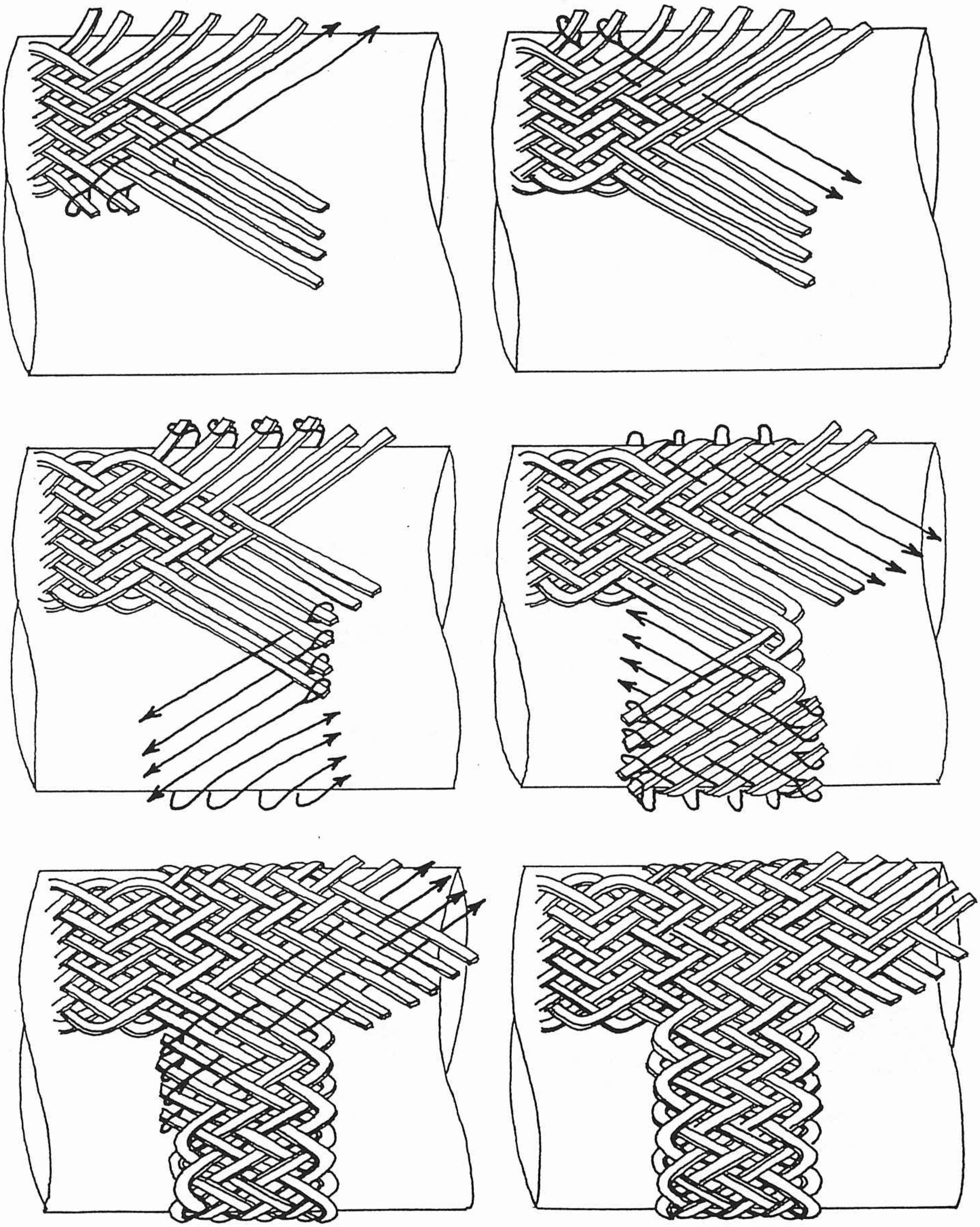


The reader is advised to study the 6/12-midkeeper ($x = 6$) diagrams on pg. 148 and the 8/14-midkeeper ($x = 6$) diagrams on pg. 149 where no coding-transitions have been used. With the codings used the results are that either the flat braid coding or the mid-keeper coding is lopsided.

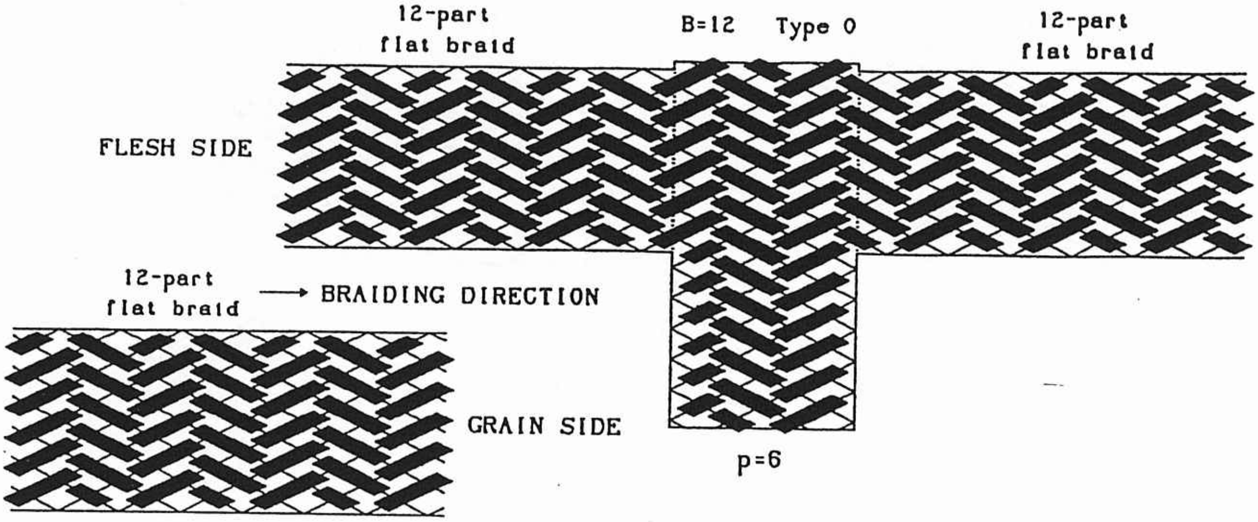
As mentioned on pg. 117, the pages 150 to 165 inclusive present some general mid-keeper string-run diagrams in which are indicated for the cylindrical braids the valid p-values for given values of x and b.

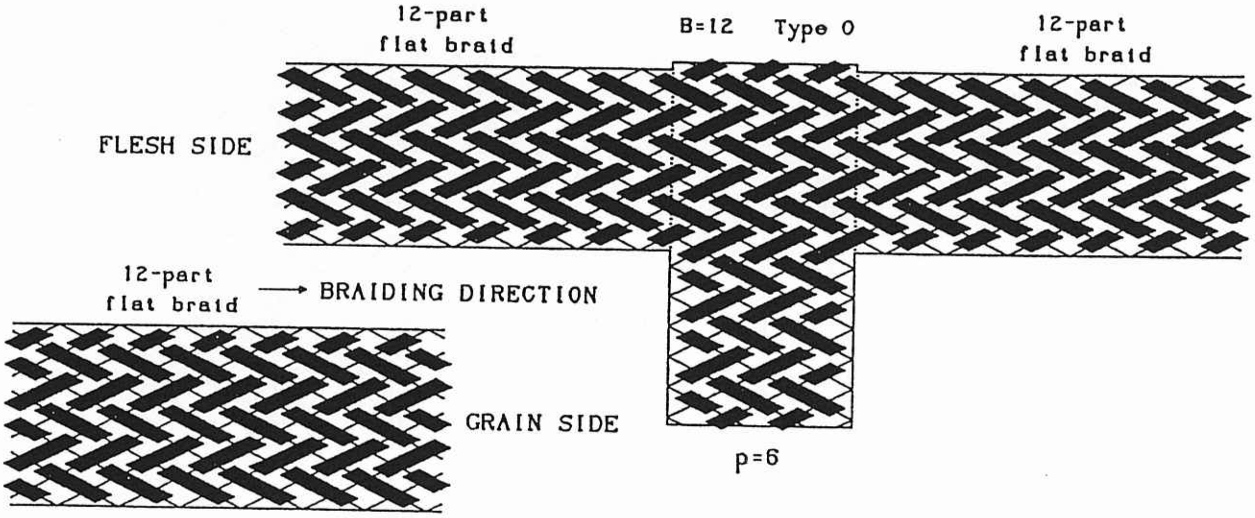


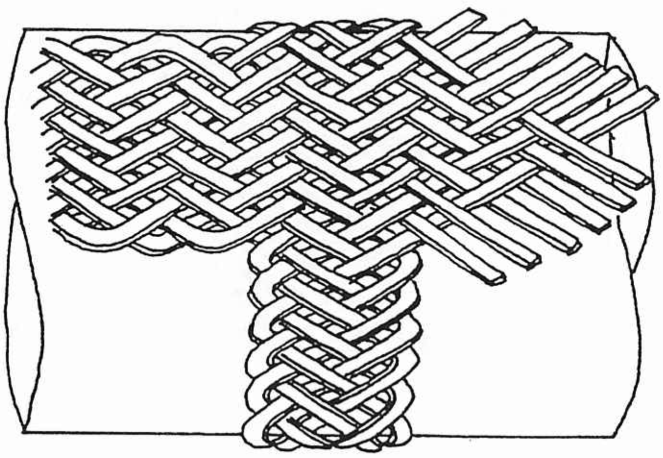
Pictorial drawings associated with pg. 145.

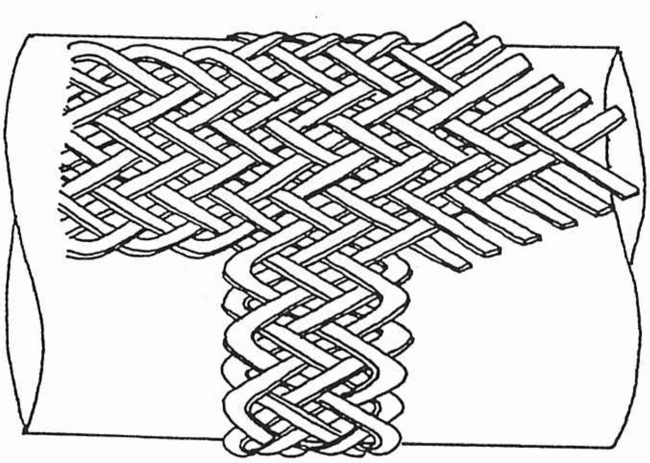


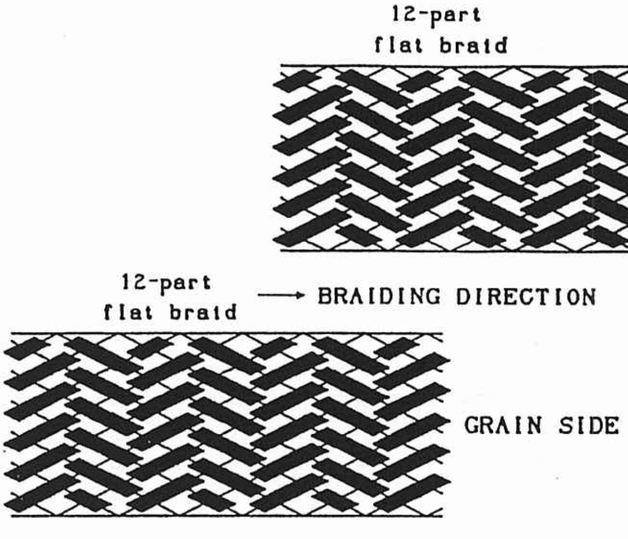
Pictorial drawings associated with pg. 145.

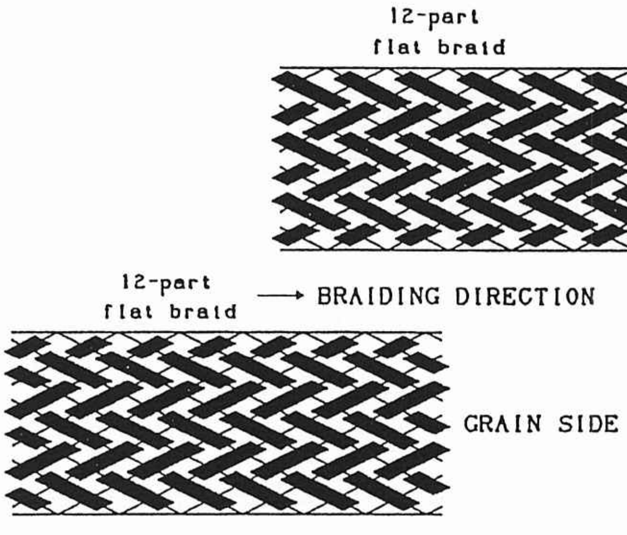

12-part
flat braid
B=12 Type 0
12-part
flat braid
FLESH SIDE
12-part
flat braid
→ BRAIDING DIRECTION
GRAIN SIDE
p=6

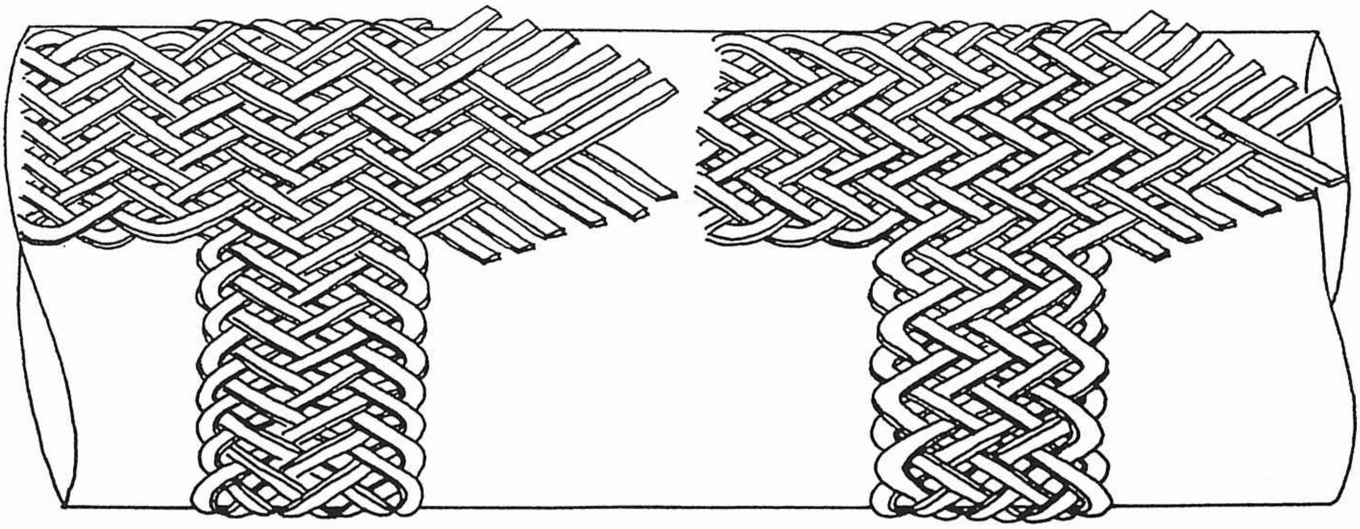

12-part
flat braid
B=12 Type 0
12-part
flat braid
FLESH SIDE
12-part
flat braid
→ BRAIDING DIRECTION
GRAIN SIDE
p=6

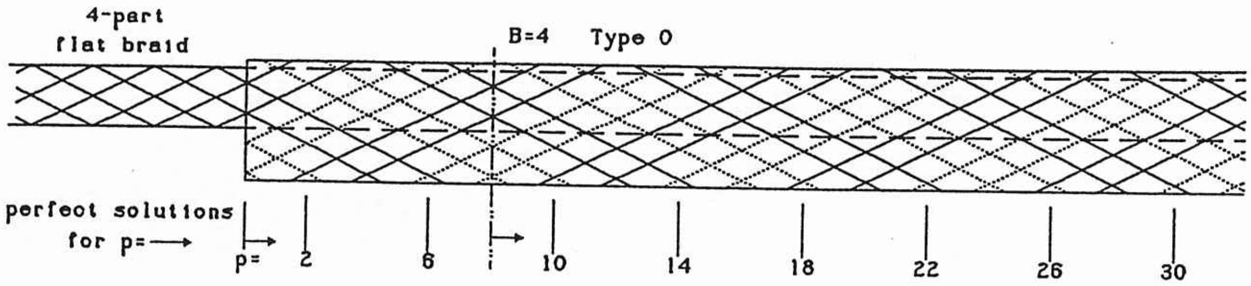


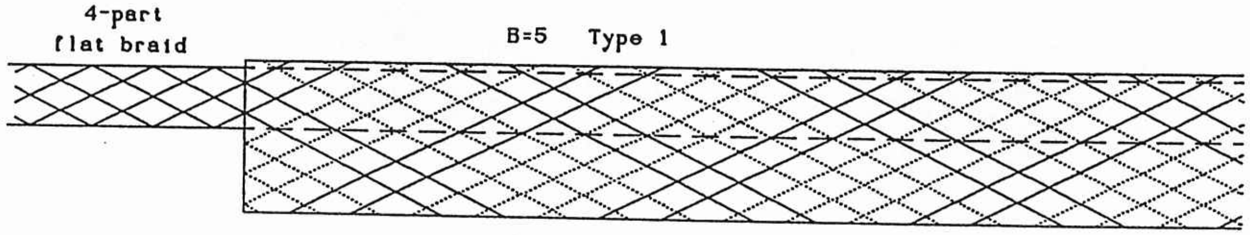



12-part flat braid
B=14 Type 2
12-part flat braid
12-part flat braid
BRAIDING DIRECTION
GRAIN SIDE
p=8

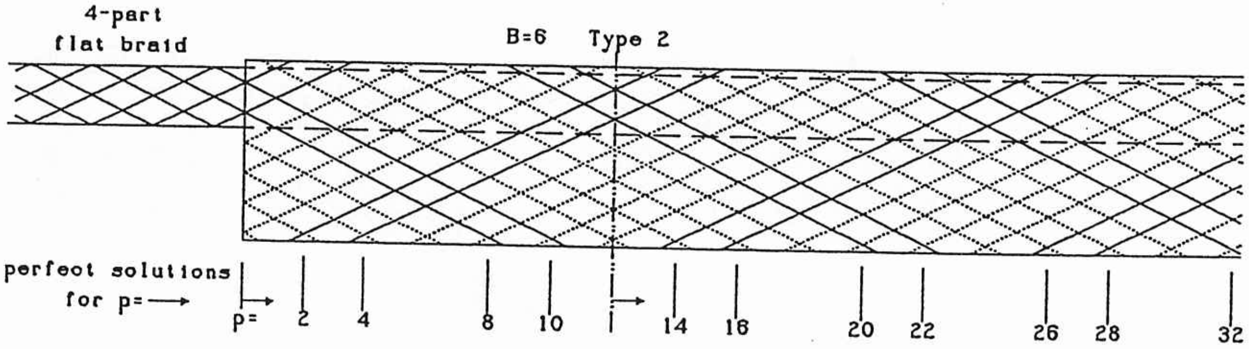

12-part flat braid
B=14 Type 2
12-part flat braid
12-part flat braid
BRAIDING DIRECTION
GRAIN SIDE
p=8

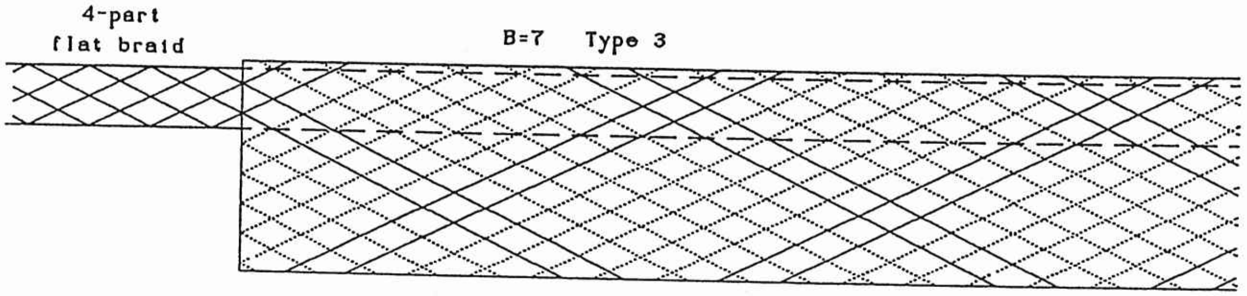




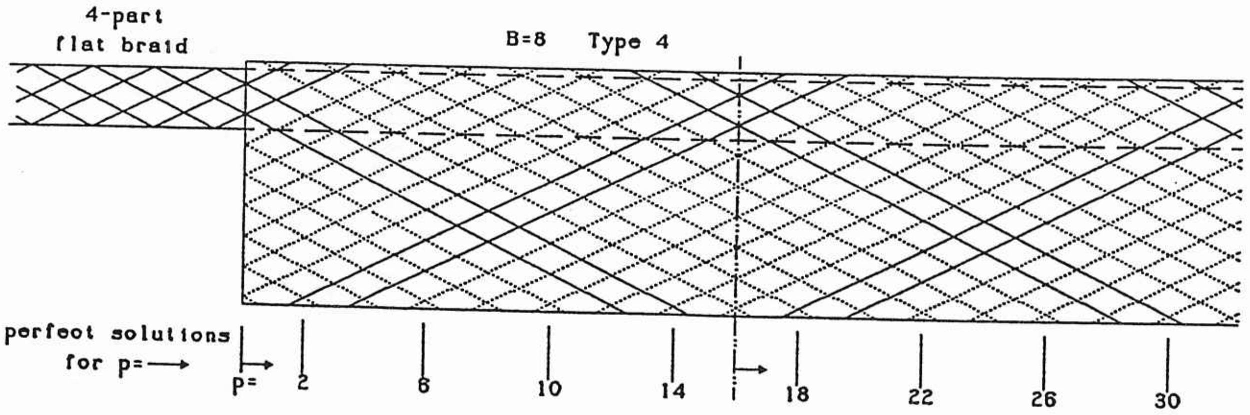


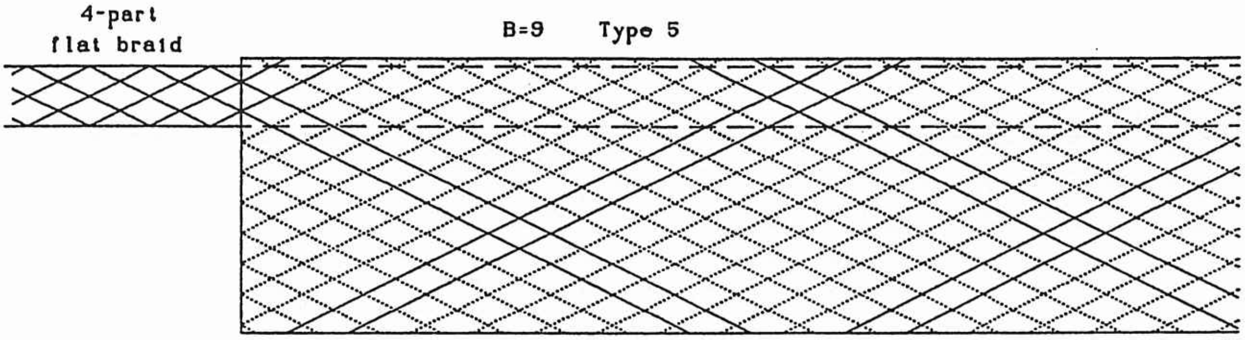
NO perfect solutions



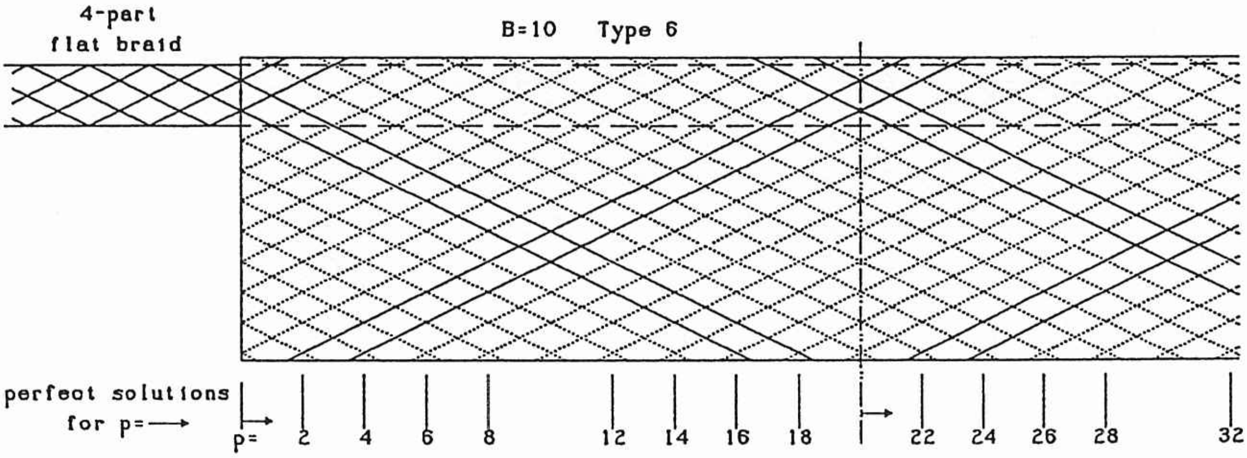


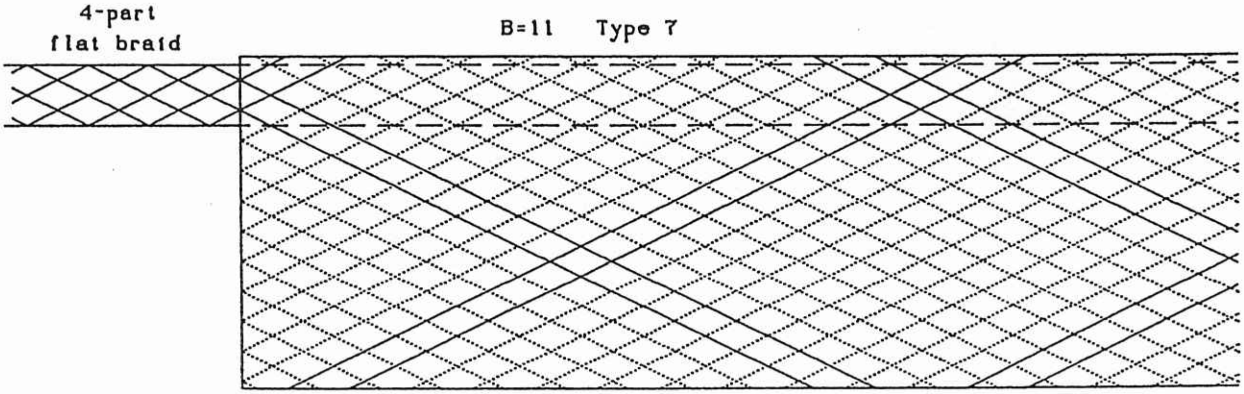
NO perfect solutions



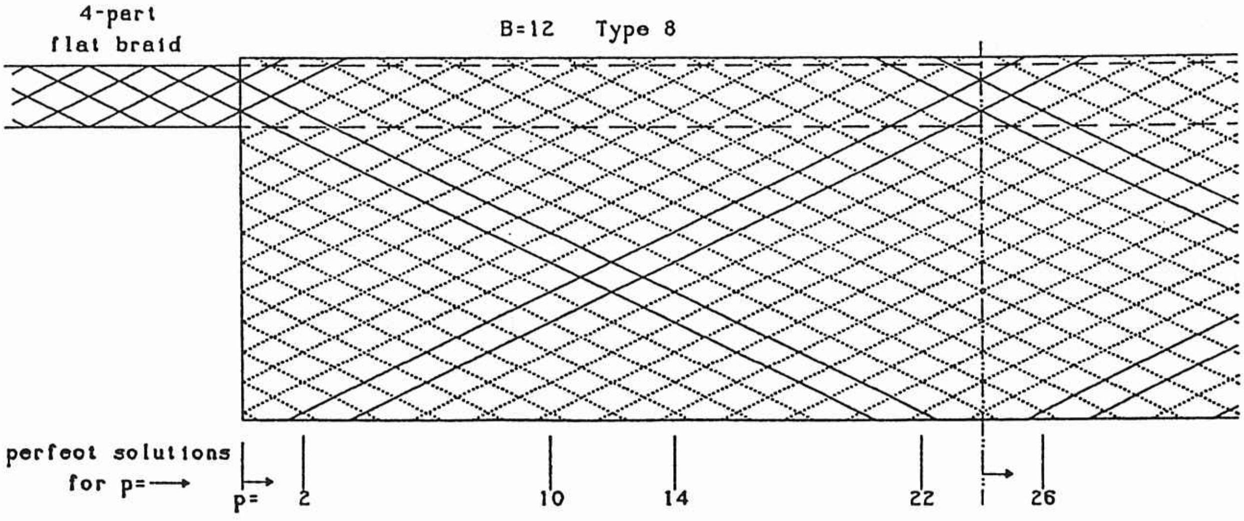


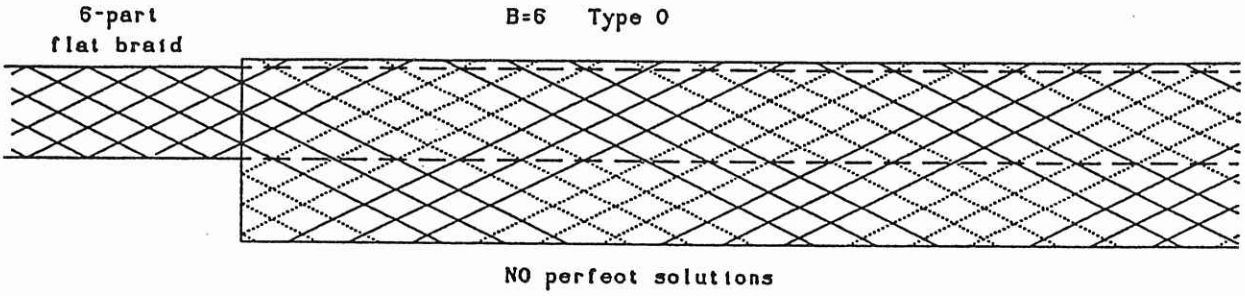
NO perfect solutions

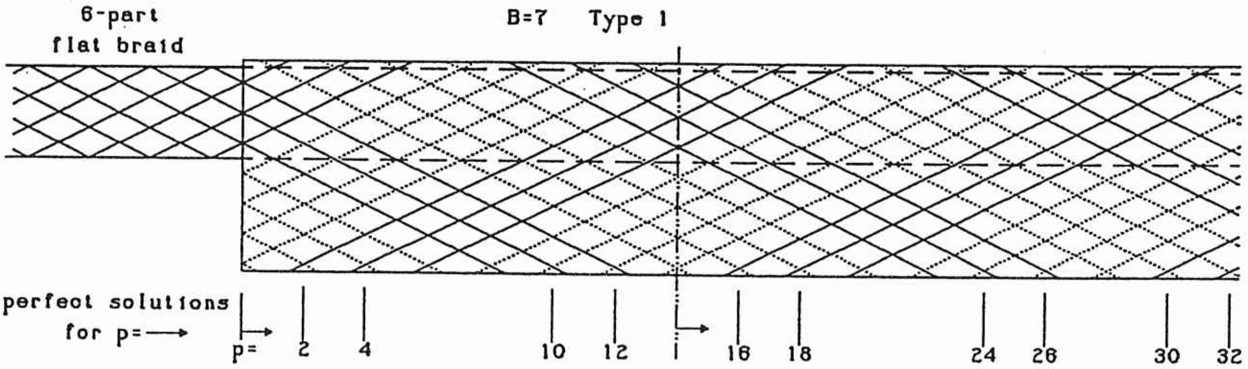


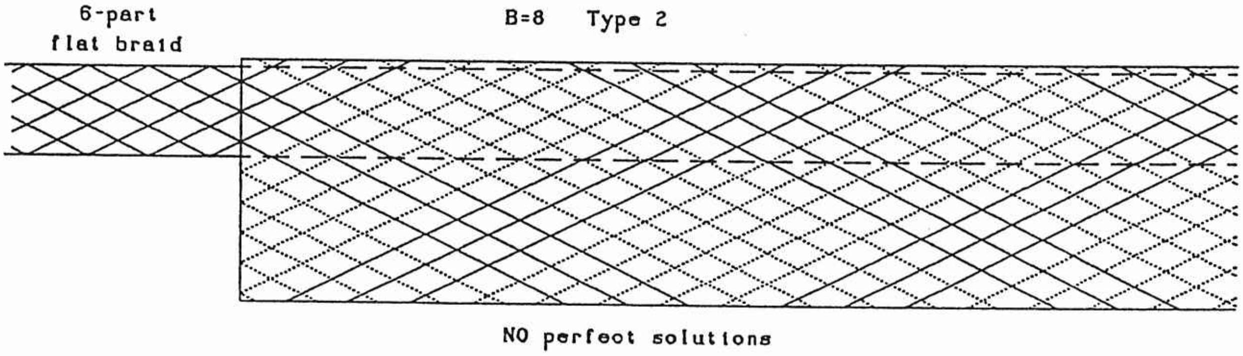


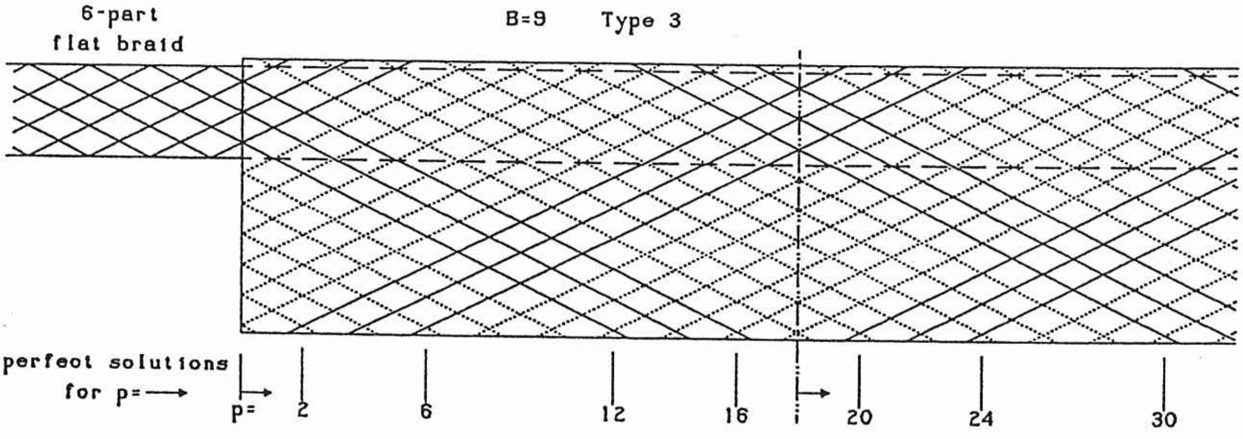
NO perfect solutions

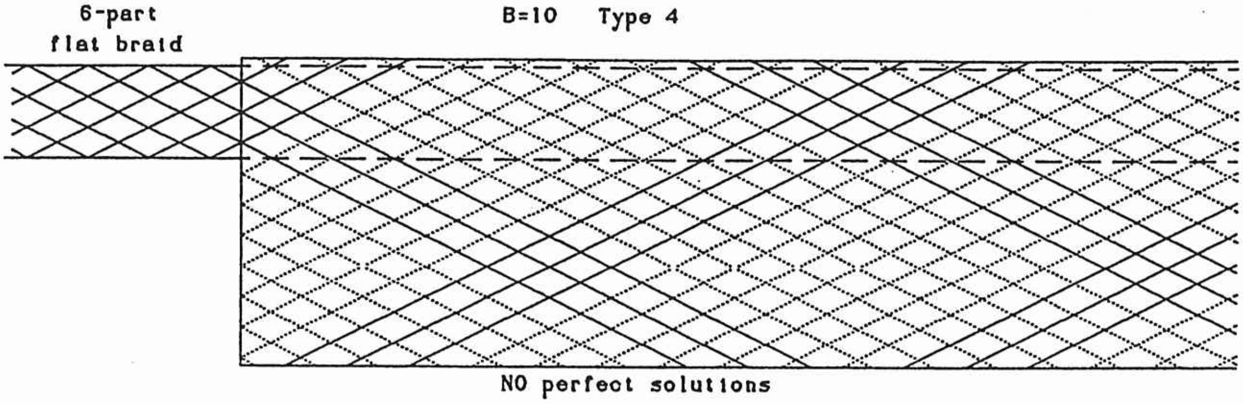


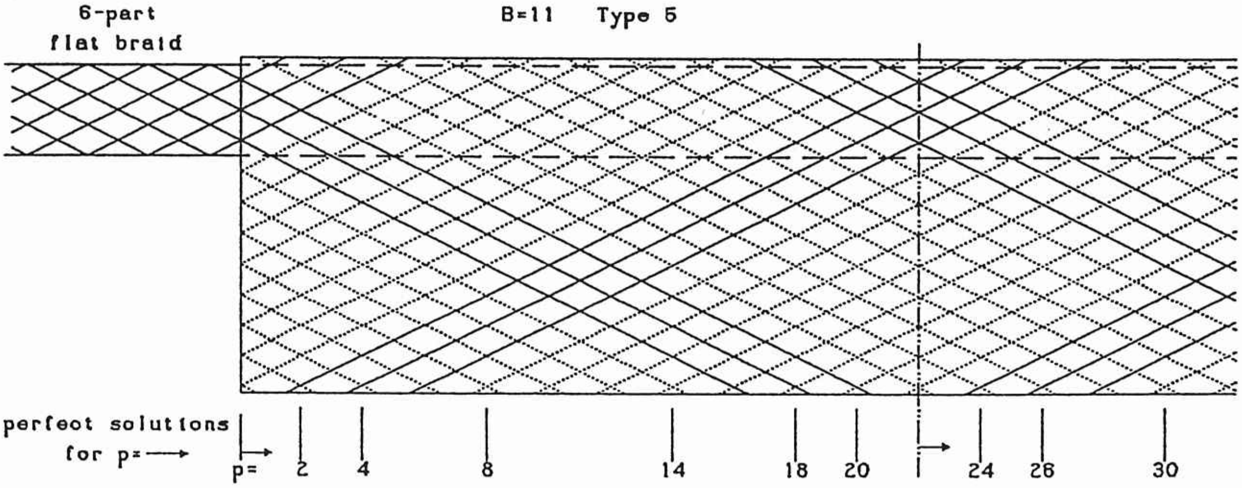

6-part
flat braid
B=6 Type 0
NO perfect solutions

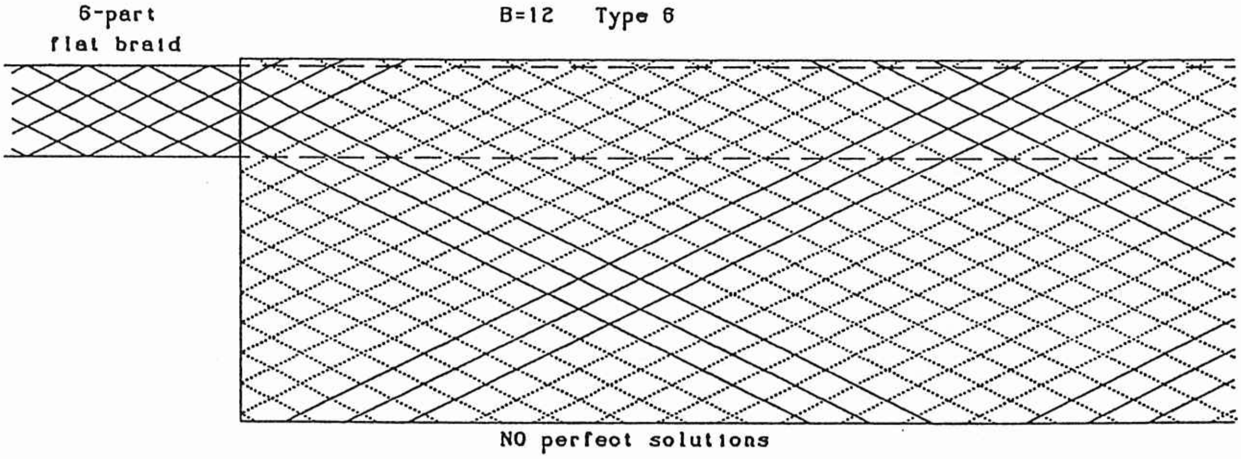

6-part
flat braid
B=7 Type 1
perfect solutions
for p=
p=

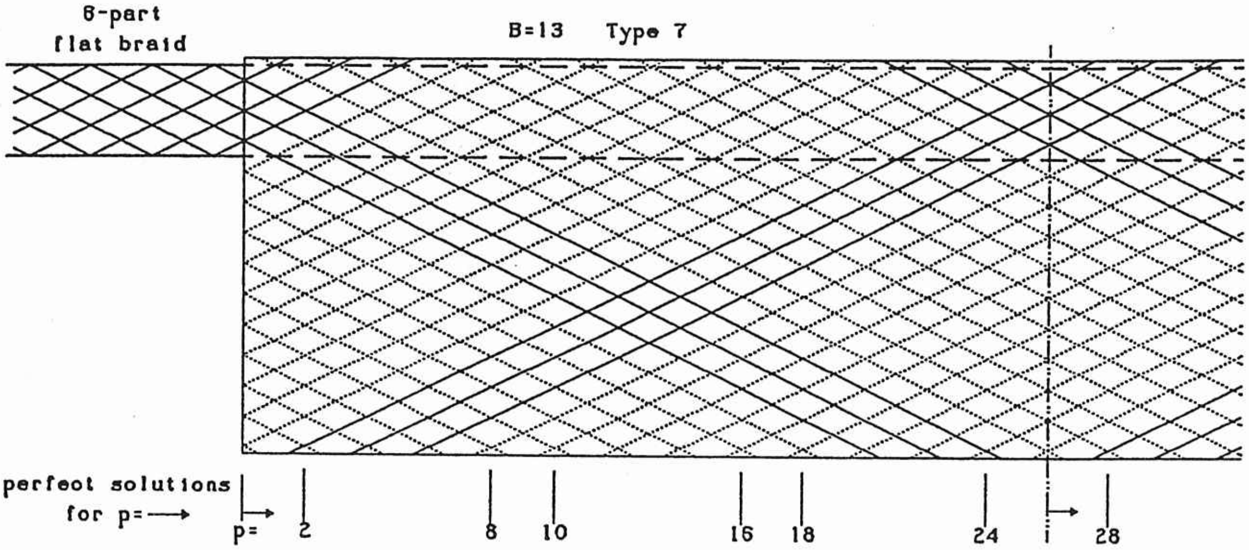

6-part
flat braid
B=8 Type 2
NO perfect solutions

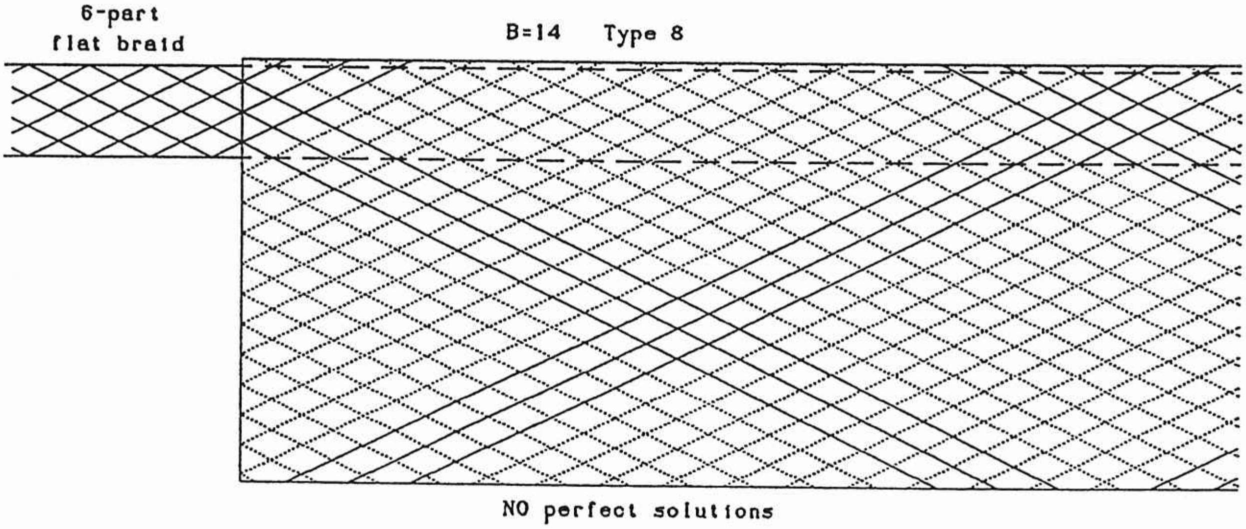

6-part
flat braid
B=9 Type 3
perfect solutions
for p=
p=
2
6
12
16
20
24
30

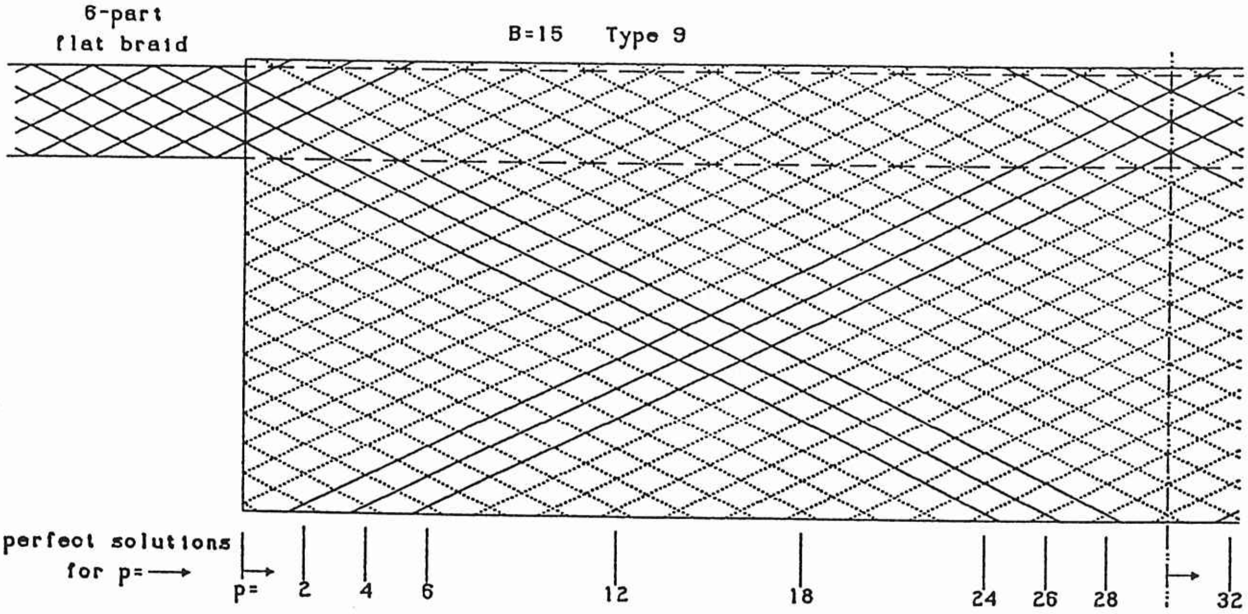

6-part
flat braid
B=10 Type 4
NO perfect solutions

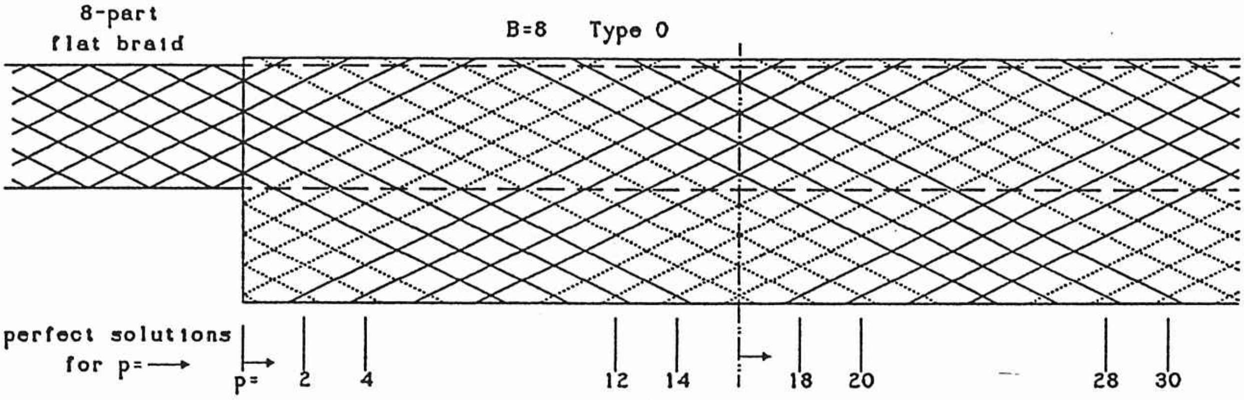

6-part
flat braid
B=11 Type 5
perfect solutions
for p=
p=

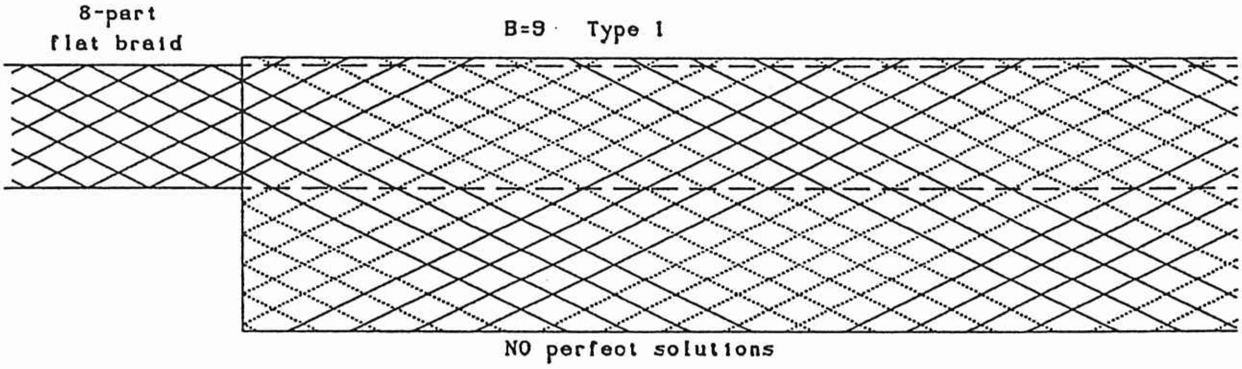

6-part
flat braid
B=12 Type 6
NO perfect solutions

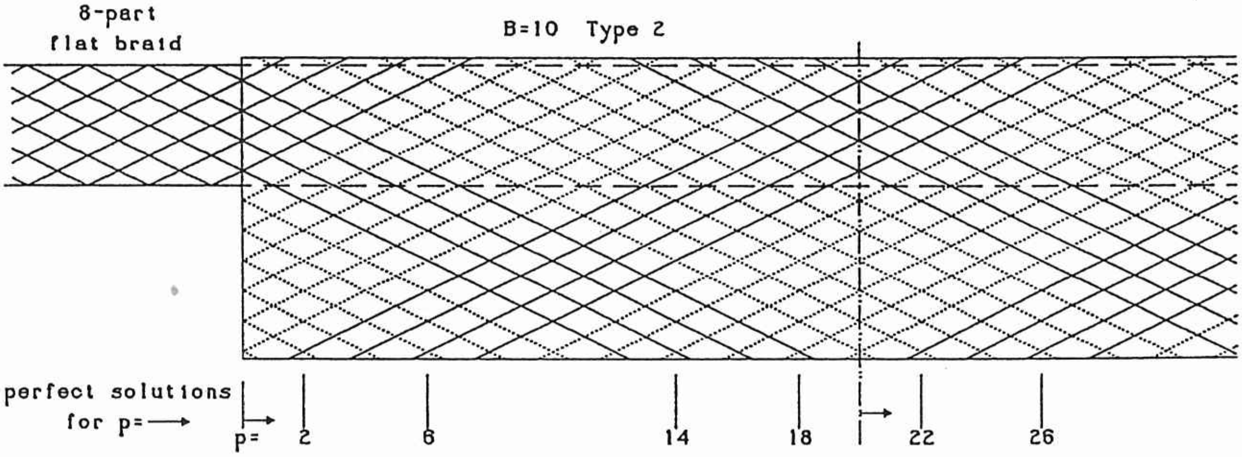

6-part
flat braid
B=13 Type 7
perfect solutions
for p=
p=
2
8
10
16
18
24
28

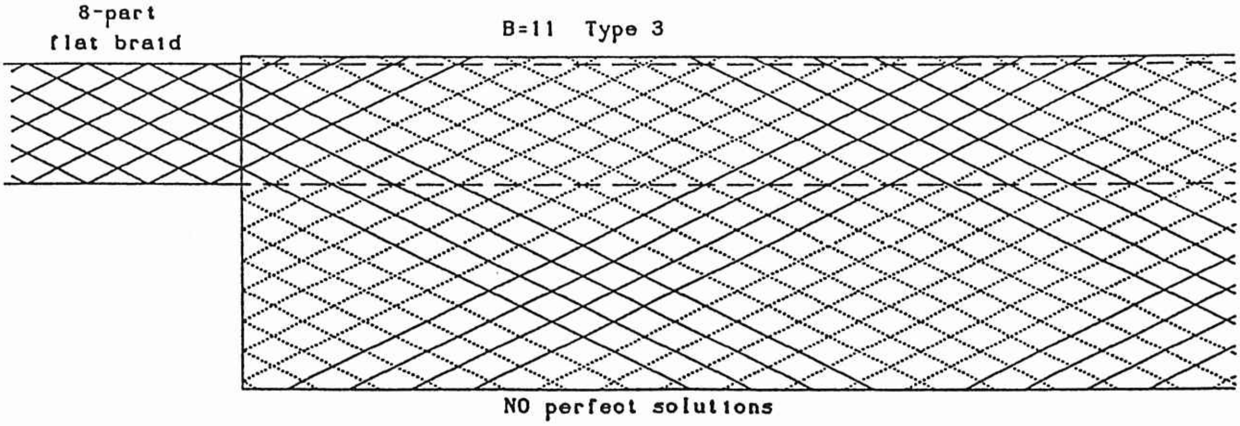

6-part
flat braid
B=14 Type 8
NO perfect solutions

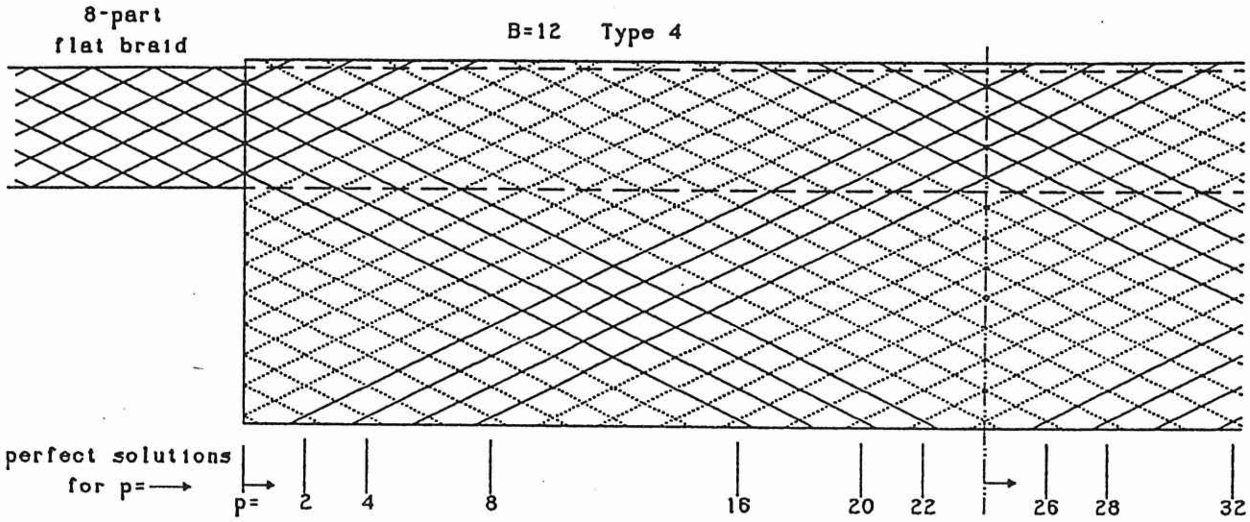

6-part
flat braid
B=15 Type 9
perfect solutions
for p=
p=

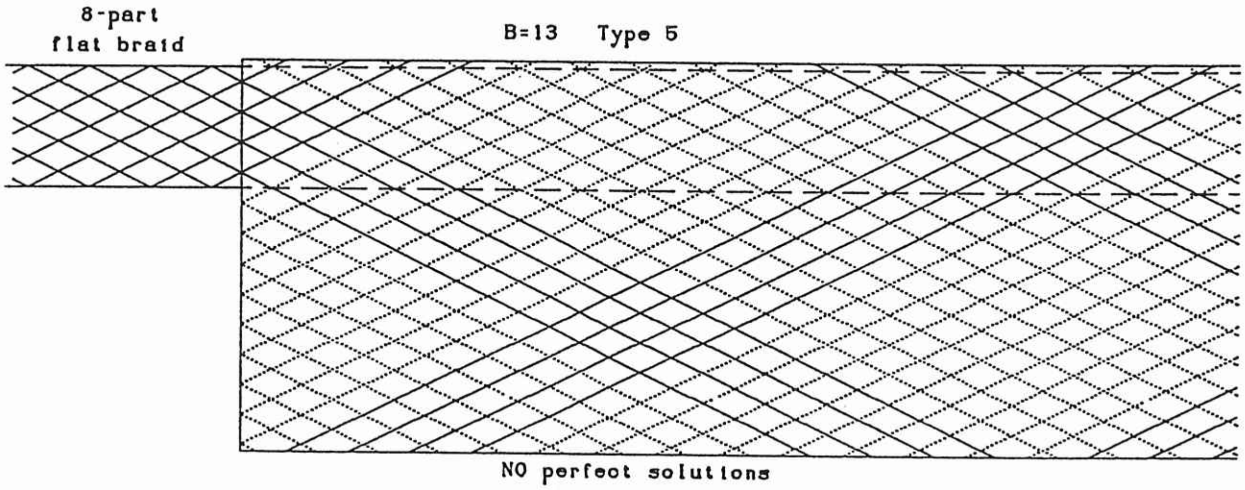

8-part
flat braid
B=8 Type 0
perfect solutions
for p=
p=
2
4
12
14
18
20
28
30

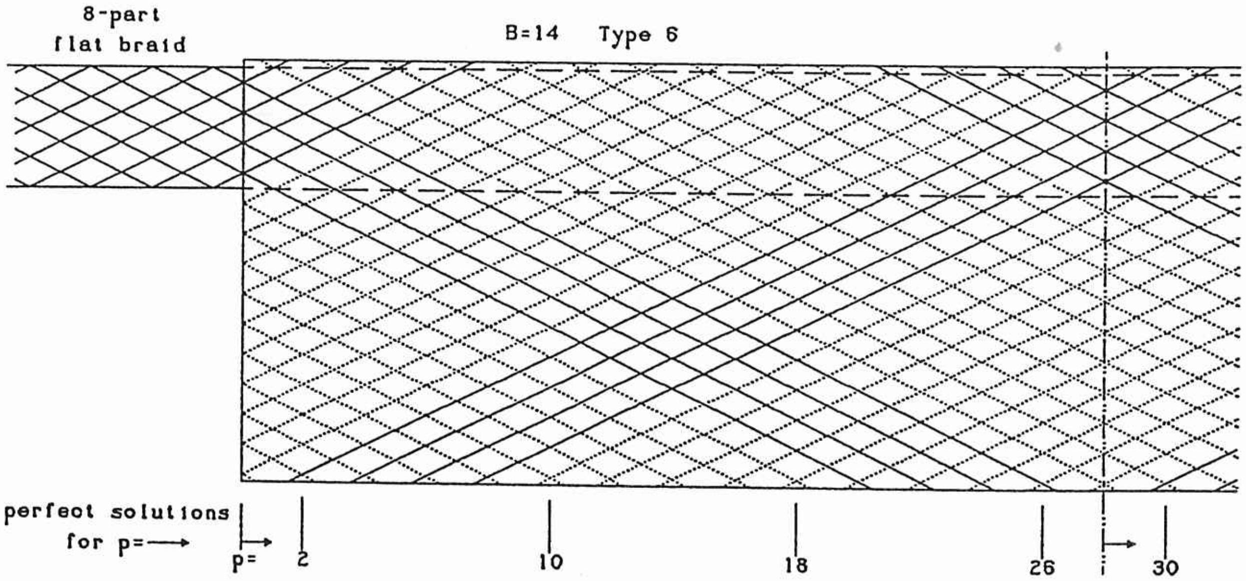

8-part
flat braid
B=9 Type 1
NO perfect solutions

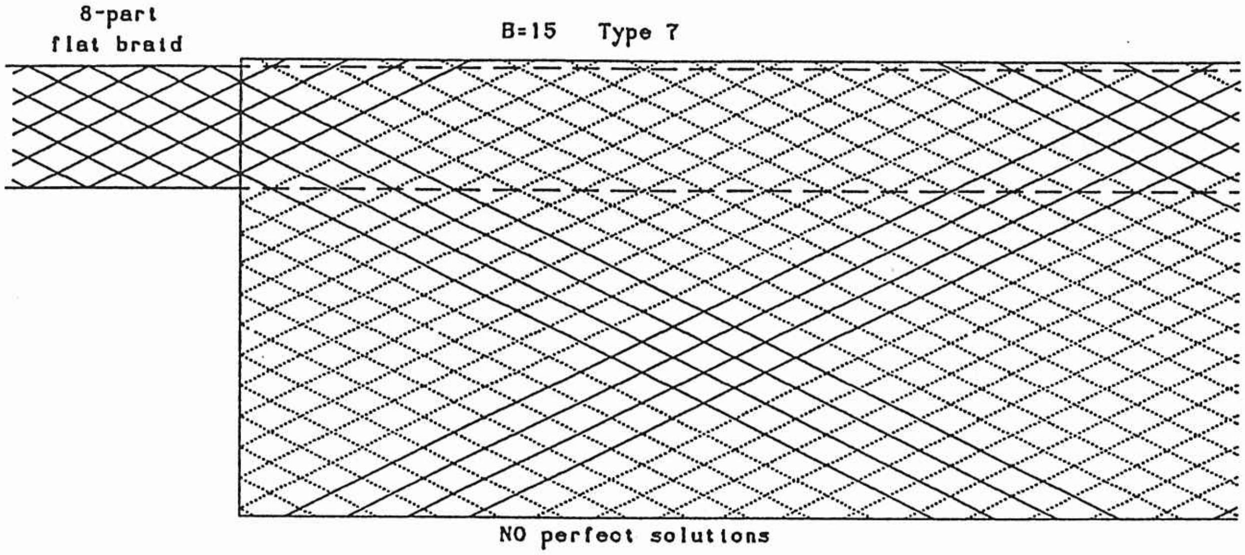

8-part
flat braid
B=10 Type 2
perfect solutions
for p=
p=
2
6
14
18
22
26

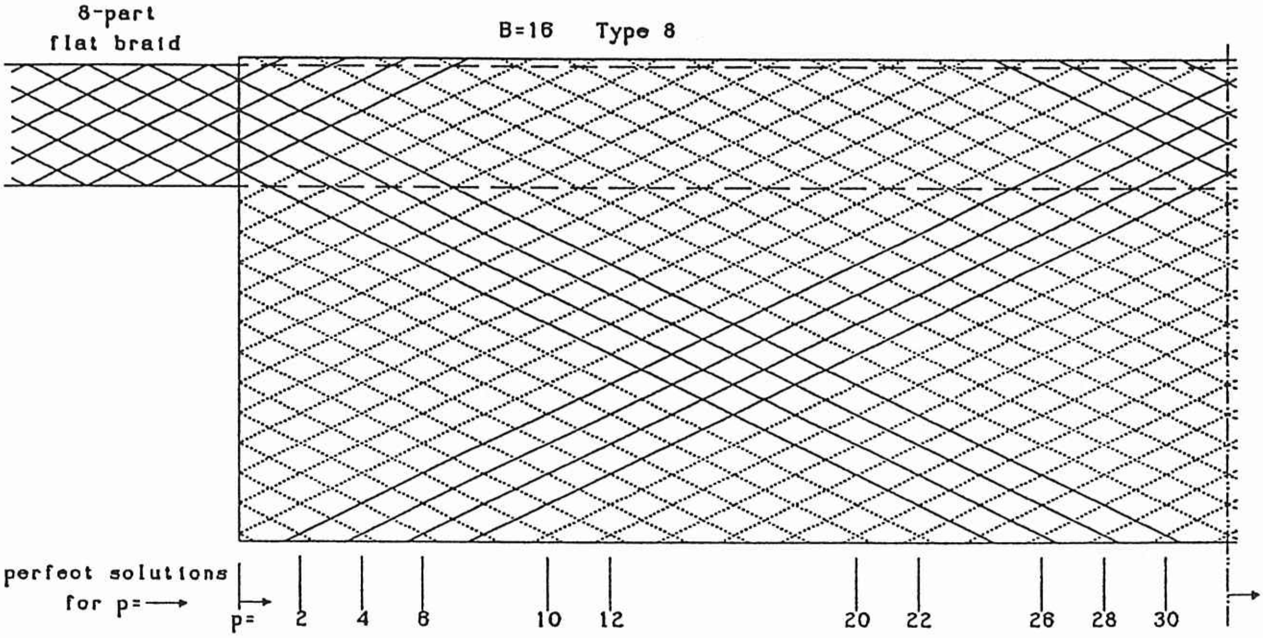

8-part
flat braid
B=11 Type 3
NO perfect solutions

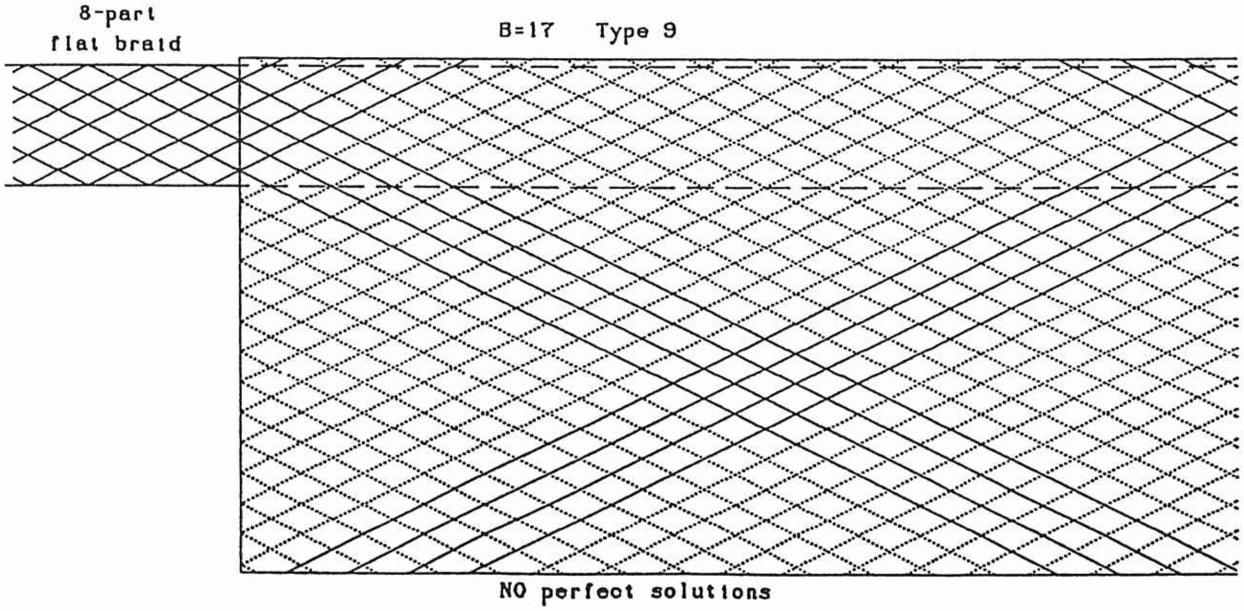

8-part
flat braid
B=12 Type 4
perfect solutions
for p=⟶
p=

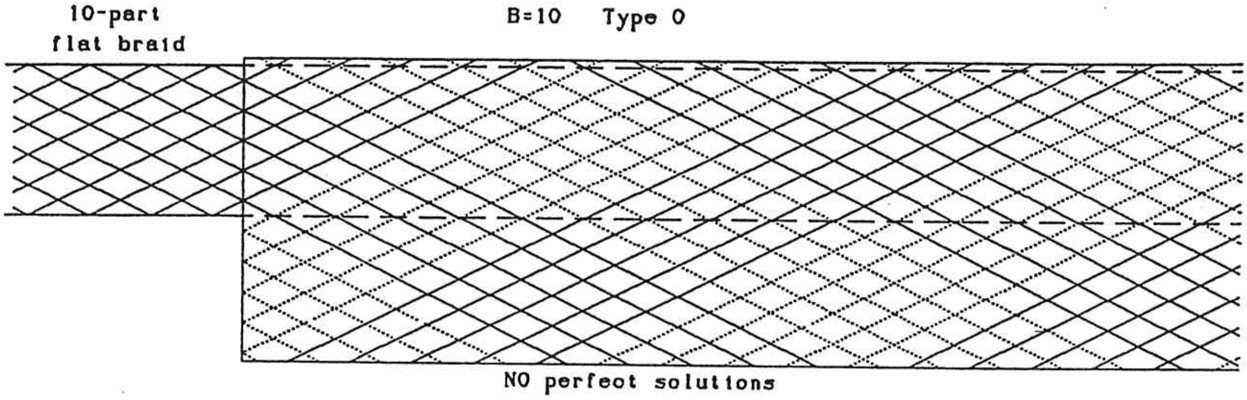

8-part
flat braid
B=13 Type 5
NO perfect solutions

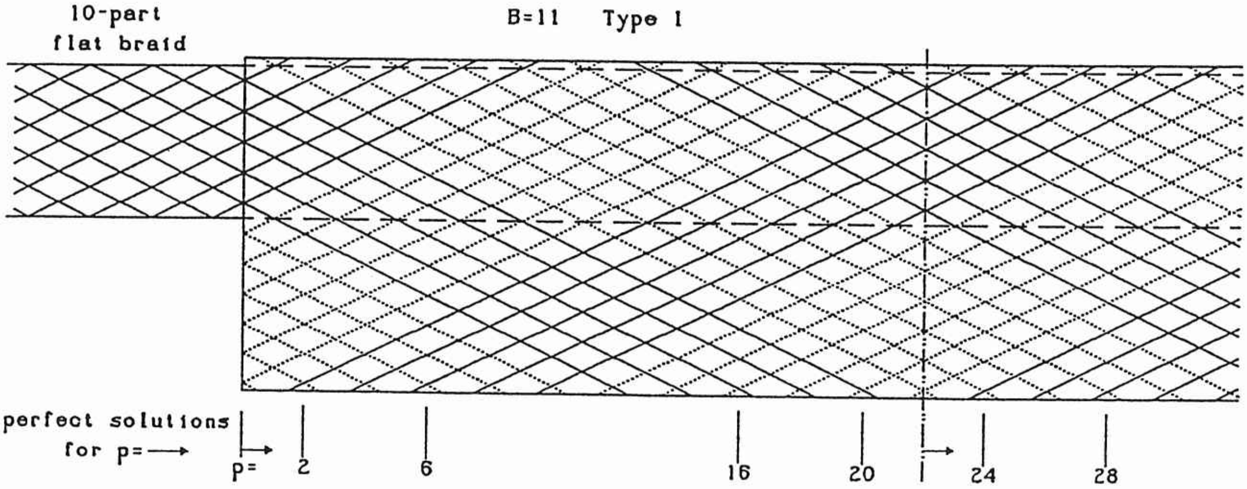

8-part
flat braid
B=14 Type 6
perfect solutions
for p=⟶
p=
2
10
18
26
30

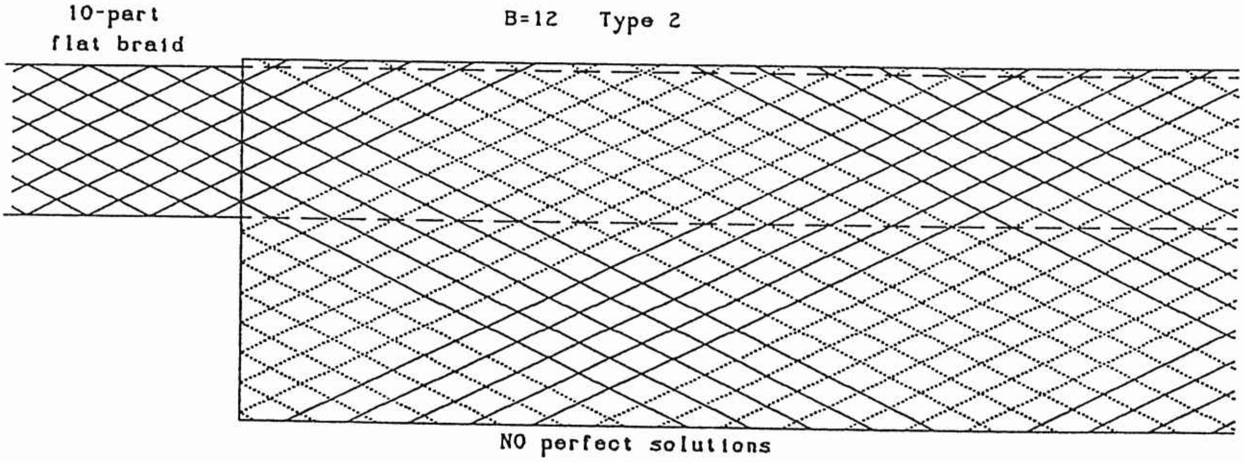

8-part
flat braid
B=15 Type 7
NO perfect solutions

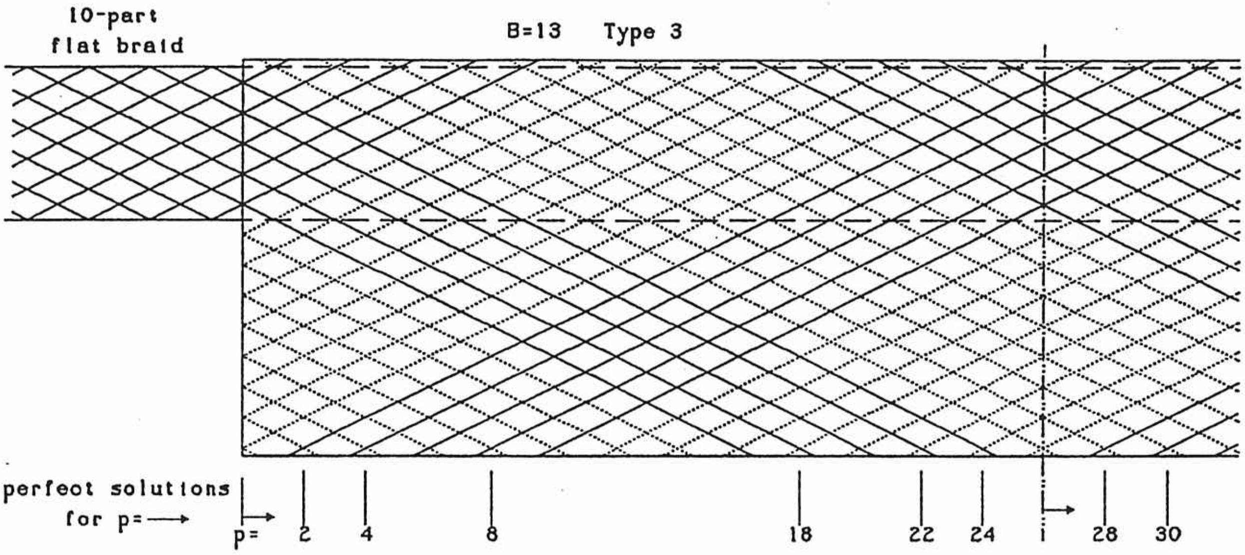

8-part
flat braid
B=16 Type 8
perfect solutions
for p=
p=

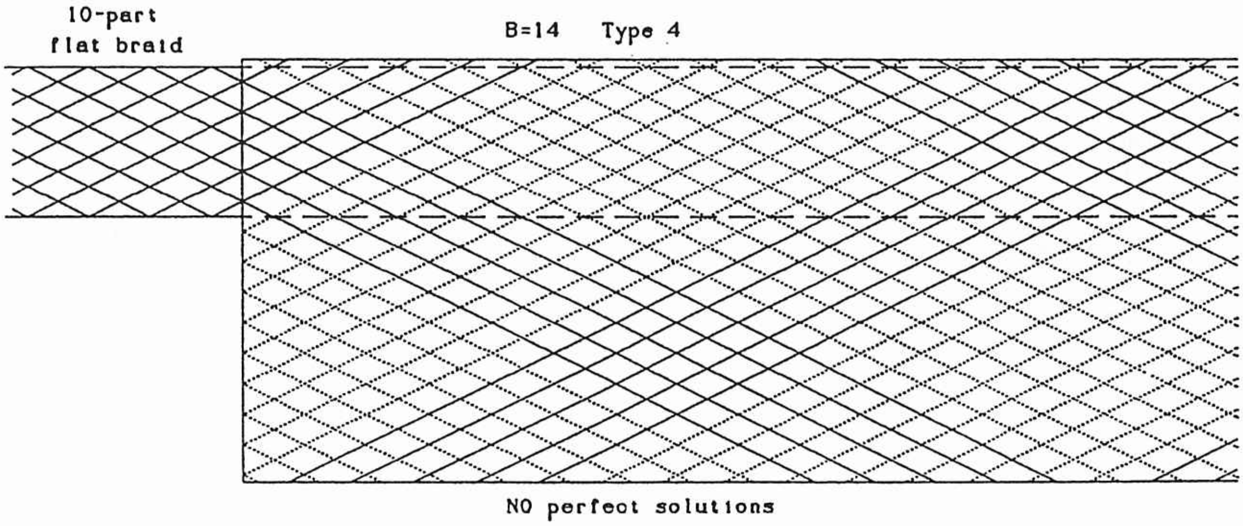

8-part
flat braid
B=17 Type 9
NO perfect solutions

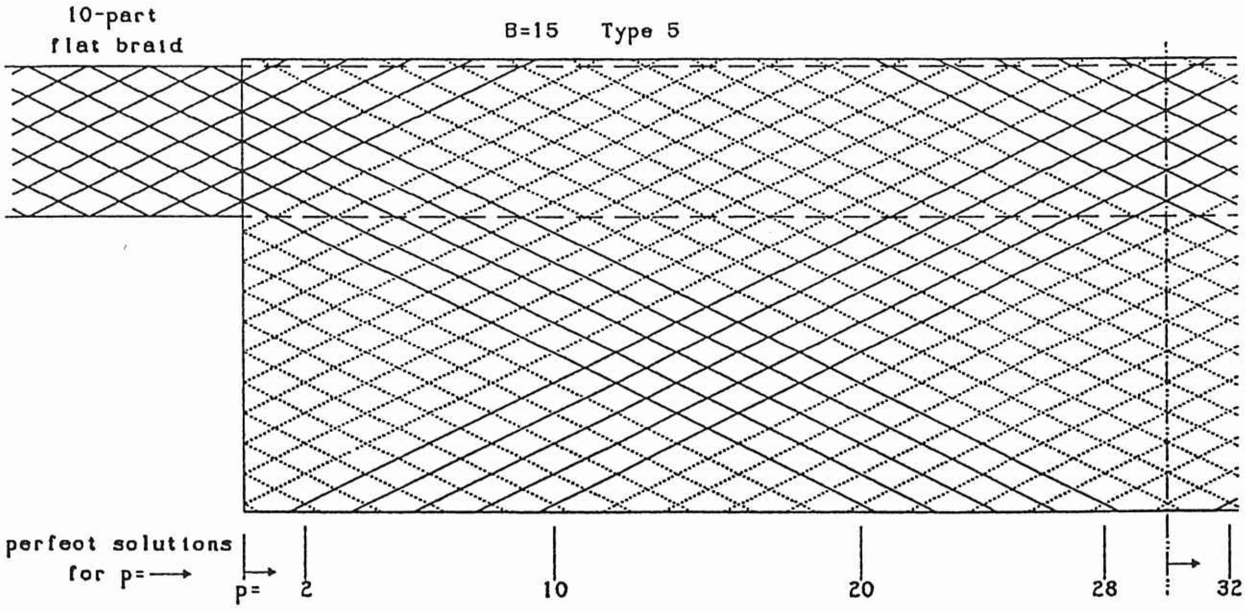

10-part
flat braid
B=10 Type 0
NO perfect solutions

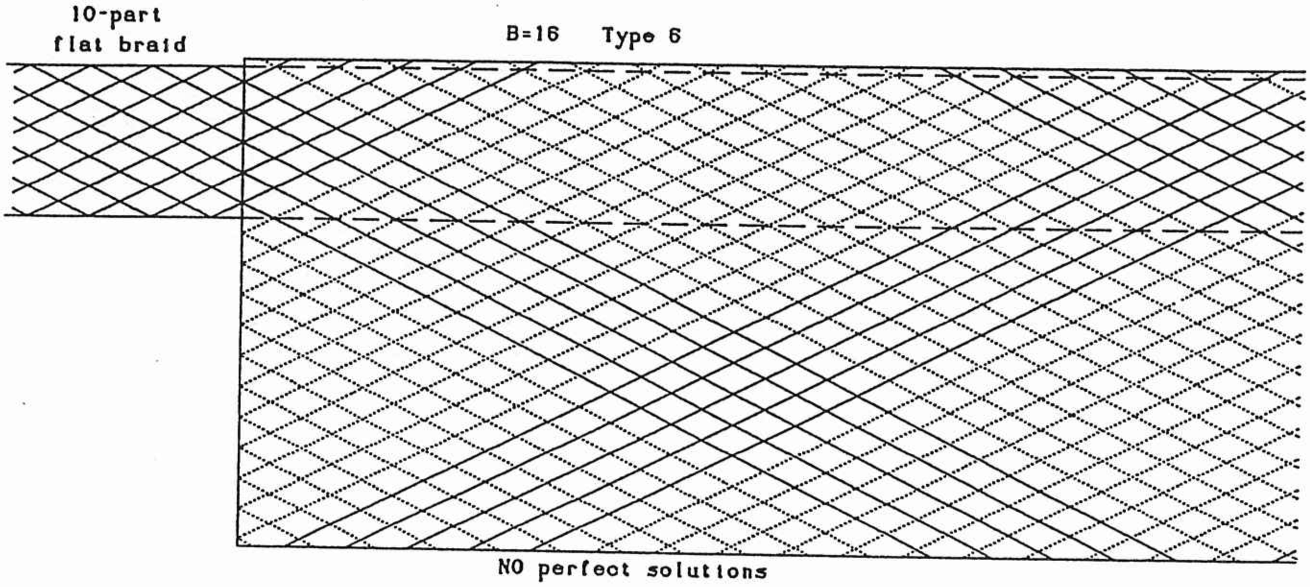

10-part
flat braid
B=11 Type 1
perfect solutions
for p=
p=
2
6
16
20
24
28

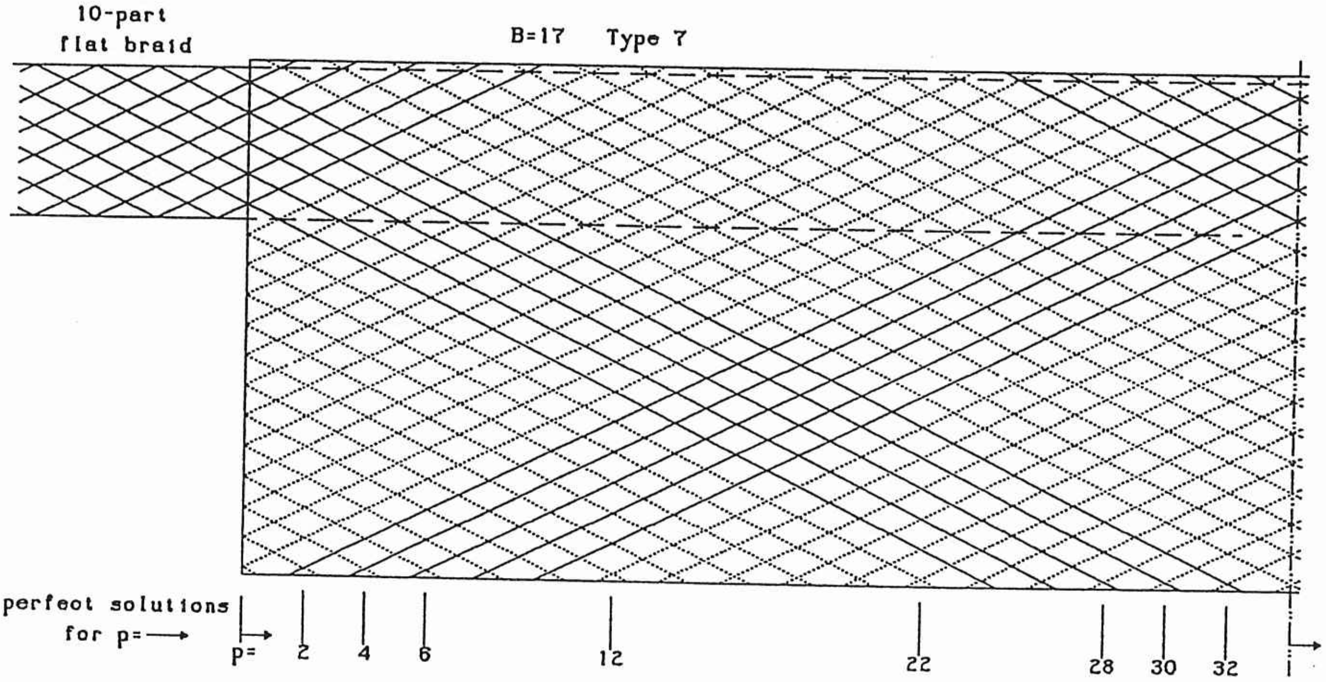

10-part
flat braid
B=12 Type 2
NO perfect solutions

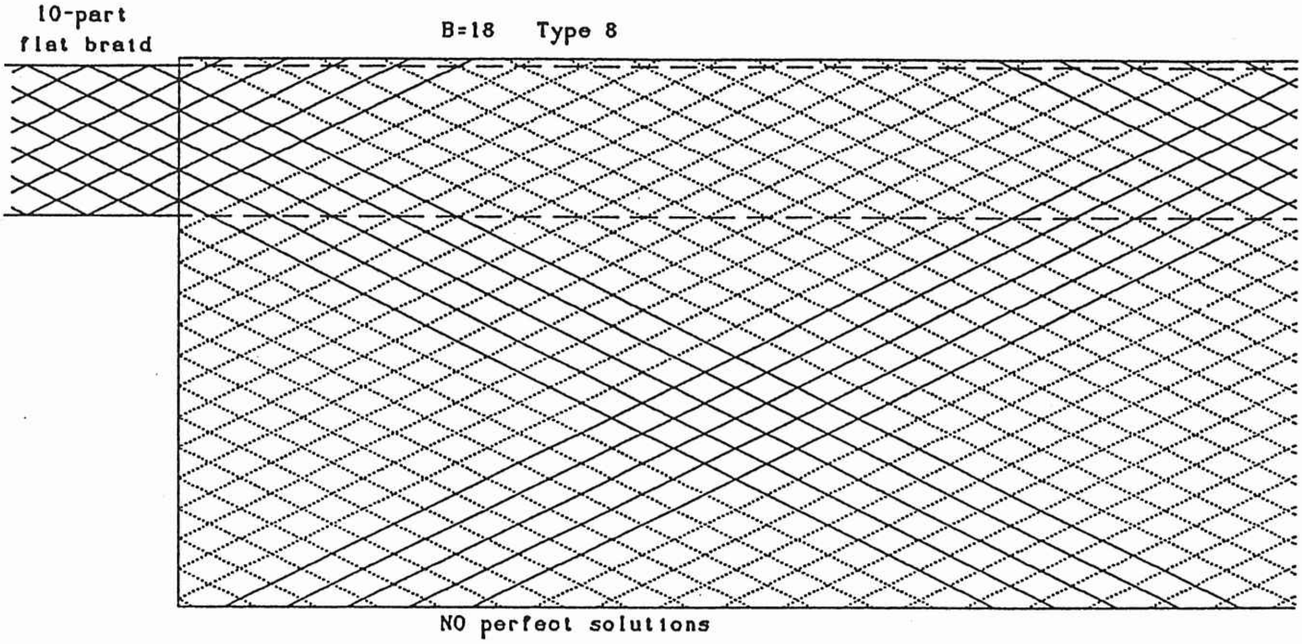

10-part
flat braid
B=13 Type 3
perfect solutions
for p=
p=
2
4
8
18
22
24
28
30

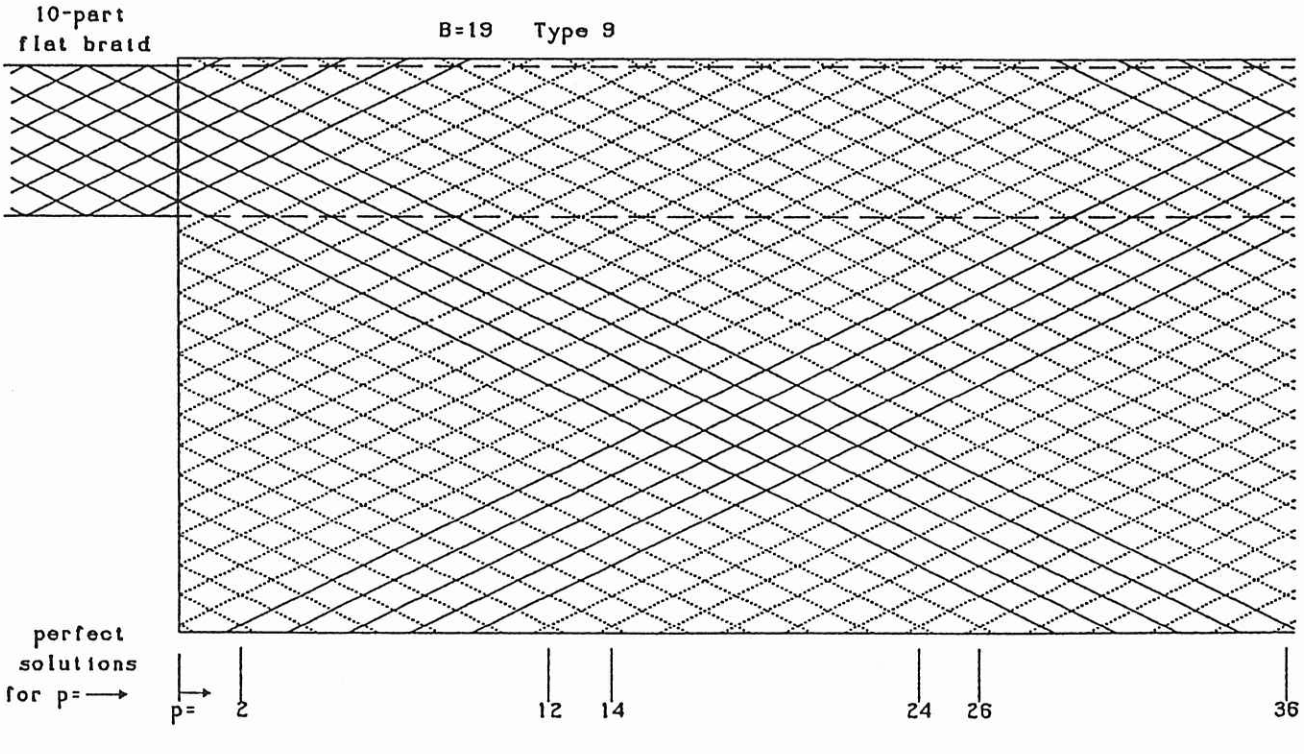

10-part
flat braid
B=14 Type 4
NO perfect solutions

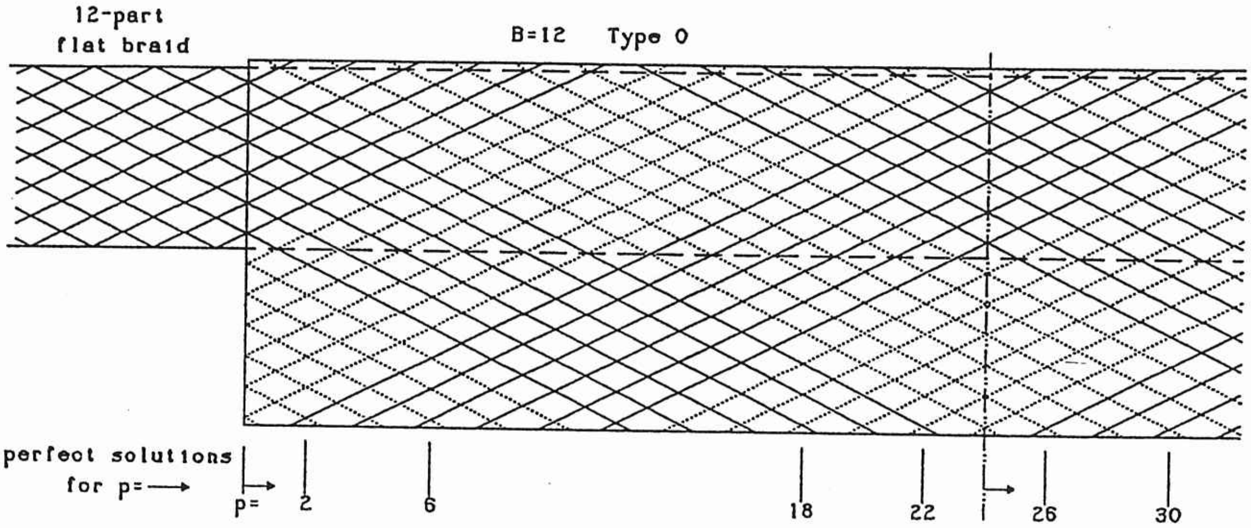

10-part
flat braid
B=15 Type 5
perfect solutions
for p=
p=
2
10
20
28
32

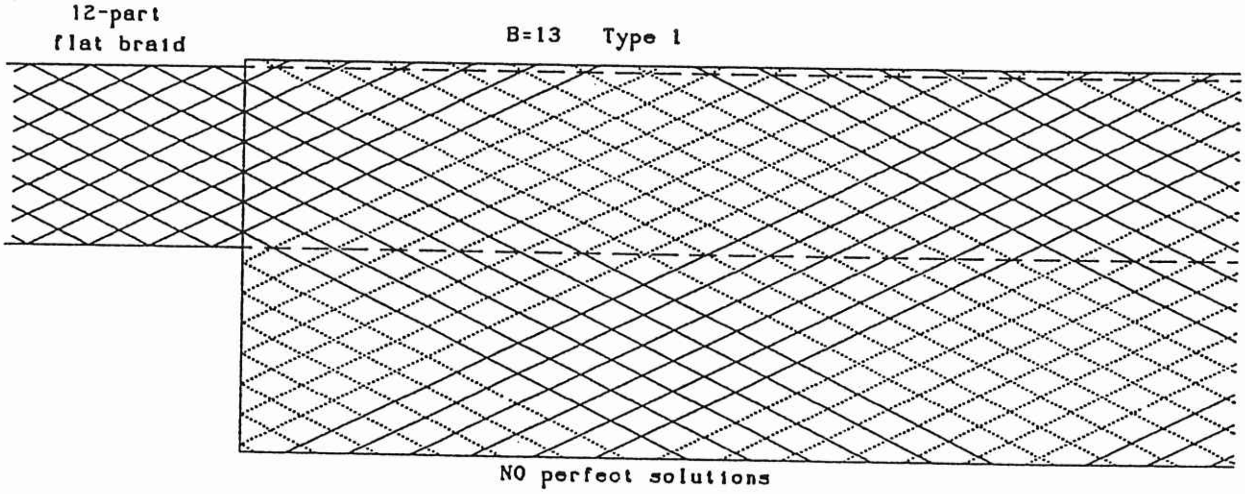

10-part
flat braid
B=16 Type 6
NO perfect solutions

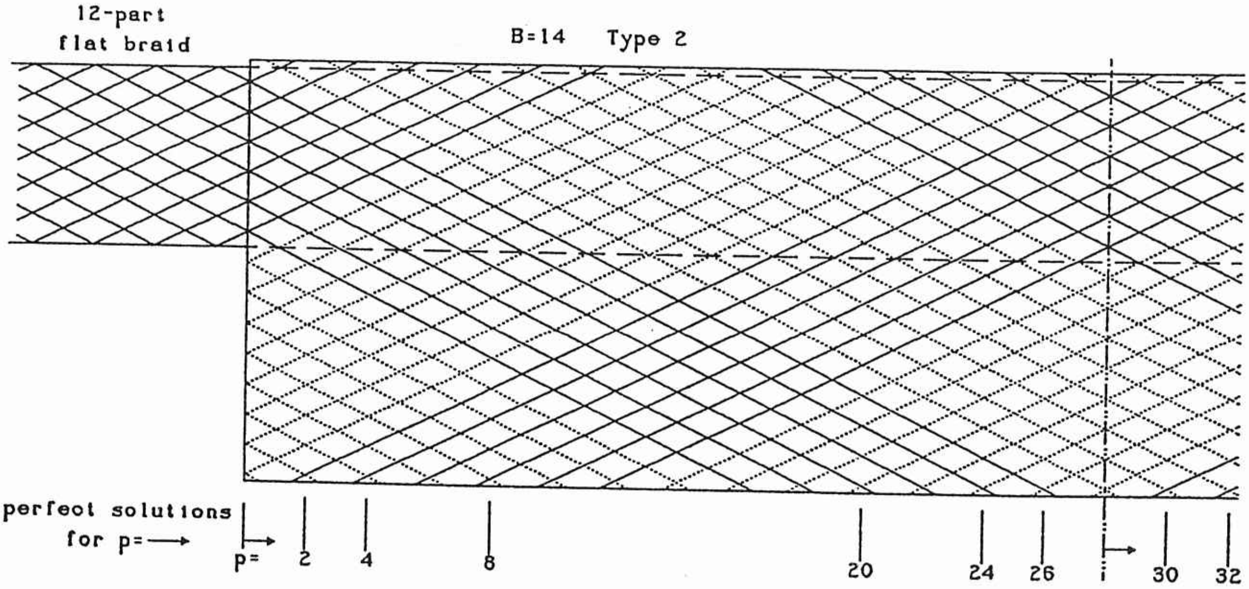

10-part
flat braid
B=17 Type 7
perfect solutions
for p=
p=
2
4
6
12
22
28
30
32

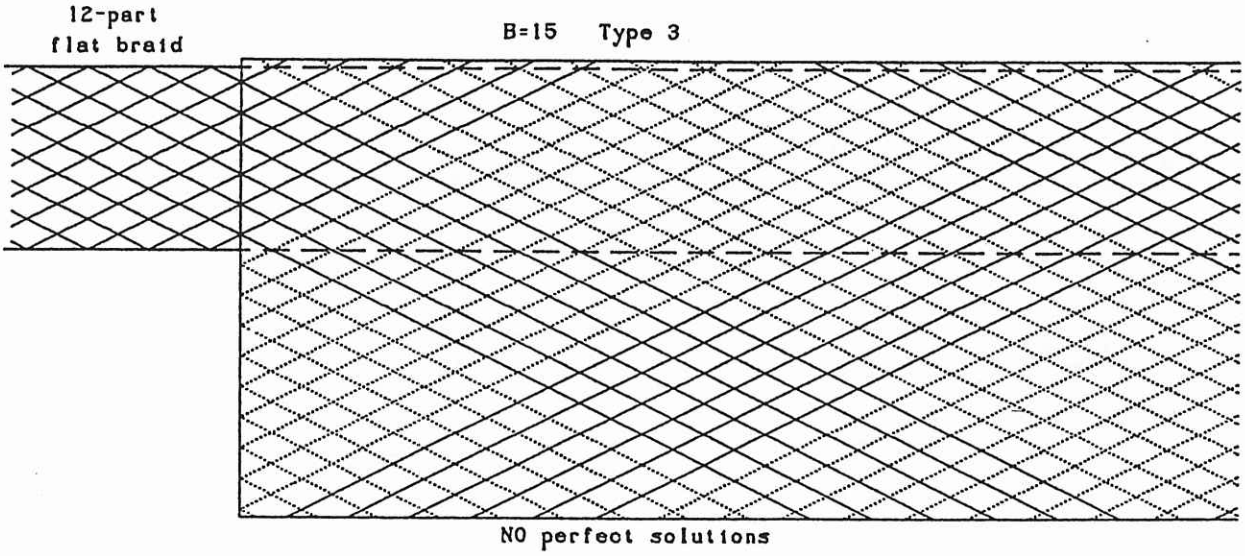

10-part
flat braid
B=18 Type 8
NO perfect solutions

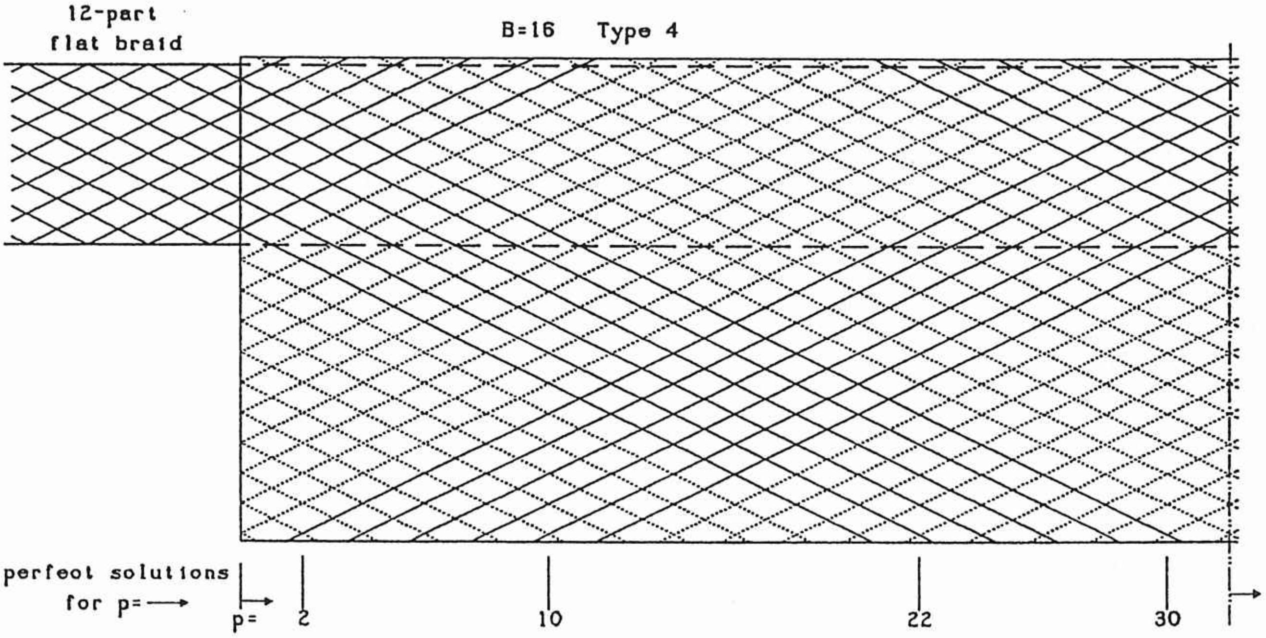

10-part
flat braid
B=19 Type 9
perfect
solutions
for p=
p=
2
12
14
24
26
36

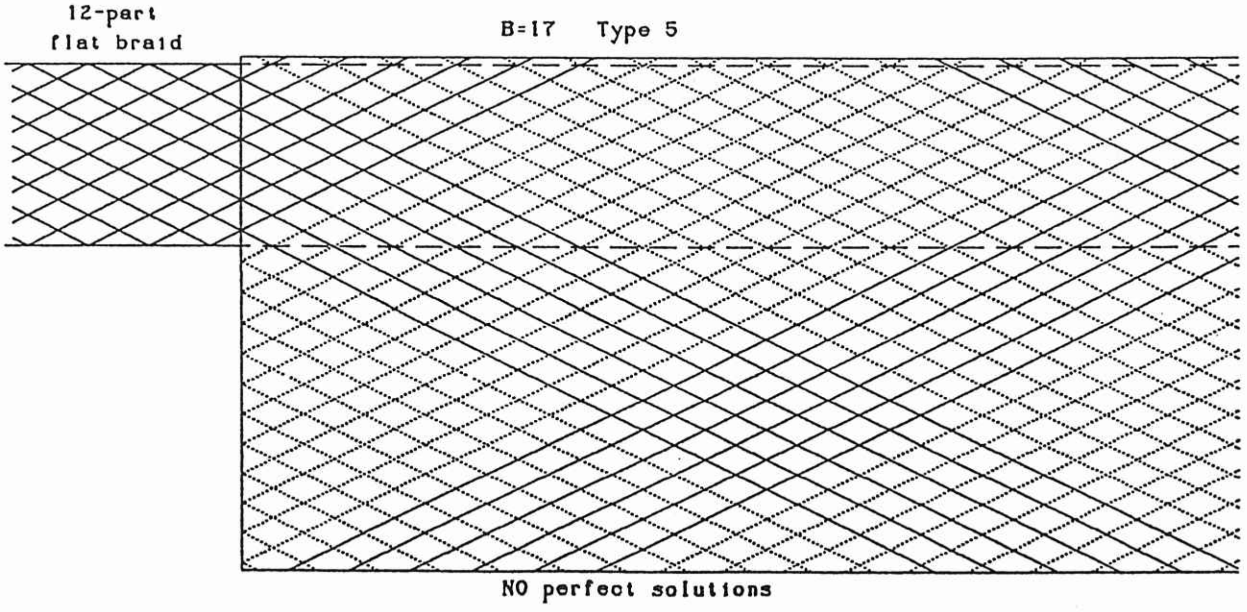

12-part
flat braid
B=12 Type 0
perfect solutions
for p=⟶
p= 2
6
18
22
26
30

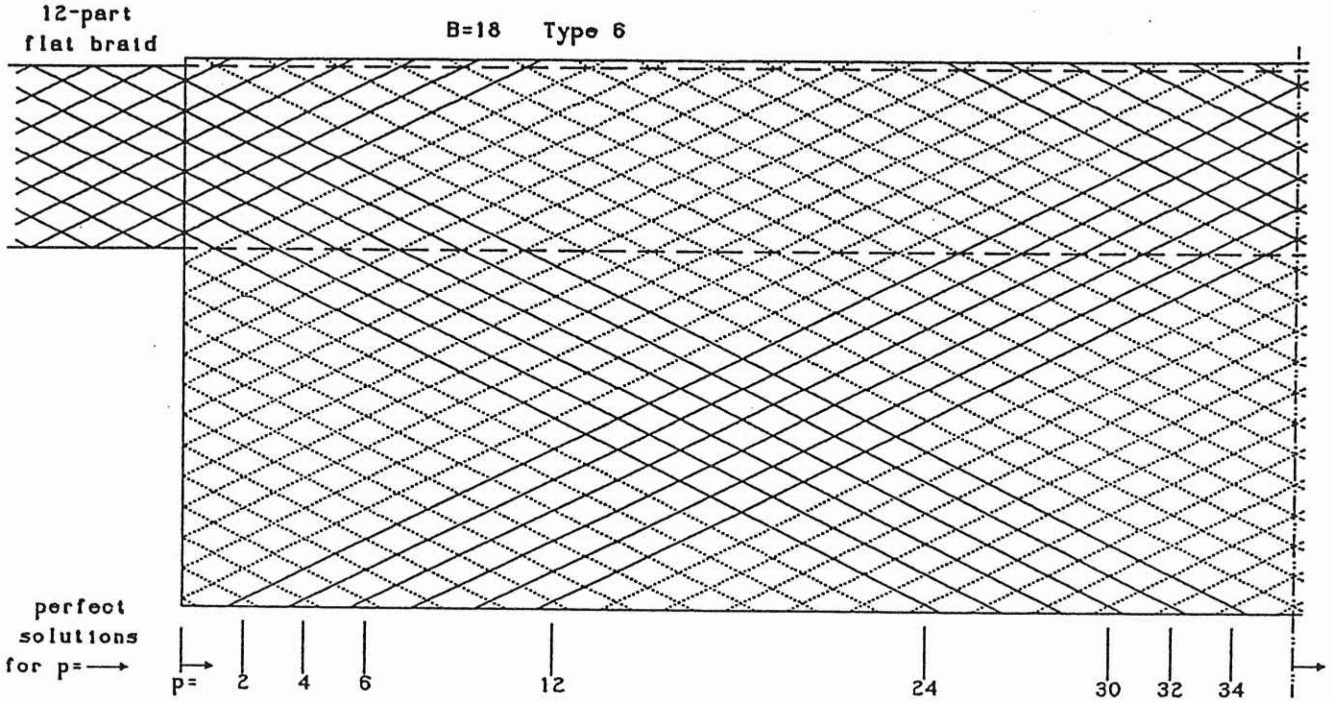

12-part
flat braid
B=13 Type 1
NO perfect solutions

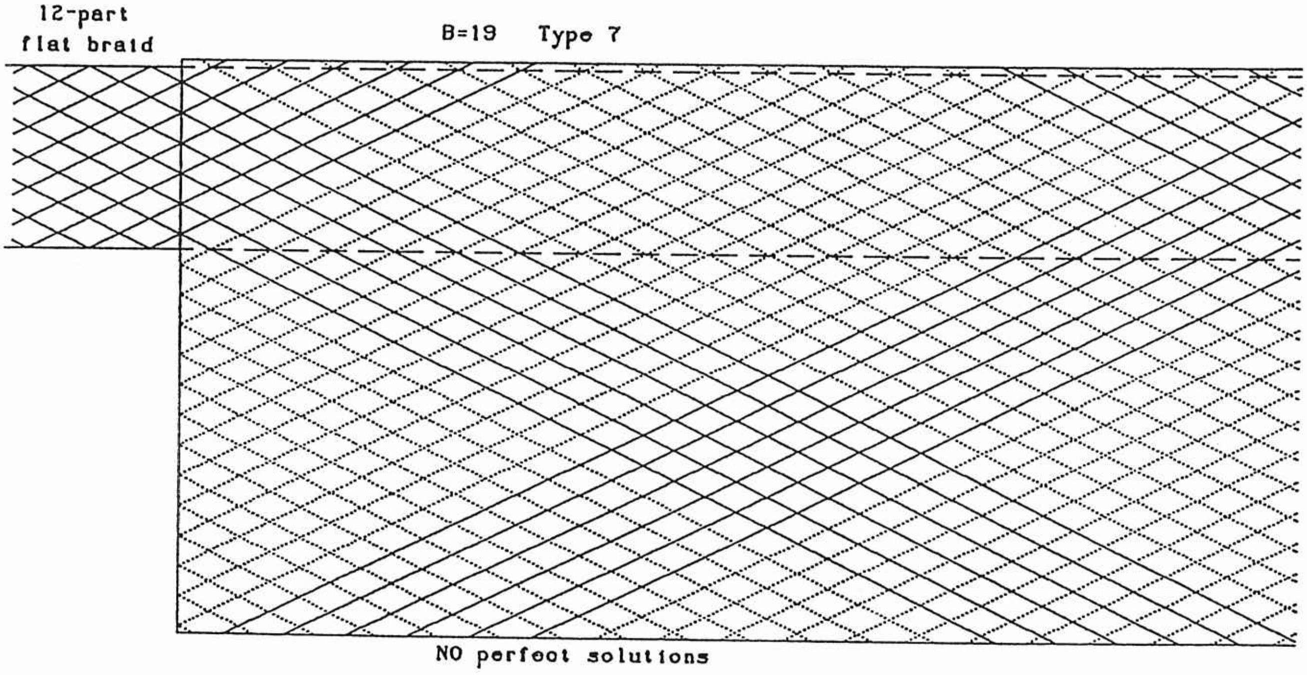

12-part
flat braid
B=14 Type 2
perfect solutions
for p=⟶
p= 2
4
8
20
24
26
30
32

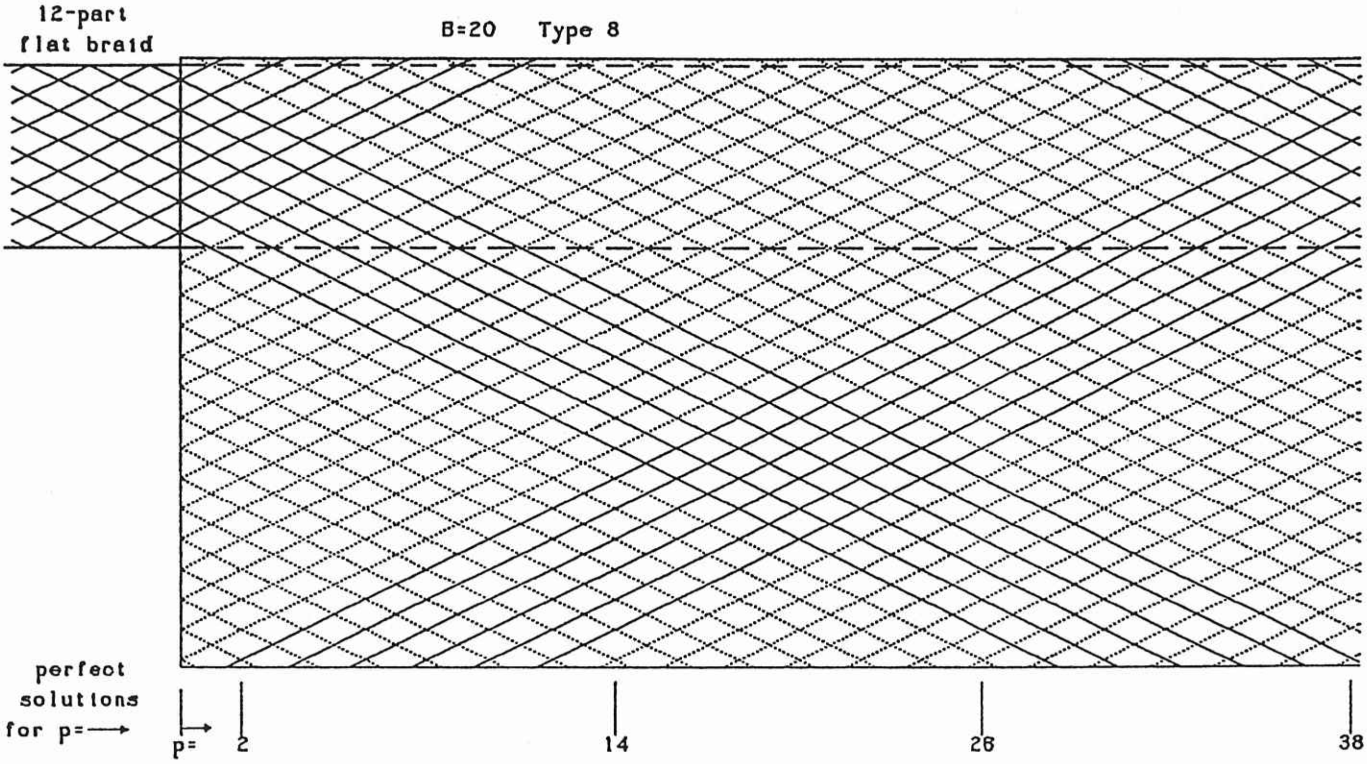

12-part
flat braid
B=15 Type 3
NO perfect solutions

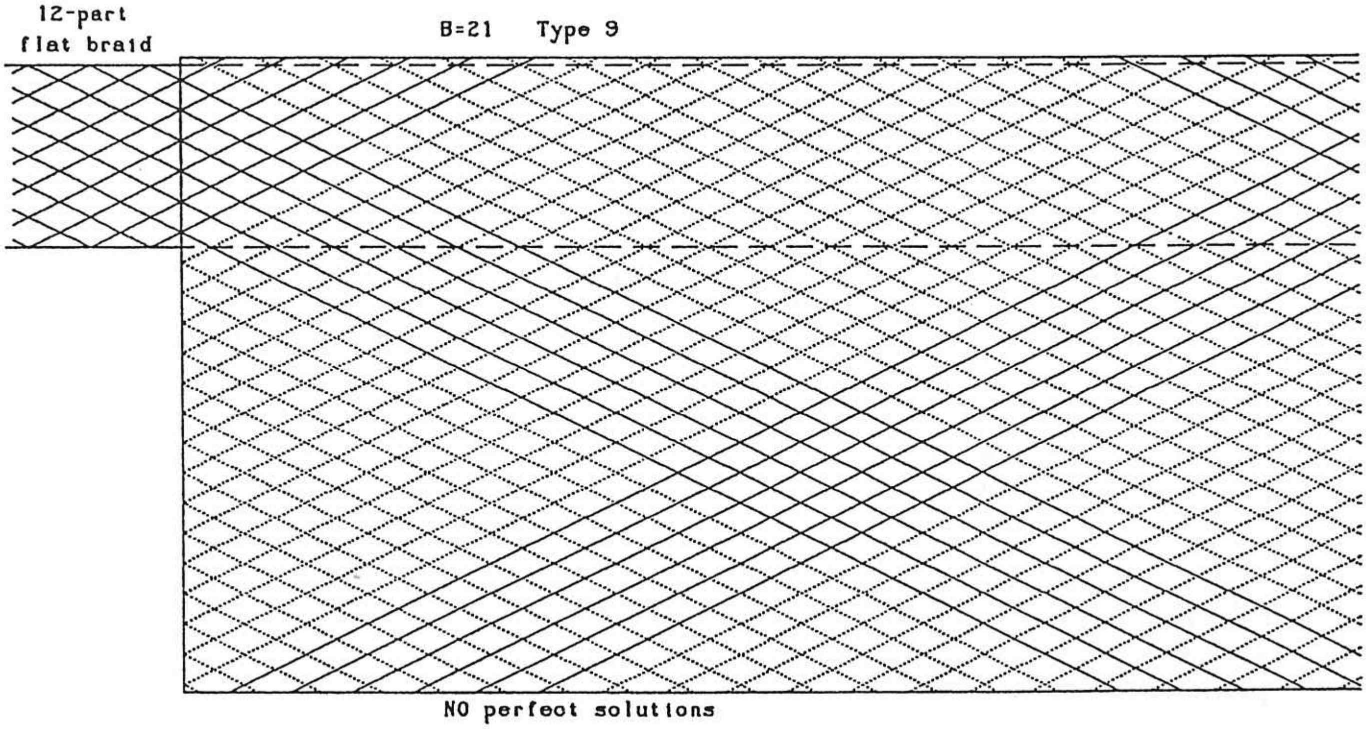

12-part
flat braid
B=16 Type 4
perfect solutions
for p=
p=
2
10
22
30


12-part
flat braid
B=17 Type 5
NO perfect solutions


12-part
flat braid
B=18 Type 6
perfect
solutions
for p=⟶
p=
2
4
6
12
24
30
32
34


12-part
flat braid
B=19 Type 7
NO perfect solutions


12-part
flat braid
B=20 Type 8
perfect
solutions
for p=⟶
p=
2
14
26
38


12-part
flat braid
B=21 Type 9
NO perfect solutions

Instead of indicating in general mid-keeper string-run diagrams the various valid values of the parameters, they can also readily and more conveniently be tabulated. These tabulations specify the various valid b-values associated with given values for x and p. The advantage of these tabulations is that the valid b-values can readily be calculated by using the following rules:

(1). All valid p-values are even.

(2). For even x-values, all valid b-values are even.

(3). For **odd** x-values, all valid b-values are **odd**.

(4). For a given x-value and p-value all valid b-values have g.c.d.$(p,b) \leq x$.

(5). For a given x-value the smallest valid b-value is $b = x$, whereby any positive even integer is a valid p-value.

(6). For a given x-value and p-value the greatest valid b-value in the range $x \leq b \leq p$ is:

$$b = x \text{ when } p \leq 2x.$$
$$b = (p - x) \text{ when } p \geq 2x.$$

(7). If $(P = p_1 \; ; \; B = b_1)$ are valid values for a given x-value, then also $(P = p_1 \; ; \; B = (b_1 + p_1))$ are valid values. This is the case of **bights-equivalency** in Regular Cylindrical Braids.

(8). If $(P = p_1 \; ; \; B = b_1)$ are valid values for a given x-value, then also $(P = (p_1 + 2b_1) \; ; \; B = b_1)$ are valid values. This is the case of **parts-equivalency** in Regular Cylindrical Braids.

(9). If $(P = p_1 \; ; \; B = b_1)$ are valid values for a given x-value, then also $(P = (2b_1 - p_1) \; ; \; B = b_1)$ are valid values.

We shall not waste space in presenting the proofs of the above rules since they are all more or less self evident.

On pgs. 167 — 169 we have presented the tables for $x = 2; x = 3; x = 4; x = 5; x = 6$ and $x = 7$. In each table are tabulated the valid b-values for each *even* p-value equal or less than 32.

In order to show how the entries in a table for a specific x-value are obtained with the aid of the above rules, we shall take the table for $x = 7$ as an Example. Rule 2 tells us that we shall only find valid b-values for *even* p-values. Since x is *odd* it follows from rule 3 that valid b-values must be *odd.*

For $p = 2$:

$b = 7$ is a valid value (rule 5; rule 6); $b = 9$ is a valid value (rule 7); $b = 11$ is a valid value (rule 7); etc.

For $p = 4$:

$b = 7$ is a valid value (rule 5; rule 6); $b = 9$ is **not** a valid value (rule 5 & 7); $b = 11$ is a valid value (rules 5 & 7); $b = 13$ is **not** a valid value (rule 7); $b = 15$ is a valid value (rule 7); etc.

For $p = 6$:

$b = 7$ is a valid value (rule 5; rule 6); $b = 9$ is **not** a valid value (rules 5 & 7); $b = 11$ is **not** a valid value (rules 5 & 7); $b = 13$ is a valid value (rule 7); $b = 15$ is **not** a valid value (rule 7); etc.

$\mathbf{x=2}$

$p=2$: $b=2, 4, 6, 8, 10, 12, \cdots$
$p=4$: $b=2, 6, 10, 14, 18, 22, \cdots$
$p=6$: $b=2, 4, 8, 10, 14, 16, 20, 22, 26, \cdots$
$p=8$: $b=2, 6, 10, 14, 18, 22, 26, 30, 34, \cdots$
$p=10$: $b=2, 4, 6, 8, 12, 14, 16, 18, 22, 24, \cdots$
$p=12$: $b=2, 10, 14, 22, 26, 34, 38, 46, \cdots$
$p=14$: $b=2, 4, 6, 8, 10, 12, 16, 18, 20, 22, \cdots$
$p=16$: $b=2, 6, 10, 14, 18, 22, 26, 30, 34, 38, \cdots$
$p=18$: $b=2, 4, 8, 10, 14, 16, 20, 22, 26, 28, \cdots$
$p=20$: $b=2, 6, 14, 18, 22, 26, 34, 38, 42, 46, \cdots$
$p=22$: $b=2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 24, \cdots$
$p=24$: $b=2, 10, 14, 22, 26, 34, \cdots$
$p=26$: $b=2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 28, \cdots$
$p=28$: $b=2, 6, 10, 18, 22, 26, 30, 34, \cdots$
$p=30$: $b=2, 4, 8, 14, 16, 22, 26, 28, 32, 34, \cdots$
$p=32$: $b=2, 6, 10, 14, 18, 22, 26, 30, 34, 38, \cdots$

$\mathbf{x=3}$

$p=2$: $b=3, 5, 7, 9, 11, 13, \cdots$
$p=4$: $b=3, 7, 11, 15, 19, 23, \cdots$
$p=6$: $b=3, 9, 15, 21, 27, 33, 39, \cdots$
$p=8$: $b=3, 5, 11, 13, 19, 21, 27, 29, 35, \cdots$
$p=10$: $b=3, 7, 13, 17, 23, 27, 33, 37, 43, 47, \cdots$
$p=12$: $b=3, 5, 7, 9, 15, 17, 19, 21, 27, \cdots$
$p=14$: $b=3, 11, 17, 25, 31, 39, \cdots$
$p=16$: $b=3, 7, 9, 13, 19, 23, 25, 29, 35, 39, \cdots$
$p=18$: $b=3, 5, 7, 11, 13, 15, 21, 23, 25, 29, \cdots$
$p=20$: $b=3, 9, 11, 17, 23, 29, 31, 37, 43, 49, \cdots$
$p=22$: $b=3, 5, 17, 19, 25, 27, 39, \cdots$
$p=24$: $b=3, 7, 9, 11, 13, 15, 17, 21, 27, 31, \cdots$
$p=26$: $b=3, 7, 11, 15, 19, 23, 29, 33, 37, \cdots$
$p=28$: $b=3, 5, 13, 15, 23, 25, 31, 33, 41, \cdots$
$p=30$: $b=3, 7, 9, 11, 19, 21, 23, 27, 33, 37, \cdots$
$p=32$: $b=3, 5, 7, 15, 17, 25, 27, 29, 35, 37, \cdots$

$\mathbf{x=4}$

$p=2$: $b=4, 6, 8, 10, 12, 14, 16, \cdots$
$p=4$: $b=4, 8, 12, 16, 20, 24, 28, \cdots$
$p=6$: $b=4, 10, 16, 22, 28, 34, 40, 46, \cdots$
$p=8$: $b=4, 12, 20, 28, 36, 44, \cdots$
$p=10$: $b=4, 6, 14, 16, 24, 26, 34, 36, \cdots$
$p=12$: $b=4, 8, 16, 20, 28, 32, 40, 44, \cdots$
$p=14$: $b=4, 6, 8, 10, 18, 20, 22, 24, 32, \cdots$
$p=16$: $b=4, 12, 20, 28, 36, 44, \cdots$
$p=18$: $b=4, 8, 10, 14, 22, 26, 28, 32, 40, \cdots$
$p=20$: $b=4, 8, 12, 16, 24, 28, 32, 36, 44, \cdots$
$p=22$: $b=4, 6, 10, 12, 16, 18, 26, 28, 32, 34, \cdots$
$p=24$: $b=4, 20, 28, 44, 48, 68, \cdots$
$p=26$: $b=4, 6, 10, 12, 14, 16, 20, 22, 30, 32, \cdots$
$p=28$: $b=4, 8, 12, 16, 20, 24, 32, 36, 40, \cdots$
$p=30$: $b=4, 8, 14, 16, 22, 26, 34, 38, 44, \cdots$
$p=32$: $b=4, 12, 20, 28, 36, 44, \cdots$

$\mathbf{x=5}$

$p=2$: $b=5, 7, 9, 11, 13, 15, 17, \cdots$
$p=4$: $b=5, 9, 13, 17, 21, 25, 29, \cdots$
$p=6$: $b=5, 11, 17, 23, 29, 35, 41, \cdots$
$p=8$: $b=5, 13, 21, 29, 37, 45, 53, \cdots$
$p=10$: $b=5, 15, 25, 35, 45, 55, \cdots$
$p=12$: $b=5, 7, 17, 19, 29, 31, 41, 43, \cdots$
$p=14$: $b=5, 9, 19, 23, 33, 37, 47, \cdots$
$p=16$: $b=5, 7, 9, 11, 21, 23, 25, 27, 37, 39, \cdots$
$p=18$: $b=5, 13, 23, 31, 41, 49, \cdots$
$p=20$: $b=5, 9, 11, 15, 25, 29, 31, 35, 45, 49, \cdots$
$p=22$: $b=5, 9, 13, 17, 27, 31, 35, 39, \cdots$
$p=24$: $b=5, 11, 13, 19, 29, 35, 37, \cdots$
$p=26$: $b=5, 7, 19, 21, 31, 33, 45, \cdots$
$p=28$: $b=5, 11, 13, 15, 17, 23, 33, 39, \cdots$
$p=30$: $b=5, 7, 13, 17, 23, 25, 35, 37, 43, 47, \cdots$
$p=32$: $b=5, 9, 15, 17, 23, 27, 37, 41, 47, \cdots$

$\mathbf{x = 6}$

$p = 2$: $b = 6, 8, 10, 12, 14, 16, 18, 20, \cdots$
$p = 4$: $b = 6, 10, 14, 18, 22, 26, 30, 34, \cdots$
$p = 6$: $b = 6, 12, 18, 24, 30, 36, \cdots$
$p = 8$: $b = 6, 14, 22, 30, 38, 46, \cdots$
$p = 10$: $b = 6, 16, 26, 36, 46, 56, \cdots$
$p = 12$: $b = 6, 18, 30, 42, 54, \cdots$
$p = 14$: $b = 6, 8, 20, 22, 34, 36, 48, \cdots$
$p = 16$: $b = 6, 10, 22, 26, 38, 42, \cdots$
$p = 18$: $b = 6, 8, 10, 12, 24, 26, 28, 30, 42, \cdots$
$p = 20$: $b = 6, 14, 26, 34, 46, \cdots$
$p = 22$: $b = 6, 10, 12, 16, 28, 32, 34, \cdots$
$p = 24$: $b = 6, 10, 14, 18, 30, 34, \cdots$
$p = 26$: $b = 6, 12, 14, 20, 32, 38, 40, \cdots$
$p = 28$: $b = 6, 22, 34, 50, 62, \cdots$
$p = 30$: $b = 6, 8, 12, 14, 16, 18, 22, 24, 36, 38, 42, \cdots$
$p = 32$: $b = 6, 14, 18, 26, 38, 46, \cdots$

$\mathbf{x = 7}$

$p = 2$: $b = 7, 9, 11, 13, 15, 17, 19, \cdots$
$p = 4$: $b = 7, 11, 15, 19, 23, 27, 31, \cdots$
$p = 6$: $b = 7, 13, 19, 25, 31, 37, 43, \cdots$
$p = 8$: $b = 7, 15, 23, 31, 39, 47, \cdots$
$p = 10$: $b = 7, 17, 27, 37, 47, 57, \cdots$
$p = 12$: $b = 7, 19, 31, 43, 55, \cdots$
$p = 14$: $b = 7, 21, 35, 49, 63, \cdots$
$p = 16$: $b = 7, 9, 23, 25, 39, 41, \cdots$
$p = 18$: $b = 7, 11, 25, 29, 43, 47, \cdots$
$p = 20$: $b = 7, 9, 11, 13, 27, 29, 31, 33, 47, 49, \cdots$
$p = 22$: $b = 7, 15, 29, 37, 51, 59, \cdots$
$p = 24$: $b = 7, 11, 13, 17, 31, 35, 37, 41, \cdots$
$p = 26$: $b = 7, 11, 15, 19, 33, 37, 41, \cdots$
$p = 28$: $b = 7, 13, 15, 21, 35, 41, 43, \cdots$
$p = 30$: $b = 7, 23, 37, 53, 67, \cdots$
$p = 32$: $b = 7, 13, 15, 17, 19, 25, 39, 45, 47, \cdots$

For $p = 8$:
$b = 7$ is a valid value (rule 5; rule 6); $b = 9$ is a **not** valid value (rules 5 & 7); $b = 11$ is **not** a valid value (rules 5 & 7); $b = 13$ is **not** a valid value (rules 5 & 7); $b = 15$ is a valid value (rule 7); etc.

For $p = 10$:
$b = 7$ is a valid value (rule 5; rule 6); $b = 9$ is a **not** valid value (rule 6); $b = 11$ is **not** a valid value (rules 5 & 7); $b = 13$ is **not** a valid value (rules 5 & 7); $b = 15$ is **not** a valid value (rules 5 & 7); $b = 17$ is a valid value (rule 7); etc.

For $p = 12$:
$b = 7$ is a valid value (rule 5; rule 6); $b = 9$ is a **not** valid value (rule 6); $b = 11$ is **not** a valid value, (rule 6); $b = 13$ is **not** a valid value (rules 5 & 7); $b = 15$ is **not** a valid value (rules 5 & 7); $b = 17$ is **not** a valid value (rules 5 & 7); $b = 19$ is a valid value (rule 7); etc.

For $p = 14$:
$b = 7$ is a valid value (rule 5; rule 6); $b = 9$ is a **not** valid value (rule 6); $b = 11$ is **not** a valid value (rule 6); $b = 13$ is **not** a valid value (rule 6); $b = 15$ is **not** a valid value (rules 5 & 7); $b = 17$ is **not** a valid value (rules 5 & 7); $b = 19$ is **not** a valid value (rules 5 & 7); $b = 21$ is a valid value (rule 7); etc.

For $p = 16$:
$b = 7$ is a valid value (rule 5; rule 8 since $b = 7$ is a valid value with $p = 2$); $b = 9$ is a valid value (rule 6; rule 9 since $b = 9$ is a valid value with $p = 2$); $b = 11$ is **not** a valid value (rule 6); $b = 13$ is **not** a valid value (rule 6); $b = 15$ is **not** a valid value (rules 5 & 7); $b = 17$ is **not** a valid value (rules 5 & 7); $b = 19$ is **not** a valid value (rules 5 & 7); $b = 21$ is **not** a valid value (rules 5 & 7); $b = 23$ is a valid value (rule 7); etc.

For $p = 18$:
$b = 7$ is a valid value (rule 5; rule 8 since $b = 7$ is a valid value with $p = 4$); $b = 9$ is **not** a valid value (rule 4); $b = 11$ is a valid value (rule 6; rule 9 since $b = 11$ is a valid value with $p = 4$); $b = 13$ is **not** a valid value (rule 6); $b = 15$ is **not** a valid value (rule 6); $b = 17$ is **not** a valid value (rule 6); $b = 19$ is **not** a valid value (rules 5 & 7); $b = 21$ is **not** a valid value (rules 5 & 7); $b = 23$ is **not** a valid value (rules 5 & 7); $b = 25$ is a valid value (rule 7); etc.

For $p = 20$:
$b = 7$ is a valid value (rule 5; rule 8 since $b = 7$ is a valid value with $p = 6$); $b = 9$ is a valid value (rule 8 since $b = 9$ is a valid value with $p = 2$); $b = 11$ is a valid value (rule 9 since $b = 11$ is a valid value with $p = 2$); $b = 13$ is a valid value (rule 6; rule 9 since $b = 13$ is a valid value with $p = 6$); $b = 15$ is **not** a valid value (rule 6); $b = 17$ is **not** a valid value (rule 6); $b = 19$ is **not** a valid value (rule 6); $b = 21$ is **not** a valid value (rules 5 & 7); $b = 23$ is **not** a valid value (rules 5 & 7); $b = 25$ is **not** a valid value (rules 5 & 7); $b = 27$ is a valid value (rule 7); etc.

For $p = 22$:
$b = 7$ is a valid value (rule 5; rule 8 since $b = 7$ is a valid value with $p = 8$); $b = 9$ is **not** a valid value (rule 8 since $b = 9$ is **not** a valid value with $p = 4$); $b = 11$ is **not** a valid value (rule 4); $b = 13$ is **not** a valid value (rule 9 since $b = 13$ is **not** a valid value with $p = 4$); $b = 15$ is a valid value (rule 6); $b = 17$ is **not** a valid value (rule 6); $b = 19$ is **not** a valid value (rule 6); $b = 21$ is **not** a valid value (rule 6); $b = 23$ is **not** a valid value (rules 5 & 7); $b = 25$ is **not** a valid value (rules 5 & 7); $b = 27$ is **not** a valid value (rules 5 & 7); $b = 29$ is a valid value (rule 7); etc.

For $p = 24$:
$b = 7$ is a valid value (rule 5; rule 8 since $b = 7$ is a valid value with $p = 10$); $b = 9$ is not a valid value (rule 8 since $b = 9$ is not a valid value with $p = 6$); $b = 11$ is a valid value (rule 8 since $b = 11$ is a valid value with $p = 2$); $b = 13$ is a valid value (rule 9 since $b = 13$ is a valid value with $p = 2$); $b = 15$ is not a valid value (rule 9 since $b = 15$ is not a valid value with $p = 6$); $b = 17$ is a valid value (rule 6); $b = 19$ is not a valid value (rule 6); $b = 21$ is not a valid value (rule 6); $b = 23$ is not a valid value (rules 6); $b = 25$ is not a valid value (rules 5 & 7); $b = 27$ is not a valid value (rules 5 & 7); $b = 29$ is not a valid value (rules 5 & 7); $b = 31$ is a valid value (rule 7); etc.

And so on.

For mid-keepers the various braiding methods so far discussed were all solely based on their grid-diagrams. This necessitated the drawing of a grid-diagram in its successive construction stages. This gives no doubt a good overview of the successive braiding stages and hence may be the most covenient braiding method for the beginner. It is, however, quite a time consuming approach, and consequently the more experienced braider would braid a mid-keeper by means of half-cycle algorithms instead. In order to enable him or her to do so, first such half-cycle algorithms will have to be worked out. The most convenient way to do this is via the intersection colum-numbers of each half-cycle.

The bights of the cylindrical mid-keeper part and transition-points between this part and the flat braids on both the left and right-hand boundaries are numbered in successive order (**index-numbers**). The index-number of a bight or transition-point on the right-hand boundary is identical to the index-number of its left-hand counter part on the same cylinder generating line. In the uppermost grid-diagram on pg. 172 the successive numbering on the left-hand boundary only is shown. It is assumed that the overall braiding direction is from left to right, hence after reaching (while braiding the flat braid) the left-hand boundary of the cylindrical mid-keeper braid, this cylindrical mid-keeper braid will be braided string by string. Since we have x transition-points (in the Example on pg. 172 the value of x is 4, and hence the transition-points are numbered $0, 1, 2$ and 3), we have $2x$ strings. First the successive from lower-left to upper-right running strings entering the successive numbered left-hand transition-points are braided, then the successive from upper-left to lower-right running strings entering the successive left-hand transition-points are braided. Each of these successive braiding-stages are depicted in the successive string-run diagrams on pg. 172. It should be noted that in actual practice these successive string-run diagrams are not drawn (they are here only presented in order to explain the calculation procedure), instead their successive half-cycle begin and end-points only are written down. When we denote the begin-point of a half-cycle by I_s and its end-point by I_e, then:

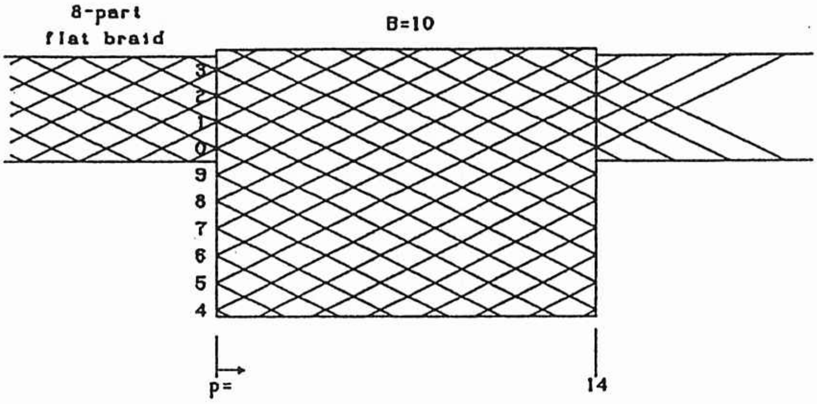
For the first set of x strings (each string with a half-cycle running from lower-left to upper-right, followed by a half-cycle from lower-right to upper-left, etc):

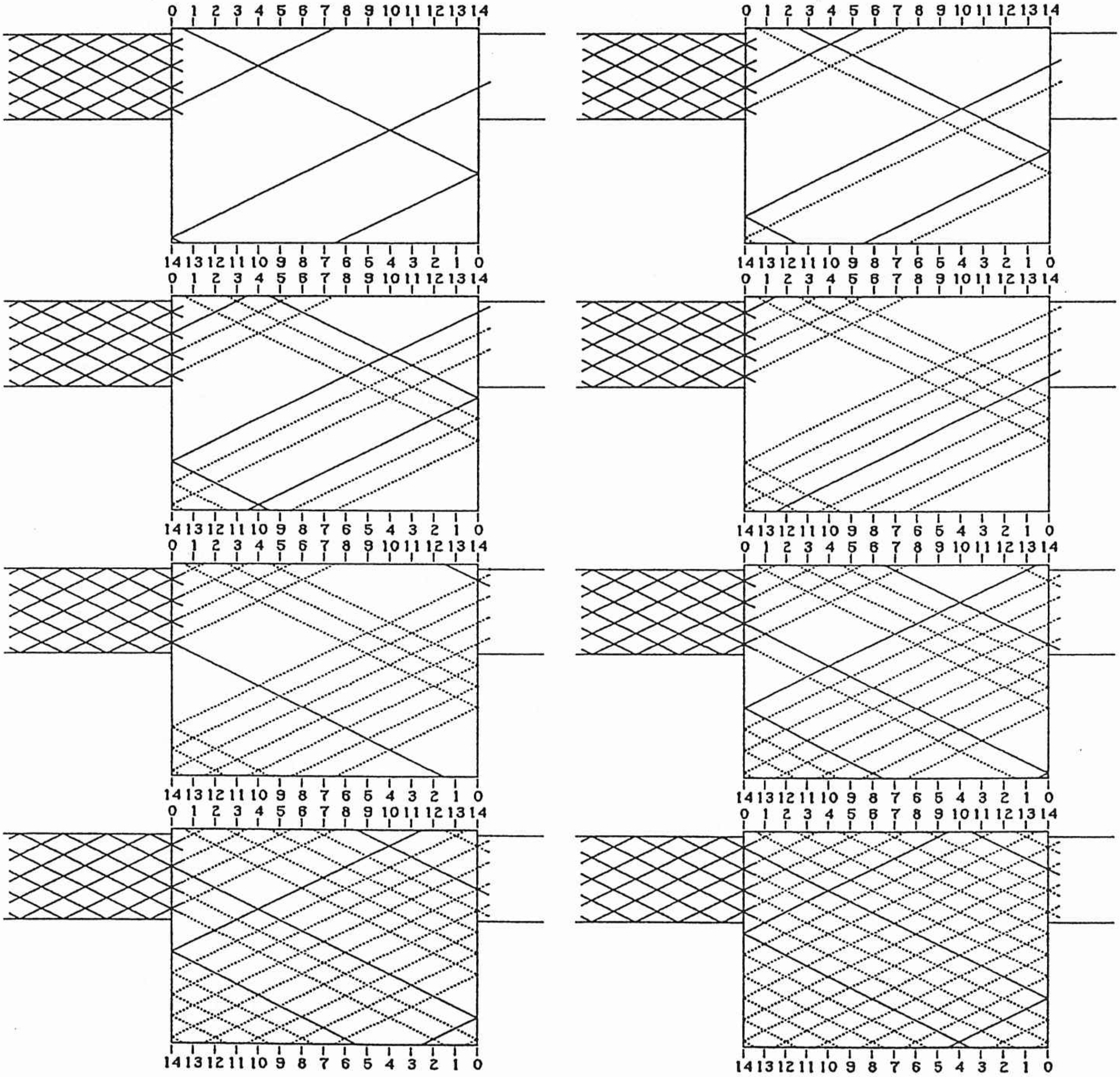
$$I_e = \left| I_s + \frac{p}{2} \right|_b .$$

For the second set of x strings (each string with a half-cycle running from upper-left to lower-right, followed by a half-cycle from upper-right to lower-left, etc):

$$I_e = \left| I_s - \frac{p}{2} \right|_b .$$

The successive half-cycles of the two sets of $x = 4$ strings in the Example on pg. 172 are therefore:


8-part
flat braid
B=10
p=


0 1 2 3 4 5 6 7 8 9 10 11 12 13 14
14 13 12 11 10 9 8 7 6 5 4 3 2 1 0

$$\left\{\begin{array}{l} 0_L - 7_R - 4_L - 1_R \\ 1_L - 8_R - 5_L - 2_R \\ 2_L - 9_R - 6_L - 3_R \\ 3_L - 0_R \end{array}\right.$$

and

$$\left\{\begin{array}{l} 0_L - 3_R \\ 1_L - 4_R - 7_L - 0_R \\ 2_L - 5_R - 8_L - 1_R \\ 3_L - 6_R - 9_L - 2_R \end{array}\right.$$

In the above, the subscript 'L' indicates the left-boundary while the subscript 'R' indicates the right-boundary of the cylindrical mid-keeper braid.

In order to formulate the rules with which the intersection column-numbers associated with a half-cycle are determined, we introduce the following notations:

For the first set of x strings (for which the left to right half-cycles run from lower left to upper right, and the right to left half-cycles run from lower right to upper left), I_{ij} with $i = 0, 1, 2, \cdots, (x-1)$ and $j = 1, 2, 3, \cdots, f$ is the index-number where a half-cycle begins, or ends, or where one half-cycle ends and the next half-cycle begins. Hence for $j = odd$ the index numbers are on the left-hand boundary, and for $j = even$ they are on the right-hand boundary of the cylindrical mid-keeper braid. Furthermore, I_{i1} is the transition index-number (on the left boundary) where string i enters, from the left, the cylindrical mid-keeper braid and runs from lower-left to upper-right; I_{if} is the transition index-number (on the right boundary) where string i leaves the cylindrical mid-keeper braid and runs from lower-left to upper-right (note that f is *even*). Also note that a string has $(f-1)$ half-cycles, and since f is even, a string has therefore an odd number of half-cycles.

For the second set of x strings (for which the left to right half-cycles run from upper left to lower right, and the right to left half-cycles run from upper right to lower left), $I_{i^*j^*}$ with $i^* = 0, 1, 2, \cdots, (x-1)$ and $j^* = 1, 2, 3, \cdots, f^*$ is the index-number where a half-cycle begins, or ends, or where one half-cycle ends and the next half-cycle begins. Hence for $j^* = odd$ the index numbers are on the left-hand boundary, and for $j^* = even$ they are on the right-hand boundary of the cylindrical mid-keeper braid. Furthermore, $I_{i^*1^*}$ is the transition index-number (on the left boundary) where string i^* enters from the left, the cylindrical mid-keeper braid and runs from upper-left to lower-right; $I_{i^*f^*}$ is the transition index-number (on the right boundary) where string i^* leaves the cylindrical mid-keeper braid and runs from upper-left to lower-right (note that f^* is *even*). Again, a string has (f^*-1) half-cycles, and since f^* is even, a string has therefore an odd number of half-cycles.

Note that for $i^* = i$, and $j^* = j$, transition index-number $I_{ij} = I_{i^*j^*}$.

The following rules for the intersection column-numbers of a half-cycle should be fairly well self evident:

(1). The intersection column-numbers of a half-cycle which begins in transition index-number I_{ij}, where $j = odd$, are obtained with the formula:

$$|-(I_{ij} - I_{cd})|_b + nb < p, \quad \text{where}$$

$$\text{for } c < i \left\{\begin{array}{l} d = odd; \\ 3 \le d \le f_c - 1; \\ n = 0, 1, 2, \cdots . \end{array}\right. \qquad \text{for } c = i \left\{\begin{array}{l} d = odd; \\ 3 \le d \le j; \\ n = 0, 1, 2, \cdots . \end{array}\right.$$

(2). The intersection column-numbers of a half-cycle which begins in transition index-number I_{ij}, where $j = even$, and $j < f_i$, are obtained with the formula:

$$|-(I_{ij} - I_{cd})|_b + nb < p, \quad \text{where}$$

$$\text{for } c < i \begin{cases} d = even; \\ 2 \leq d \leq f_c; \\ n = 0, 1, 2, \cdots . \end{cases} \qquad \text{for } c = i \begin{cases} d = even; \\ 2 \leq d \leq j; \\ n = 0, 1, 2, \cdots . \end{cases}$$

Note that the half-cycle which begins in transition index-number I_{01} has a free run, and that there are no half-cycles which begin in transition index-numbers I_{if}.

(3). The intersection column-numbers of a half-cycle which begins in transition index-number $I_{i^*j^*}$, where $j^* = odd$, are obtained with the formulae:

$$|I_{i^*j^*} - I_{cd}|_b + nb < p,$$
$$|I_{i^*j^*} - I_{c^*d^*}|_b + nb < p; \quad \text{where}$$

$$\begin{cases} 0 \leq c \leq x - 1; \\ d = odd; \\ 1 \leq d \leq f_c - 1; \\ n = 0, 1, 2, \cdots . \end{cases} \quad \text{for } c^* < i^* \begin{cases} d^* = odd; \\ 3 \leq d^* \leq f^*_{c^*} - 1; \\ n = 0, 1, 2, \cdots . \end{cases} \quad \text{for } c^* = i^* \begin{cases} d^* = odd; \\ 3 \leq d^* \leq j^*; \\ n = 0, 1, 2, \cdots . \end{cases}$$

(4). The intersection column-numbers of a half-cycle which begins in transition index-number $I_{i^*j^*}$, where $j^* = even$, and $j^* < f^*_{i^*}$, are obtained with the formulae:

$$|I_{i^*j^*} - I_{cd}|_b + nb < p,$$
$$|I_{i^*j^*} - I_{c^*d^*}|_b + nb < p; \quad \text{where}$$

$$\begin{cases} 0 \leq c \leq x - 1; \\ d = even; \\ 2 \leq d < f_c; \\ n = 0, 1, 2, \cdots . \end{cases} \quad \text{for } c^* < i^* \begin{cases} d^* = even; \\ 2 \leq d^* \leq f^*_{c^*}; \\ n = 0, 1, 2, \cdots . \end{cases} \quad \text{for } c^* = i^* \begin{cases} d^* = even; \\ 2 \leq d^* \leq j^*; \\ n = 0, 1, 2, \cdots . \end{cases}$$

In the grid-diagrams the intersection column-numbers calculated in accordance with the above formulae are read from left to right for the left to right half-cycles, and the intersection column-numbers for the right to left half-cycles are read from right to left.

Thus for our Example on pg. 172 we obtain for the successive half-cycles the following intersection column-numbers :

$$\begin{aligned}
h_{01} &= 0_L \longrightarrow 7_R: && \text{Free run.} \\
h_{02} &= 7_R \longrightarrow 4_L: && 10. \\
h_{03} &= 4_L \longrightarrow 1_R: && 10. \\
h_{11} &= 1_L \longrightarrow 8_R: && 3, 13. \\
h_{12} &= 8_R \longrightarrow 5_L: && 9, 3, 13, 10. \\
h_{13} &= 5_L \longrightarrow 2_R: && 9, 10. \\
h_{21} &= 2_L \longrightarrow 9_R: && 2, 12, 3, 13. \\
h_{22} &= 9_R \longrightarrow 6_L: && 8, 2, 12, 9, 3, 13, 10. \\
h_{23} &= 6_L \longrightarrow 3_R: && 8, 9, 10. \\
h_{31} &= 3_L \longrightarrow 0_R: && 1, 11, 2, 12, 3, 13. \\
h_{0^*1^*} &= 0_L \longrightarrow 3_R: && 10, 6, 9, 5, 8, 4, 7. \\
h_{1^*1^*} &= 1_L \longrightarrow 4_R: && 1, 11, 7, 10, 6, 9, 5, 8.
\end{aligned}$$

$h_{1^*2^*} = 4_R \longrightarrow 7_L$: $7, 6, 5, 1, 11, 10.$
$h_{1^*3^*} = 7_L \longrightarrow 0_R$: $7, 3, 13, 6, 2, 12, 5, 1, 11, 4, 10.$
$h_{2^*1^*} = 2_L \longrightarrow 5_R$: $2, 12, 8, 1, 11, 7, 10, 6, 9, 5.$
$h_{2^*2^*} = 5_R \longrightarrow 8_L$: $8, 7, 6, 2, 12, 1, 11, 5, 10.$
$h_{2^*3^*} = 8_L \longrightarrow 1_R$: $8, 4, 7, 3, 13, 6, 2, 12, 5, 1, 11, 10.$
$h_{3^*1^*} = 3_L \longrightarrow 6_R$: $3, 13, 9, 2, 12, 8, 1, 11, 7, 10, 6, 5.$
$h_{3^*2^*} = 6_R \longrightarrow 9_L$: $9, 8, 7, 3, 13, 2, 12, 6, 1, 11, 5, 10.$
$h_{3^*3^*} = 9_L \longrightarrow 2_R$: $9, 5, 8, 4, 7, 3, 13, 6, 2, 12, 1, 11, 10.$

After arranging the intersection column-numbers in ascending order, we obtain:

$h_{01} = 0_L \longrightarrow 7_R$: Free run.
$h_{02} = 7_R \longrightarrow 4_L$: $10.$
$h_{03} = 4_L \longrightarrow 1_R$: $10.$
$h_{11} = 1_L \longrightarrow 8_R$: $3, 13.$
$h_{12} = 8_R \longrightarrow 5_L$: $3, 9, 10, 13.$
$h_{13} = 5_L \longrightarrow 2_R$: $9, 10.$
$h_{21} = 2_L \longrightarrow 9_R$: $2, 3, 12, 13.$
$h_{22} = 9_R \longrightarrow 6_L$: $2, 3, 8, 9, 10, 12, 13.$
$h_{23} = 6_L \longrightarrow 3_R$: $8, 9, 10.$
$h_{31} = 3_L \longrightarrow 0_R$: $1, 2, 3, 11, 12, 13.$
$h_{0^*1^*} = 0_L \longrightarrow 3_R$: $4, 5, 6, 7, 8, 9, 10.$
$h_{1^*1^*} = 1_L \longrightarrow 4_R$: $1, 5, 6, 7, 8, 9, 10, 11.$
$h_{1^*2^*} = 4_R \longrightarrow 7_L$: $1, 5, 6, 7, 10, 11.$
$h_{1^*3^*} = 7_L \longrightarrow 0_R$: $1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13.$
$h_{2^*1^*} = 2_L \longrightarrow 5_R$: $1, 2, 5, 6, 7, 8, 9, 10, 11, 12.$
$h_{2^*2^*} = 5_R \longrightarrow 8_L$: $1, 2, 5, 6, 7, 8, 10, 11, 12.$
$h_{2^*3^*} = 8_L \longrightarrow 1_R$: $1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13.$
$h_{3^*1^*} = 3_L \longrightarrow 6_R$: $1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13.$
$h_{3^*2^*} = 6_R \longrightarrow 9_L$: $1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13.$
$h_{3^*3^*} = 9_L \longrightarrow 2_R$: $1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13.$

The intersection column-numbers can now for each half-cycle be converted into their corresponding coding in accordance with the coding of the grid-diagram.

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